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DYNAMICAL SYSTEMS IN MACROECONOMICS

Alternative Approaches to the Analysis of Macroeconomic Fluctuations

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1. Introduction

The main purpose of this work is to analyse the most recent approaches to the study of macroeconomic fluctuations. To this end, the equilibrium approach and the 'fixed' price approach are outlined, discussed, and, to the extent to which this is possible, compared with the classical ('non-linear') approach (e.g., the basic contributions to business cycle theory by Goodwin, Kaldor, Hicks, and Kalecki). To make any kind of comparison between these different approaches, however, does not seem to be an easy task because, although they are all labelled 'business cycle theories', in fact even the subject under study appears to be different. For example, whereas for the classical approach the main problem is to explain (endogenously) the turning points of the cycle, for the equilibrium approach this problem is not important at all because the turning points of the cycle may be very easily accounted for by random shocks.

In spite of this difficulty, however, a problem which is interesting to discuss with regard to the most recent approaches and which also has important implications for the classical approach is the problem of the role played by 'ad hoc' assumptions.

First, given Grandmont's recent contribution (1985), it is relevant to analyse this problem within the equilibrium framework. To understand better the role of this recent contribution, some aspects of the 'standard' equilibrium model are reviewed in section 2. Then, Grandmont's model is anaysed in section 3. Finally, in section 4, the same problem is considered with regard to models of the 'fixed' price type by Blad (1981) and Blad and Zeeman (1982).

2. Equilibrium models of the business cycle with imperfect information and random disturbances

There are two features of the 'standard' equilibrium model that are useful to call to mind.

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First, in the 'standard' formulation of the model [e.g., in Barro (1980, 1981)] both demand for and supply of output, and thus also the equilibrium values, depend on the unperceived part of money growth. In this regard, the basic assumption is that of imperfect current information: if m_t, the current rate of growth of money, were included in the information set, then the equilibrium values would be equal to the natural values and all monetary movements would be neutral [see Barro (1981, p. 49)]. This gives a first impression of the crucial role played in the 'standard' equilibrium model by the ('*ad hoc*') *specification of the information set*. This impression is, on the other hand, reinforced if we consider the problem of the persistence of fluctuations: although the 'standard' model cannot account for effects of monetary shocks on output that last more than one period, serially correlated output movements may be very easily accounted for by assuming a variable lag in the acquisition of information.

Second, in 'standard' formulations of the model [e.g., in Lucas (1973,1976)], supply in any market z is assumed to be equal to the sum of a permanent component and of a cyclical component where the latter may be very easily expressed [see Sargent (1979, pp. 203–213, 324–331)] as a weighted sum of current and past *random*, *exogenous shocks* in the absence of which output is equal to its permanent value.

This problem, together with all problems related to the ('ad hoc') specification of the information set, may well account for the recent developments in equilibrium business cycle theory the main purpose of which is to show how the existence of fluctuations in an equilibrium model does not rely on random monetary disturbances or special specifications of the information set.

3. Equilibrium models of the business cycle with perfect foresight and absence of random disturbances

The most important effort in this direction is that made by Grandmont (1985).

Grandmont's model is an overlapping-generations model where agents live two periods, are identical and have perfect foresight, and the economy is subject to no shocks of any sort. Given that agents belong to two different generations, they must solve two different maximization problems with the crucial difference that the maximization problem of the 'young' agent involves expected magnitudes. Indeed, the results of the latter problem [see Grandmont (1985, pp. 999–1003)], taking account of the fact that the 'young' agent can hold a non-negative money balance, are values of the excess demands for the good as functions of the ratio between the actual and the expected price $[z_\tau(p/p^e), \tau = 1,2]$ and of the demand for money as a function of the two prices $[m^d(p, p^e)]$.

Given the results of the maximization problem solved by the 'old' agent, Grandmont finds that the following market clearing conditions must hold:

$$z_1(p_t/p^e_{t+1}) - (c - L) = 0, \quad \text{for the goods market,} \tag{1}$$

$$m^d(p_t, p^e_{t+1}) = M, \quad \text{for the money market,} \tag{2}$$

where $(c - L)$ is the optimal excess demand for goods by the 'old' agent and M is the outstanding stock of money.

