Growth Accounting:

- Measurement versus explanation
- The pitfalls of measurement

1st approach: Growth Accounting (Solow 1957)

- **the first is a pure accounting approach, no causation is involved**

- assumptions:
- neoclassical production function $Y_t = B_t K_t^{\alpha} L_t^{(1-\alpha)}$ *(here Cobb Douglas)*
- perfect competition

- a relation is postulated between factor growth and output
- no explanation is offered at this stage of the causes of factor growth

Assume all firms have access to the same technology
Technology is described by *the* Cobb Douglas *production function*

$$Y = BK^{\alpha} L^{(1-\alpha)}$$

deviding both sides by L *we obtain output per capita:*

$$y = \frac{Y}{L} = \frac{BK^{\alpha}L^{1-\alpha}}{L} = \frac{BK^{\alpha}L^{1-\alpha}}{L^{\alpha}L^{1-\alpha}} = Bk^{\alpha}$$

Where: $k = \frac{K}{L}$ = capital per worker

- given the production function

$$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha}$$

- B_t tells us how productive the factors capital and labour are:

$$B_t = \text{ total factor productivity (TFP)}$$

$$\log Y_t = \log B_t + \alpha \log K_t + (1-\alpha) \log L_t$$

$$\frac{\partial(\log Y_t)}{\partial t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{B}_t}{B_t} + \alpha \frac{\dot{K}_t}{K_t} + (1-\alpha)\frac{\dot{L}_t}{L_t}$$

- applying the same transformation to per-capita output:

$$g = \frac{\dot{y}_t}{y_t} = \frac{\dot{B}_t}{B_t} + \alpha \frac{\dot{k}_t}{k_t}$$

- data concerning $g =$ growth tate of per capita GDP and per-capita physical capital are obtaind by national statistics

- direct data on α and B are missing, but are estimated through the following argument:

- the gross marginal product of capital is:

$$\frac{\partial Y}{\partial K} = B\alpha K^{\alpha-1}L^{1-\alpha} = B\alpha k^{\alpha-1}$$

- assume the economy is perfectly competitive: then the gross marginal product of capital = user cost of capital = $r + \delta$

$$\frac{(r+\delta)K}{Y} = \frac{[B\alpha K^{\alpha-1}L^{1-\alpha}]K}{Y} = \frac{B\alpha K^{\alpha}L^{1-\alpha}}{Y} = \alpha$$

- if the economy is competitive, α is the share of GDP which goes to capital; $(1-\alpha)$ is the share of GDP going to wage income
- $\alpha \approx 0.3$ in most economies
- We obtain a residual estimate of TFP growth:

$$\frac{\dot{B}_t}{B_t} = g - \alpha \frac{\dot{k}_t}{k_t}$$

TFP growth (left-hand side) is <u>the residual</u> of GDP per-capita growth g which is left unexplained by the growth of capital per person.

2° approach: adopt the causal explanation of neoclassical model (here Solow)

- Assume: competitive markets + equilibrium axiom → no demand limits to growth
- Assume: labor-augmenting tecgnological progress

a. The immediate causes of growth are those shaping factor dinamics:

- Accumulation of capital stock K(t): falls in the domain of economics
- Accumulation of labour force L(t): falls mainly in the domain of demography
- Accumulation of technology A(t): falls in the domain of science and engineering

b. In the long run, technologcal progress is the ultimate source of growth:

after the economy has reached its steady state path y_t^*, whatever growth of GDP per capita is observed, it is explained by technological progress

$$\frac{\dot{\boldsymbol{y}}^*}{\boldsymbol{y}^*} = \boldsymbol{g}$$

3. Two different measures of technological progress?

The first approach suggests: $\frac{\dot{B}}{B} = g - \alpha \frac{\dot{k}}{k}$

In the long run, $\frac{\dot{k}}{k} = \frac{\dot{k}^*}{k^*} = g$

long-run contribution of technological progress to growth is:

$$\frac{\dot{B}}{B} = g - \alpha \frac{\dot{k}^*}{k^*} = (1 - \alpha)\, g$$

The second approach suggests $\frac{\dot{A}}{A} = \frac{\dot{y}^*}{y^*} = g$

long-run contribution of technological progress to growth is g

Interpretation:

- If A_t is constant, capital accumulation causes fall of MPK

- growth of A_t at rate g avoids fall of MPK: it enables <u>persistent</u> growth of k^* at rate g, while MPK at k^* remains constant.

