Endogenous technology growth

- The key to understanding *technology* is that R&D and technology adoption are purposeful activities.
- This lecture, focus on technological change and R&D.

Two forms of technology growth

1. Growth in the number (Variety) of inputs
2. Growth in the quality (productivity) of inputs

many varieties of machines

- final output Y_t produced with m varieties of *machines* and labour
- a machine is a couple $(v, A_v) \in R_+^2$

interpretation:

- v identifies machine type
- A_v is the quality (technology level = productivity index) of the machine v

Variety Growth

- The simplest models of endogenous technological change are those in which R&D expands the variety of inputs or machines used in production (Romer, 1990).

  - technology level A_v is constant: $A_v = 1$ all v
  - new 'machine varieties' **last for ever**... never displaced (simplicity!)

  LARGER MACHINE VARIETY implies a greater division of labour among workers assisting those machines

- **Models with expanding input varieties**: *product + process innovation*
  Research leads to the creation of new varieties of inputs (machines) and greater input variety increases the 'division of labour'

- **Models with expanding variety of consumption goods**: *product innovation* (Grossman Helpman 1991)
  Because consumers have a *love for variety,* wellbeing (real income) increases

Quality Growth

- When the quality of each input variety *v* improves, if improvement is large enough, the better quality displaces the worse from market.

  Quality ladder → *Schumpeterian creative destruction*
  economic life of machines is: *finite* and *uncertain*

Variety + Quality Growth

- 2nd generation Schumpeterian growth models

Common features to models of quality-ladder growth of technology

1. **A new idea is non rival and is embodied in a 'machine' of higher quality**
2. **R&D costs are paid as fixed costs upfront**

- If sectors producing machines of given quality were perfectly competitive →

  Machine price p = marginal cost of production

  → no compensation for the R&D cost

  → **No innovation under perfect competition**

- under constant returns to scale and perfect competition:

  $Y = wL + (r + \delta)K$. After remunerating L and K with their marginal products, nothing is left for remunerating resources invested in R&D.

Qualification: ‘competitive growth’ (M. Boldrin, D. Levine 2003)

a. **Non rivalry of ideas**: accepted. But ideas are always embodied in a physical support (human brain or material support), which is always rival.

b. **Replication of innovative ideas requires one item of the ‘prototype’** as input (replication and imitation increase the demand for ‘prototypes’)

c. **In the short run, the supply of prototypes is limited**. If the time rate of replication (production of copies) of new ideas is finite, and prototypes are excludable, they can be priced: → innovators earn a **short run** strictly positive profit, without any deviation from perfect competition.

d. **If indivisible R&D costs are not too high**, short-run profit may be sufficient to provide R&D incentives in a perfectly competitive environment.

e. The proposition: NO privately financed innovation without some monopoly power is NOT true ‘in principle’, but may be ‘empirically’ true, if indivisibility of R&D expenditure is ‘large enough’.

● Taking the above remarks into account, interpret the following as resting on a twofold assumption:

- Replication of ideas is fast enough
- Indivisibility of R&D investment is large enough

Schumpeterian quality ladder model: a synthesis (Aghion and Howitt, 2005, 2009)

But confront with Grossman Helpman (1991) and Acemoglu (2009, ch.14)

Introduction:

1 Competitive final output sector

1 Monopolistic machine (innovation good) sector

1 R&D sector

Labor supply = constant

$$Y_t = A_t^{1-\alpha} x_t^{\alpha} (L)^{1-\alpha}$$

machine x is a intermediate good produced in monopolistic 'sector'
only one (intermediate) sector in this example

A_t = *technology level of machine* x_t
L = *total employment = employment per (intermediate) sector*

- The x's depreciate fully after use.
- They can be interpreted as generic inputs, intermediate goods, machines, or capital.
- Thus machines are *not* additional state variables.

1 unit of x produced with 1 unit of K
$x_t \, r_{Kt}$ = *cost of producing* x_t
$r_{Kt} = r_t + \delta$ = *user cost of capital*
δ = exponential depreciation rate of K

1

1 unit of x produced with 1 unit of K
K is not used in other production activities
In equilibrium: $x = K$ total output of machines = total demand of capital =
= total supply of capital

$$Y_t = x_t^{\alpha} A_t^{1-\alpha} L^{1-\alpha}$$

Can be written as

$$Y_t = K_t^{\alpha} \left(A_t \cdot L \right)^{1-\alpha}$$

General intuition 1

Innovators' profit = Revenue from R&D activity – R&D cost

1 R&D revenue comes from the monopoly profit gained in selling innovation goods
2 Innovator's cost is the R&D cost needed to get a new idea
3 This will depend on:
- economy-wide characteristics such as efficiency of national innovation system
- nature of 'returns to R&D' that depend on the 'production function' in the R&D sector

General intuition 2

1 Monopoly profit results from the property right on the idea embodied in a ‘machine’. This machine is sold at a price larger than its unit production cost r_{Kt}

2 At given level of employment, and interest rate, monopoly power will result in a higher monopoly price p* of a machine and in a lower monopoly output x* of the machine

3 However, if employment L is higher, the monopoly output x* of the machine is also higher (a larger number of workers will ‘use’ the machine), while monopoly price p* and the profit per-unit of machine $p^* - r_K$ are unchanged, if r_K is unchanged.