In order to derive the dynamics of the system from conditions (1) and (2), the crucial step is the specification of the mechanism by means of which the expected price for the next period is computed by the 'young' agent; i.e., the specification of an expectation function Ψ relating p^e_{t+1} to the present and past information the 'young' agent has on the structure of the economy. According to the specification chosen by Grandmont (1985, p. 1008), this information is composed only of past prices so that we can write

$$p^e_{t+1} = \Psi(p_t, \dots, p_{t-T}), \tag{3}$$

where the function Ψ, for the case of a periodic competitive equilibrium, is assumed to be consistent with periodicity k:

$$\Psi(p_t, \dots, p_{t-T}) = p_{t+1-k} = p_{t+1}.$$

In other words, although Grandmont's analysis is all developed in terms of the expectation function, the mechanism by means of which expectations are formed, and possibly revised, plays no role because the 'young' agent is assumed to possess perfect foresight. In this case, it is possible to obtain from (1) and (2) [see Grandmont (1985, p. 1010)]

$$z_1(\vartheta_t) + z_2(\vartheta_{t-1}) = 0, \qquad \vartheta_t = p_t/p_{t+1}, \tag{4}$$

where $z_2(\vartheta_{t-1})$, under the assumptions made by Grandmont (1985, pp. 999–1000), has an inverse[1] so that we can write

$$\vartheta_{t-1} = z_2(-z_1(\vartheta_t)) = \Phi(\vartheta_t). \tag{5}$$

As shown by Grandmont (1985, pp. 1005–1006), eq. (5) describes cycles of period k which are the outcome of a *backward dynamics* and from which a

[1]This is true [see Grandmont (1985, p. 1005)] for $\vartheta > \bar{\vartheta}$, where $\bar{\vartheta}$ is the inverse of the marginal rate of substitution at the endowment level.

sequence ($p_1^*, \ldots, p_k^*$), determining a periodic competitive equilibrium, can be easily obtained. The problem is, however, to understand what it means to study, in an overlapping generations model where the expectations of the 'young' agent about the next period is (and must be) the basic ingredient, the dynamics of the system by means of a backward equation.[2] The impression is that this is possible in Grandmont's model because of 'manipulations' of the expectation function.

Apart from the undeniable mathematical interest of Grandmont's paper, it seems possible to draw the following conclusion from the analysis we have sketched.

Grandmont's attempt to build an equilibrium model in which cycles are endogenous has the important object of showing that economic policy has a role. However, although this result is obtained, his criticism of the reliance of equilibrium models on exogenous shocks does not seem to me to be sufficiently thorough-going. Indeed, if the problem in equilibrium models is to discover general rules about comovements among different aggregative time series, the reason why these comovements start is not relevant at all.

The important reason why the equilibrium framework does not seem to be the proper one for the analysis of business cycle phenomena lies elsewhere; furthermore Grandmont's model helps us in drawing this conclusion: the assumptions of market clearing and rational expectations leave little room for explaining fluctuations in economic activity. The only possibility that still remains open is to make 'ad hoc' assumptions about the information set or – and this amounts to the same thing – about the expectation function.

4. Disequilibrium models: 'Fast' versus 'slow' dynamics

Given our purposes, we are interested in dynamic formulations of the model [e.g., Blad (1981), Blad and Zeeman (1982)]. For these formulations, the expression 'fixed' price model is not appropriate; I will continue to use it, however, in order to underline their common descent from Malinvaud's model (1977).