- persistent growth of k^* at rate g contributes to GDP per-capita growth with the component $\boldsymbol{\alpha g}$

$$\alpha \frac{\dot{k}^*}{k^*} = \boldsymbol{\alpha g}$$

is the indirect contribution of technology to GDP per-capita growth

Technologcal progress is the unique ultimate causal source of long-run growth

$$g = \frac{\dot{A}}{A}$$

• its direct contibution to GDP per-capita growth is **TFP growth**, that is, $\frac{\dot{B}}{B} = (1-\alpha)g$

• its indirect contibution to GDP per-capita growth is αg. This occurs by enabling persistent capital accumulation.

More formally

$Y = A^{(1-\alpha)} K^{\alpha} L^{(1-\alpha)} = B K^{\alpha} L^{(1-\alpha)}$ $\qquad B = A^{(1-\alpha)}$

$y = Bk^{\alpha}$ $\qquad g = \frac{\dot{y}}{y} = \frac{\dot{B}}{B} + \alpha \frac{\dot{k}}{k}$

TFP = B = $A^{(1-\alpha)}$ $\qquad \frac{\dot{B}}{B} = TFPgrowth = \frac{\dot{A}}{A}(1-\alpha)$

$$g = \frac{\dot{y}}{y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha \frac{\dot{k}}{k}$$

Early growth accounting measures of the 'Solow residual' $\hat{x}$:

- Solow's (1957) applied this framework to US data: a large part of the growth was due to technological progress.
- From early days, however, a number of pitfalls were recognized.
  - Moses Abramovitz (1956): dubbed the $\hat{x}$ term "the measure of our ignorance".
  - If we mismeasure g_L and g_K we will arrive at inflated estimates of $\hat{x}$.

Main problems with the growth accounting exercise

- deviations from perfect competition
- mis-measurement
- measurement of capital inputs:
  - in the theoretical model, capital corresponds to the final good used as input to produce more goods.
  - in practice, capital is machinery, need assumptions about how relative prices of machinery change over time.
  - typical assumption was to use capital expenditures but if machines become cheaper would severely underestimate g_K

Reasons for mis-measurement:

● **heterogeneity** of labor and capital inputs

● **changes in input quality** through time

● **changes in relative prices** through time, partly reflecting:

- unequal productivity growth across industries

In particular, higher productivity growth in the up-stream industries, translates into lower prices of the inputs used in the down-stream industries: this may cause wrong measurement (under-evaluation) of capital inputs used in down-stream industries

Other reasons for mis-measurement:
over-statement of capital formation through public investment

● because of corruption, and government inefficiency, capital formation may be greatly overstated by official statistics

● see discussion between Young (1995) and Hsieh (2002) on the relative weight of capital accumulation and TFP growth in Esat Asian miracle

Young (1995): East Asia miracle is more a story of '*factor growth*' than of *TFP growth*

Hsieh (2002): Young's growth accounting method overstates capital formation in at least one important case (Singapore) and underestimates TFP growth for Singapore, South Korea and Taiwan.

Dual approach to growth accounting

$$Y = BK^{\alpha} L^{(1-\alpha)}$$

with competition + constant returns to scale

total capital rental: $KR = K\,MPK = K\,\alpha BK^{\alpha-1} L^{(1-\alpha)} = \alpha Y$

total wage bill: $Lw = (1-\alpha)Y$

$$Y \quad = \alpha Y + (1-\alpha)Y \quad = RK + wL$$

R = r + ∂ = user cost of K

w = wage rate

$$Y = RK + wL = \alpha Y + (1-\alpha)Y$$

$$\dot{Y} = \dot{R}K + \dot{K}R + \dot{L}w + \dot{w}L$$ = time derivative of previous equation

$$\dot{Y} = \frac{\dot{R}}{R}RK + \frac{\dot{K}}{K}RK + \frac{\dot{w}}{w}wL + \frac{\dot{L}}{L}wL$$

$$\dot{Y} = \frac{\dot{R}}{R}\alpha Y + \frac{\dot{K}}{K}\alpha Y + \frac{\dot{w}}{w}(1-\alpha)Y + \frac{\dot{L}}{L}(1-\alpha)Y$$