4 Thus we may write monopoly profit as an increasing function of L and a decreasing function r_K .

5 In a full employment equilibrium, the cost of capital r_K is inversely related with the supply of capital (in efficiency units).

6 Other things equal, higher monopoly profit, higher innovation

Machine production:

7 Cost minimization by competitive firms in Y sector implies:

8 machine price p_t = value marg. product of machine x_t

$$p_t = \alpha\, x_t^{(\alpha-1)} A_t^{(1-\alpha)} L^{(1-\alpha)}$$

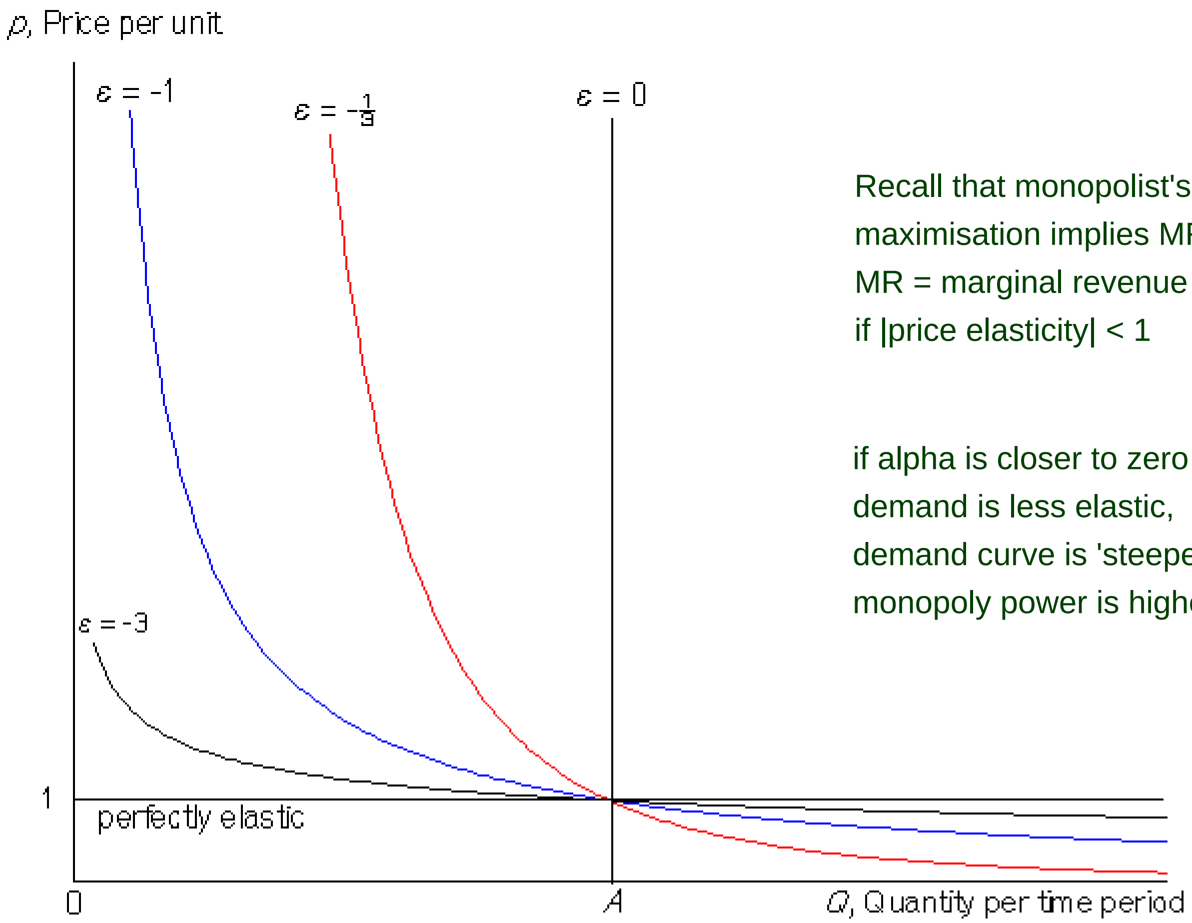
$$x_t = \alpha^{1/(1-\alpha)} A_t\, L\, p_t^{-1/(1-\alpha)}$$ machine demand is higher

if productivity A_t is higher
if employment L is higher
if price is lower

$-1/(1-\alpha)$ = *price elasticity of demand*

higher α → higher price elasticity of demand
→ **lower market power**

The figure below shows Ap^{ε} where $\varepsilon = -\frac{1}{1-\alpha}$



Recall that monopolist's profit maximisation implies MR = MC and MR = marginal revenue < 0 if |price elasticity| < 1

if alpha is closer to zero demand is less elastic, demand curve is 'steeper' monopoly power is higher

Machine production:

Monopoly price $p_t = \alpha\, x_t^{(\alpha-1)} A_t^{(1-\alpha)} L^{(1-\alpha)}$

Monopoly Revenue $p_t\, x_t = \alpha\, x_t^{(\alpha-1)} A_t^{(1-\alpha)} L^{(1-\alpha)} x_t$

Monopoly profit $\alpha\, x_t^{\alpha} A_t^{(1-\alpha)} L^{(1-\alpha)} - r_{Kt}\, x_t$

where $r_{Kt} = r_t + \delta$ = user cost of capital at time t

Max: monopoly profit with respect to x_t leads to

Marginal Revenue	=	Marginal cost
$\alpha^2 x_t^{\alpha-1} A_t^{(1-\alpha)} L^{(1-\alpha)}$	=	r_{Kt}

machine output is inversely related to the user-cost of capital

Marginal Revenue = Marginal cost

$$x_t = \alpha^{2/(1-\alpha)} A_t \, L \, r_{Kt}^{-1/(1-\alpha)}$$

full monopoly output

recalling $p_t = \alpha \, x_t^{(\alpha-1)} (A_t L)^{(1-\alpha)}$

$$p_t = \frac{1}{\alpha} r_{Kt}$$

full monopoly price

does not depend on productivity A_t

$$\frac{p_t}{r_{Kt}} = \frac{price}{unit.cost} = \frac{1}{\alpha}$$

monopoly mark up

Unconstrained monopoly profit

$$\pi_t = p_t\, x_t - r_{Kt}\, x_t = \left(\frac{1}{\alpha} - 1\right) r_{Kt}\, x_t$$

$$\pi_t = \left(\frac{1}{\alpha} - 1\right) \alpha^{2/(1-\alpha)} [A_t\, L]\, r_{Kt}{}^{-\alpha/(1-\alpha)}$$

$\pi^A{}_t = \pi_t / A_t = f(L, r_{Kt})$ *productivity adjusted profit*

$f'_L > 0$ **monopoly profit higher if labor supply higher (size effect)**

$f'_r < 0$ **monopoly profit lower with higher user cost of capital** continuum of machine inputs $[0, m]$ m = number of machines

machine $\boldsymbol{x_i}$ *produced in monopolistic 'sector' i*

$A_{i,t}$ = *technology level of machine* $x_{i,\,t}$

Output Y_{it} produced with machine x_{it} and employment L_i

$$Y_{i,t} = A_{i,t}^{1-\alpha} x_{i,t}^{\alpha} (L_i)^{1-\alpha}$$

employment is uniformly distributed across machines, $L_i = \frac{L}{M}$

$L\,/\,m$ = *average employment per (intermediate) sector*

$$Y_{i,t} = A_{i,t}^{1-\alpha} x_{i,t}^{\alpha} \left(\frac{L}{M} \right)^{1-\alpha}$$

Summing over machines:

$$Y_t = \left(\int_0^m A_{it}^{1-\alpha} x_{it}^{\alpha} \partial i \right) \left(\frac{L}{m} \right)^{1-\alpha}$$

To interpret the production function suppose for simplicity:

$$\boldsymbol{A_{it} = A_t} \textit{ all } i$$

then in equilibrium:

$$\boldsymbol{x_{it} = x_t} \textit{ all } i$$

and output Y_t is

$$Y_t = mA_t^{1-\alpha} x_t^{\alpha} \left(\frac{L}{m} \right)^{1-\alpha} = (mx)_t^{\alpha} A_t^{1-\alpha} L^{1-\alpha}$$

Total demand for K to produce machines is $\boldsymbol{mx = K}$

$$Y_t = K_t^{\alpha} A_t^{1-\alpha} L^{1-\alpha}$$

For the sake of simplicity we initially assume:
m = constant = 1
L = constant

As in the one machine case, the monopoly price fixed by the producer of machine i is constrained by

machine price $p_{i,t}$ *= value marg. product of machine* $x_{i,t}$

$$p_{i,t} = \alpha\, x_{i,t}^{-(1-\alpha)}\, A_{i,t}^{(1-\alpha)}\, L^{(1-\alpha)}$$

the preceding analysis applies here with the only difference that productivity $A_{i,t}$ may now differ across machines monopoly profit maximization by each machine producer

$p_{i,t} = p_t = \frac{1}{\alpha} r_{Kt}$ *uniform full monopoly price*

$\pi_{it} = \left(\frac{1}{\alpha} - 1\right) \alpha^{2/(1-\alpha)} (A_{it} L)\, r_{Kt}{}^{-\alpha/(1-\alpha)}$ *monopoly profit depends on* A_{it}

$\pi^A{}_t = \pi_{it} / A_{it} = f(L, r_{Kt})$ *productivity adjusted profit uniform all i*

$f'_L > 0$ **monopoly profit higher, if employment higher (size effect)**
$f'_r < 0$ **monopoly profit lower with higher user cost of capital**

perfect enforcement of monopoly rights:

→ *no competition* ***within*** *sectors*
→ *only competition* ***between*** *sectors*

price elasticity of demand: $-1/(1-\alpha)$*= elasticity of substitution between machines*