In Blad's model, where, as in Malinvaud's original model, there are no inventories, the state of a rationed economy is described by a point in the product space $X \times C$ where $X = X(x, y, z)$ is the quantity space – with the x, y and z coordinates measuring the excess demand for goods, the extent to which notional output exceeds actual output, and the excess supply of labour,

[2] A famous example of a business cycle model in which the dynamics of the system is described by a backward equation is the model by Kalecki (1935). However, in Kalecki's model, where the problem of expectations is not explicitly considered, there is an important reason that explains the existence of a backward dynamics: current investment decisions are assumed to depend on all past accumulation as described by all past 'history' of investment decisions.

respectively – and $C = C(p, w)$ is the price–wage plane which is considered as a parameter space.[3]

For the dynamics in quantities and in wages and prices, a formulation is chosen according to which the latter react 'slowly' to excess demand and supply whereas the former react 'fast'.[4] Given that excess demand and supply are different in the different regions, the slow dynamics in prices and wages must be specified separately for each of the three regions[5] and this gives rise to problems in the analysis of the crossing of the boundary between the R and the K regions: there, both the vector field and its direction are discontinuous and, as a consequence, the analysis developed by Blad in this paper does not apply. For all other cases, it does apply, however, and the problem is to define the dynamics in quantities so as to satisfy the requirement that quantities react 'fast' and smoothly to changes in the parameter.[6] The formulation chosen by Blad satisfies the requirement while making difficult any economic interpretation: first, new coordinates are chosen; second, a potential function is given; third, the dynamics of quantities is assumed to be equal to the gradients of the potential. Given this formulation, it is easy to show that the 'fast' dynamics has two points of equilibrium with an 'exchange of stability' which allows one to describe the evolution of the economy in terms of regime switchings.

The problem with this kind of approach may be summed up as follows. The solution based on the consideration of 'fast' vs. 'slow' dynamics does not apply to the case of the crossing of the boundary between the R and the K regions. Even in the case in which it does apply, however, the 'fast' dynamics in quantities is introduced in several 'ad hoc' ways and, in some cases, without the possibility of any economic interpretation. The solution to the remaining case is given by Blad and Zeeman (1982) in another paper along very different lines and, again, the argument is entirely developed in terms of specific examples.

This means that with this approach there is a lack of general results. The impression is that this is the unavoidable consequence of the kind of exercise which is undertaken and which consists in the introduction of (macro-)

[3]With this notation, an equilibrium in the K(eynesian) region is represented by a point $e_k = (p, w, 0, y, z) \in X \times C$; an equilibrium in the C(lassical) region by a point $e_c = (p, w, x, 0, z) \in X \times C$; and an equilibrium in the R(epressed inflation) region by a point $e_r = (p, w, x, y, 0) \in X \times C$.

[4]This means that Malinvaud's assumptions constitute the limiting case of Blad's assumptions.

[5]The result of a specification in terms of excess demand and supply is the following: (1) $(p, w) \in K$: $\dot{p} = -K_p y < 0$, $\dot{w} = -K_w z < 0$; (2) $(p, w) \in C$: $\dot{p} = c_p x > 0$, $\dot{w} = -C_w z < 0$; (3) $(p, w) \in R$: $\dot{p} = R_p x > 0$, $\dot{w} = R_w z > 0$, where all coefficients are positive [see Blad (1980, p. 129)].

[6]In the simplified case analysed by Blad (1981, pp. 130–139), a specific trajectory, which crosses the boundary between the C and the K regions, is considered with the parameter γ representing the (p, w) value along it.

dynamic considerations in a static (and micro-) framework, i.e., in a framework that is not macrodynamic from the very outset.

5. Concluding remarks

In the equilibrium approach, the assumptions of market clearing and rational expectations leave little room for the explanation of fluctuations in economic activity: the only possibility that still remains is to make very special (possibly 'ad hoc') assumptions about either the information set or the expectation function. A criticism of the same type applies with regard to the 'fixed' price approach which reduces itself to the analysis of only very special (possibly 'ad hoc') cases.

As far as the understanding of fluctuations is concerned, the impression one has is that these models do not really constitute any significant advance over 'old fashioned' business cycle theories and this seems to indicate that it is important to go back to the analysis of 'macroeconomic' foundations of economic fluctuations.

In doing this, the problem is not that of overcoming 'ad hoc' assumptions in the sense of assumptions lacking microeconomic foundations but that of overcoming standard assumptions of the classical approach such as, for example, the 'ad hoc' choice of values of the parameters – necessary, e.g., to obtain the representation of a limit cycle – and the assumptions according to which the economy is closed and without 'government'.

Given the present situation of almost all advanced capitalist economies, these are assumptions which are no longer acceptable.

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