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{R}}{R} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{w}}{w} + (1-\alpha)\frac{\dot{L}}{L}$$

$$TFPgrowth = \frac{\dot{Y}}{Y} - \alpha\frac{\dot{K}}{K} - (1-\alpha)\frac{\dot{L}}{L} = \alpha\frac{\dot{R}}{R} + (1-\alpha)\frac{\dot{w}}{w}$$

Remark: Young (1995) employs direct method and

Young (1995) data on factor accumulation account for:

- changes in labor participation ↑
- changes in education attainment ↑

his definition of L is close to a concept of 'human capital'

→ growth rate of L > growth rate of population

On this ground, we may have:

Growth rate of GDP per capita = $\frac{\dot{y}}{y} > \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$

Direct method

b. A. Young (1995) uses the following data on the postwar growth of the East Asian "Tigers":

	Annual Growth Rate of:				
	Period	Output	Capital	Labor	Labor Share
Hong Kong	1966–91	7.3	8.0	3.2	0.628
Singapore	1966–90	8.7	11.5	5.7	0.509
South Korea	1966–90	10.3	13.7	6.4	0.703
Taiwan	1966–90	9.4	12.3	4.9	0.743

capital share = 1 - labor share

Direct method: $$TFPgrowth = \frac{\dot{Y}}{Y} - \alpha \frac{\dot{K}}{K} - (1-\alpha)\frac{\dot{L}}{L}$$

Hong Kong	2.314
Singapore	0.152
South Korea	1.173
Taiwan	2.59

Dual method:

c. For the same countries and periods, Hsieh (2002) provides the following data on factor prices:

	Annual Growth Rate of:			
	Period	Interest Rate	Wages	Labor Share
Hong Kong	1966–91	−1.1	4.1	0.628
Singapore	1968–90	1.6	2.7	0.511
South Korea	1966–90	−4.0	4.4	0.703
Taiwan	1966–90	−0.4	5.3	0.739

Dual method: $$TFPgrowth = \alpha \frac{\dot{R}}{R} + (1-\alpha)\frac{\dot{w}}{w}$$

Comparison of Young (1995) and Hsieh (2002) FTP growth

	Dual	*Direct*
Hong Kong	2.166	2.314
Singapore	2.162	0.152
South Korea	1.905	1.173
Taiwan	3.812	2.59

In the case of Taiwan and Singapore, the dual method yields a remarkably higher evaluation of TFP growth. Dual-method measurement of TFP growth is much higher also for South Korea.

A Deeper contrast between the direct and the dual approach to growth accounting

Direct method: $$TFPgrowth = \frac{\dot{Y}}{Y} - \alpha\frac{\dot{K}}{K} - (1-\alpha)\frac{\dot{L}}{L}$$

Here α is the output elasticity of capital in the neoclassical production function

Because α is unknown, it is given a numerical measure through the restriction that in a competitive economy:

$$\alpha = \frac{MPK\ K}{Y} = \frac{(r+\delta)K}{Y}$$

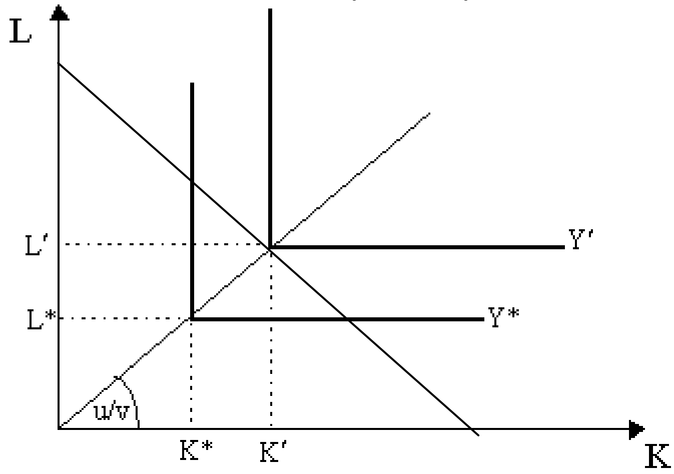
objections to the direct method (1):

the interpretation $(r + \delta) = MPK$ is unwarranted because it cannot be coherently supported by the neoclassical notion of an aggregate factor of production ‘capital’ defined in value terms (capital-theory critique).

objections to the direct method (2):

the interpretation $(r + \delta) = MPK$ is unwarranted because it is based on neoclassical production function that overstates substitutability between K and L at a point in time. Real-world production functions have the form:

Y = min(uK, vL)



To produce any output Y, the efficient input ratio is $\frac{L}{K} = \frac{u}{v}$ where $\frac{MPK}{MPL}$ is not well defined, and the marginal increment of one unit of K, or L, yields no increase in output.