● *lower α → higher market power*

price/cost ratio = $r_{Kt}\,(1/\alpha)\;/\;r_{Kt}$ = **1/α = mark-up**

Equilibrium in market for K:

define: $A_t = \int A_{it}\, \partial i$ = **average productivity**

demand for K = $K_t^{\,d} = \int x_{it}\, \partial i = \int [A_{it}\, \alpha^{2/(1-\alpha)} L\, r_{Kt}^{\;-1/(1-\alpha)}]\, \partial i$

$$= A_t\, \alpha^{2/(1-\alpha)} L\, r_{Kt}^{\;-1/(1-\alpha)}$$

equilibrium:

demand for K is inversely related with r_K

supply is pre-determined at t $\quad K_t = K_t^{\,d} = A_t\, \alpha^{2/(1-\alpha)} L\, r_{Kt}^{\;-1/(1-\alpha)}$

efficiency units $\quad K^{AL}_t = K_t / LA_t = \alpha^{2/(1-\alpha)}\, r_{Kt}^{\;-1/(1-\alpha)}$

cost of capital r_{Kt} inversely related with K^{AL}_t

$r_{Kt} = \alpha^2\, K^{AL}_t{}^{-(1-\alpha)}$

Assume that innovator's market power is less than perfect

1 unit of x_{it} produced by:

- *innovator using 1 unit of K → marginal cost = r_{Kt}*
- *competitive fringe of imitators using $\chi > 1$ units of K*
- *difference $\chi - 1$ may be interpreted 'imitation cost –in terms of K input- per unit of machine output'*

$$1/\alpha > \chi > 1$$

→ *profit maximizing by competitive imitators:*
marginal cost = $\chi\, r_{Kt}$ = imitators competitive price

- *constrained monopoly price* $p_{it} \leq \chi\, r_{Kt}$ = *imitators price*

Constrained monopoly profit

- *max.* π_{it} *subject to* $p_{it} \leq \chi\, r_{Kt}$

  $\rightarrow$ $p_{it} = \chi\, r_{Kt}$

constrained monopoly profit **increases with** χ

- *If* χ *increases:*
- *constrained price increases, and gets closer to full-monopoly price;*
- *competition falls, monopoly output* x_{it} *falls, but profit* π_{it} *increases.*

Innovations

- **Abstract from free entry in R&D**: *in each period and each sector* 1 and only 1 outsider has an innovation opportunity
- The **time period is so short that at most 1 innovation arrives** within the period in each sector *i*.

- **If innovation arrives at time t**: $A_{it} = \gamma A_{i,\, t-1}$ $\gamma > 1$
  γ = size of proportional innovation step

- **If innovation does not arrive at time t**: $A_{it} = A_{i,\, t-1}$

- $A^*_{it} = \gamma A_{i,\, t-1}$ = **target productivity** of potential innovator in i

Conditional on R&D investment R_{it} at time t in sector i

$A_{it} = \gamma A_{i,t-1}$ *with probability* $\mu_{it} = \lambda \phi(R_{it} / A^*_{it})$
$A_{it} = A_{i,t-1}$ *with probability* $1 - \mu_{it}$

higher target productivity $A^*_{it} \rightarrow$ *higher R&D effort* R_{it} *needed to innovate with a given probability* μ_{it}

$R_{it} / A^*_{it} = n_{i,t}$ = *productivity adjusted R&D*

λ = *efficiency of innovation system*

μ *is a probability* $\rightarrow$ $0 \leq \lambda \phi(n_{i,t}) \leq 1$

$\phi_n' > 0$ $\phi_{nn}'' < 0$ $\lim_{n \to \infty} \phi_n' = 0$
$\phi_{nn}'' < 0$ *implies decreasing returns to R&D effort*

- $Q = \mu_{it}\,\pi_{it} - R_{i,t} = \lambda\phi(R_{it}/A^*_{it})\,\pi_{it} - R_{i,t}$ **expected innovation pay-off**

Problem: **Max. Q with respect to $R_{i,t}$**

- 1^{st} *order condition for* *interior maximum* $R_{i,t}^* > 0$

  marg benefit = $(\lambda\,\phi_n'(n_{it})\,/\,A^*_{it})\,\pi_{it} = 1$ = *marg. cost*

optimum (Ri* / Ai*) depends on profit/productivity: hence is uniform across i

$$\lambda\,\phi'(n_{it})\,\pi^A_t = 1$$

$$\phi'(n_t^*) = \frac{1}{\lambda\pi_t^A} \qquad (1)$$

Recalling that $\phi'(n)$ *is a decreasing function of n (decreasing returns to R&D),* $\phi'(n)$ *is maximum at n = 0 :*

- If $\phi'(0) \leq \frac{1}{\lambda\pi_t^A}$ there is no n* > meeting (1). This implies: $n_t^* = 0$

- If $\phi'(0) > \frac{1}{\lambda\pi_t^A}$, then because $\lim_{n\to\infty}\phi_n' = 0$, *there exists* $n_t^* > 0$ meeting (1), and: $n_t^* = n(\lambda, \pi^A)$

  $n_\lambda' > 0 \qquad n_\pi' > 0$

$$n_t{*} = n(\lambda, \pi^A) \qquad\qquad n_\lambda' > 0 \quad n_\pi' > 0$$

$$R_t{*} = A{*}_{it}\, n(\lambda, \pi^A) = \gamma\, A_{it-1}\, n(\lambda, \pi^A)$$

Optimal R&D at t is higher:

- if improvement step γ is larger

- if efficiency λ of innovation system is higher

- if productivity adjusted monopoly profit is higher

- if efficiency A_{it-1} is higher

Notice that in the particular case in which the Inada-like condition

$$\lim_{n \to 0} \phi_n' = +\infty$$

holds, we have $n{*}_t > 0$ no matter how small λ and π^A_t. Because the condition is not plausible, we expect that in countries where R&D efficiency is too low, or intellectual-property protection too weak, we have $n{*} = 0$.

Expected Productivity growth

$A_{it} = \gamma A_{i,t-1}$ *with probability* μ_t

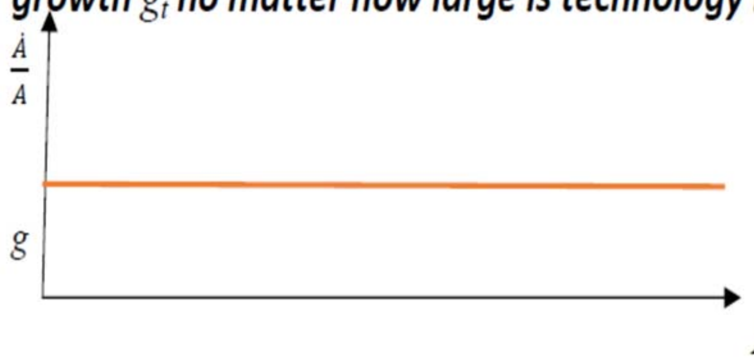
$A_{it} = A_{i,t-1}$ *with probability* $1 - \mu_t$

$E(A_{it}) = \mu_t \gamma A_{i,t-1} + (1 - \mu_t) A_{i,t-1}$

$E(A_{it} - A_{i,t-1}) = \mu_t (\gamma - 1) A_{i,t-1}$

$g_t = E(A_{it} - A_{i,t-1}) / A_{i,t-1} = \mu_t (\gamma - 1)$ uniform across sectors

constant productivity adjusted R&D effort $\frac{R_i}{A_i^*} = n^*$ ***causes a constant technology growth*** g_t ***no matter how large is technology level*** A_t



expected technology growth g as a function of technology level A, at constant R&D effort n*

g_t *increases with* K^{AL}_t *through R&D incentives*

higher capital per unit of efficiency K^{AL}_t

$\longrightarrow$ ***lower capital cost*** r_{Kt}

$\longrightarrow$ ***higher monopoly profit*** π^A_t

$\longrightarrow$ ***higher R&D effort*** n^*_t

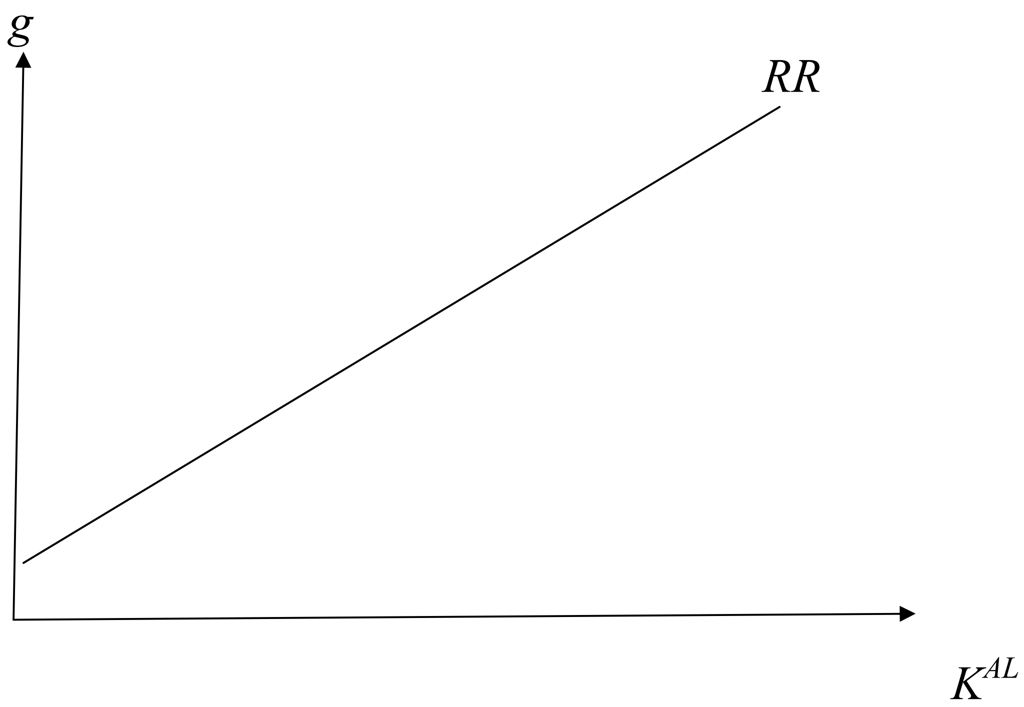
$\longrightarrow$ ***higher innovation probability*** μ_t