Dual method:

does not rely on 'marginal product of capital'

$$Y = (r + \delta)K + wL \ = RK + wL = \alpha Y + (1-\alpha)Y$$

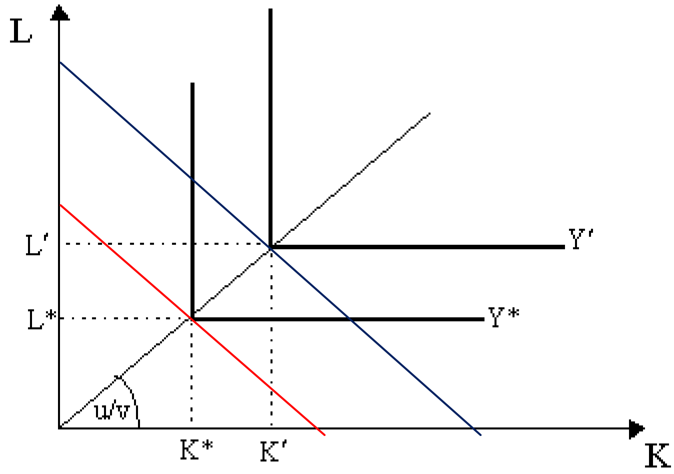
consistent with 'conflict view' of distribution:

if technology available is constant → inverse relation between R and w:

- wage + capital costs to produce output Y fully absorb output value
- to efficiently produce Y' > Y* proportionally larger inputs K and L are needed, and the change in total cost fully absorbs change in output, at constant w and R

example: $Y = \min(uK, vL)$

$Y^* = uK^* = vL^*$ $K^*R + wL^* =$ total cost $C(Y^*) = Y^*$



$Y' = \beta Y^*$ $\beta > 1$

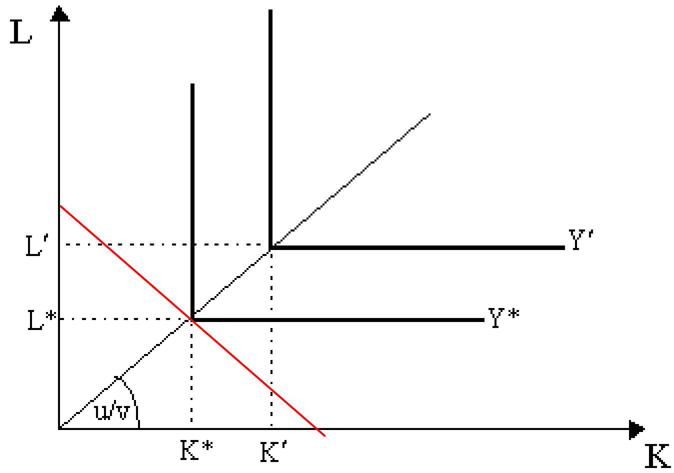
$Y' = uK' = vL'$ $K' = \beta K^*$ $L' = \beta L^*$

$K'R + wL' =$ total cost $C(Y') = Y'$

by proportionally expanding the intuts K and L the ratio K/L remains constant and the output Y grows in the same proportion.

Technological progress causes $Y = \beta \min(uK, vL)$ $\beta > 1$

Larger output Y' is now produced using the same inputs previously used in producing Y*: inward shift of the isoquants



With inputs K* and L*, output is now Y' > Y* = inward shift of isoquants

At constant R and w $Y^* = K^*R + wL^* =$ total cost $C(Y') < Y'$

Competition drives firms' net profit to zero and income Y is allocated to K and L

There is scope for increasing w, or R, or both!!

Dual method:

We can measure technological progress as

$$TFPgrowth = \alpha \frac{\dot{R}}{R} + (1-\alpha)\frac{\dot{w}}{w}$$

weighted average of the rate of increase of the wage rate, and the rate of increase of the user cost of capital.

R = riskless interest rate + risk compensation + depreciation