In steady state:

$g = g^A(K^{AL}) \quad g^{A\,\prime} > 0$

graph of $g = g^A(K^{AL})$ upward sloping curve RR in g, K^{AL} plane

RR shifts upwards if λ, χ, or L are higher



Capital accumulation equation:

$$K_{t+1} - K_t = s_t Y_t - \delta K_t$$

$$in\ equilibrium\ Y_t = K_t^{\alpha} L^{1-\alpha} A_t^{1-\alpha}$$

$$K_{t+1} - K_t = s_t K_t^{\alpha} L^{1-\alpha} A_t^{1-\alpha} - \delta K_t$$ *transitional relation*

$$g^K{}_t = (K_{t+1} - K_t) / K_t \quad = s_t (K_t)^{\alpha-1} A_t^{1-\alpha} L^{1-\alpha} - \delta$$
$$= s_t (K^{LA}{}_t)^{\alpha-1} - \delta \qquad (2)$$

*In **steady state**, the growth rate of K_t is:*

$$g^K = s\, K_{AL}{}^{\alpha-1} - \delta \qquad 0 < \alpha < 1 \qquad (3)$$

s = *steady state propensity to save is constant*

graph steady-state g^K: downward sloping curve KK in g, K_{AL} plane

Remark:

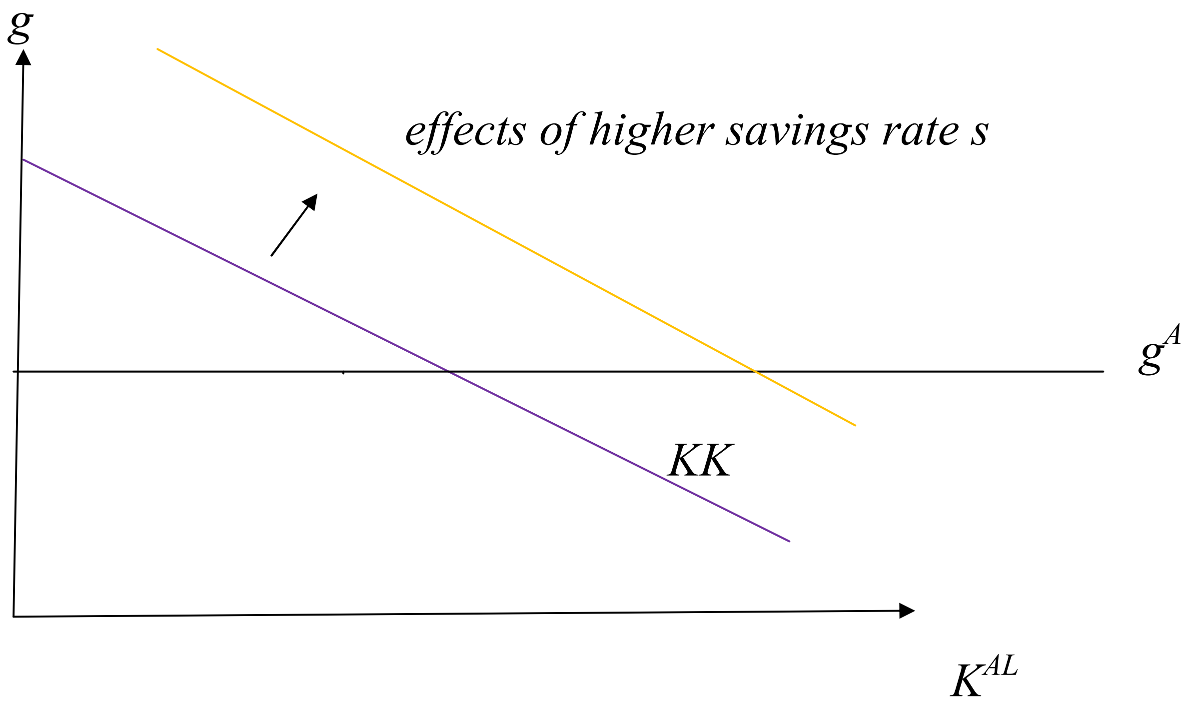
*If **s** = constant reflects a steady-state restriction, K_{AL} in equation (3) above must be interpreted steady-state efficiency-units of capital.*
*In the Solow model **s** is effectively constant to the effect that the steady-state relation (3) is identical to the transitional relation (2).*

Remark: steady state in the neoclassical model: n = 0 (constant L)

$g^K = s\, K_{AL}^{\alpha-1} - \delta$ *steady state capital in efficiency units*

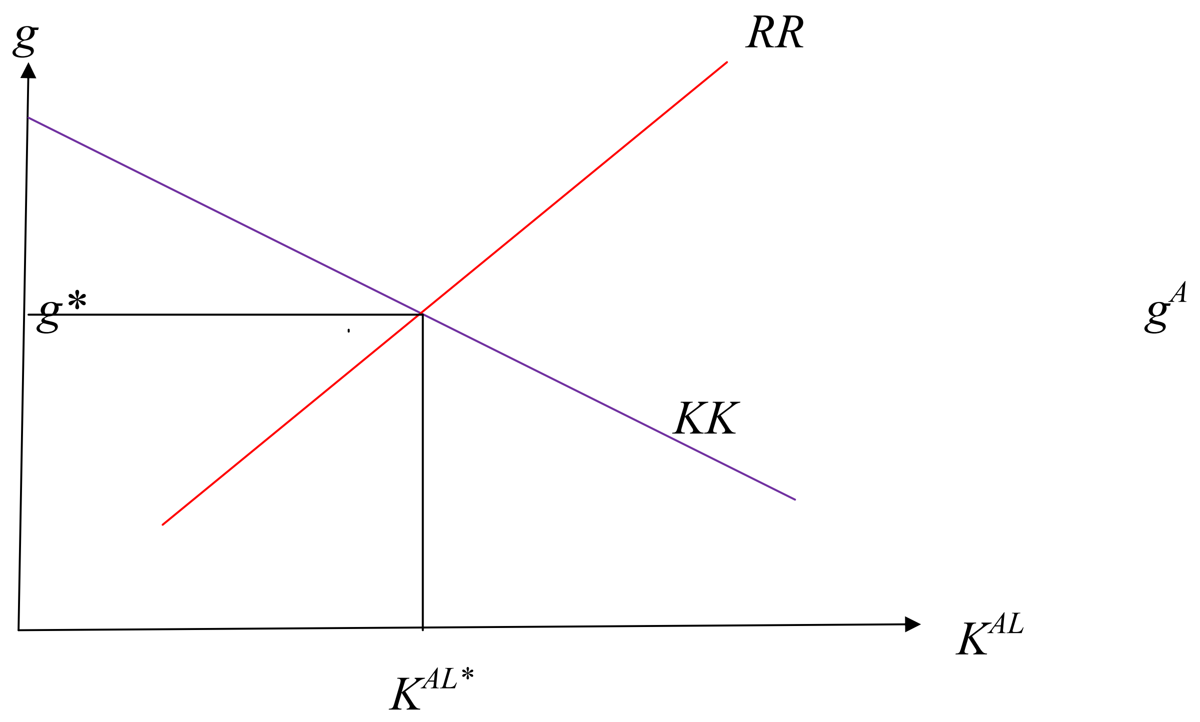
$g^A =$ *exogenous technological progress*

in steady state: $\boldsymbol{g^A = g^K}$ $g^A = s\, K_{AL}^{\alpha-1} - \delta$

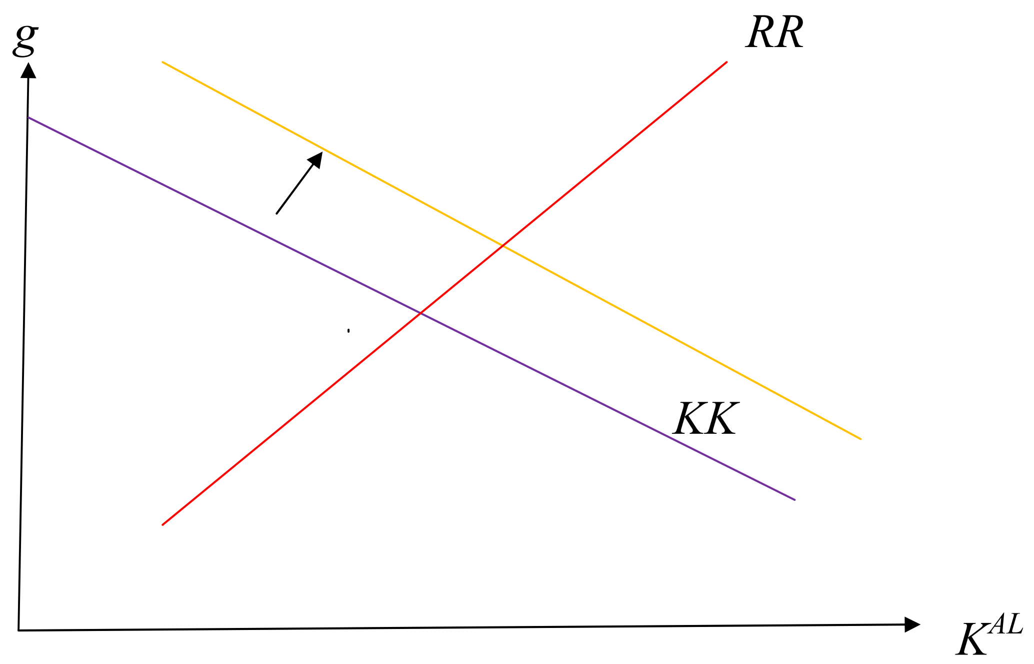


Steady state in the Schumpeterian model with K

K^{AL} and g are constant through time



Steady state in the Schumpeterian model with K: effects of a higher s

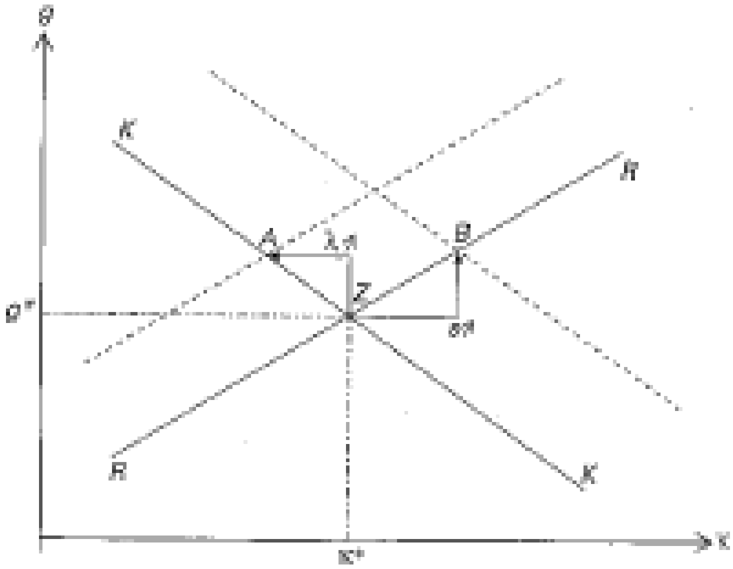


Steady state Schumpeterian growth with physical K:

$g^A = g^A\,(K^{AL})$ **curve RR** *shifts up if* **π ↑ and if λ↑**

$g^K = s\,K_{AL}{}^{\alpha-1} - \delta$ **curve KK** *shifts up if* s ↑

in steady state: $g^A = g^K$



Preliminary conclusions:

a. ***Lower competition, higher patent protection (higher χ) → higher g***
b. ***Higher productivity of R&D (higher λ) → higher g***
c. ***monopoly profit increases with L : Scale effect on g***
d. ***higher savings ratio, higher K^{AL}, lower cost of K, higher g***

Are such conclusions robust?

Scale effect on g not corroborated by evidence (C. Jones 1995 JPE)

Recall that the scale effect on g depends on the fact that the profit to be made on the output of one 'machine' depends linearly on the number of workers using that 'machine'.
This number is higher If L (total number of workers) is higher.

Scale effect on *g* can be removed through variety + quality growth

to te effect that in the long run L / m = constant

$$Y_t = \left(\int_0^m A_{it}^{1-\alpha} x_{it}^{\alpha} \partial i\right)\left(\frac{L}{m}\right)^{1-\alpha}$$

population grows exogenously at rate g_L

change in variety is (linear specification):

$$m_{t+1} - m_t = \psi L_t - \varepsilon m_t$$

more heads L, … more new ideas
variety change may depend on variety level

Variety growth:

$$g_{m,\,t} = (m_{t+1} - m_t) \,/\, m_t = \psi(L_t \,/\, m_t) - \varepsilon$$

$g_{m,\,t} < g_L \rightarrow \quad (L_t \,/\, m_t) \uparrow \qquad and \quad g_{m,\,t} \uparrow$

$g_{m,\,t} > g_L \rightarrow \quad (L_t \,/\, m_t) \downarrow \qquad and \quad g_{m,\,t} \downarrow$

Variety *m grows through time and* $g_{m,\,t}$ *converges to* g_L

In steady state, L/m = constant
monopoly profit depends on L/m
the scale effect is removed

conclusion that less competition is growth promoting …
relies on 2 crucial assumptions …

no tacit knowledge: best practice knowledge available to outsiders

innovations are radical: every new machine displaces the previous type of machine from the market

Implications:

1. In equilibrium, only 1 monopolist producer in each intermediate sector
2. R&D only by outsiders: the innovation probability for the incumbent monopolist is not greater, and if successful, she loses her previous monopoly rents (Arrow 1962 '*replacement effect*')

3 (potential) sources of Pareto inefficiency

- Monopoly output of innovation goods (price > marginal cost)
- Outsiders investing in R&D ignore that 1 innovation destroys current monopoly rents.
- Innovators are only interested in private benefits, and ignore that the social benefits of innovations may outlast the private ones.

A compensation schema

- The blue and green inefficiencies are avoided forcing:

- The current innovator pays the incumbent monopolist for her loss of rents
- the current innovator will be compensated in the future →
  → she correctly computes innovation benefits outlasting market demand.

- The productivity adjusted present value of the second payment is lower, because it takes place later in time

Interpretation

- inter-temporal equilibrium prices are present value prices:
- market value of innovator’s gain for missing payment to incumbent monopolist > market value loss from missing future recovery of rents (when innovation will be displaced)

- → market incentive to R&D is ‘too high’ ??

- *Possibly, but after consideration of inefficiency from monopoly …*

Market R&D > or < Pareto optimal R&D