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Abstract

This paper contributes to the recent macro-dynamics literature on demand-led growth, that borrows insights from the idea expressed long ago by J. Hicks (1950) that Harrodian instability may be tamed by a source of autonomous expenditure in the economy. Contrary to the other contributions in this literature, autonomous expenditure is not exogenous, but is driven by a flow of profit-seeking R&D and innovation expenditures, that raise labour productivity through time. If the state of distribution, hence the wage share, is exogenously fixed and constant, the model gives rise to a macro-dynamics in a two dimensional state space, that may converge to, or give rise to limit cycles around, an endogenous growth path. An exogenous rise of the profit share exerts negative effects on long-run growth and employment, showing that growth is wage led.

Keywords: wage-led growth; endogenous autonomous expenditure; labour-saving technological progress: limit cycles.

JEL classifications: E11; E12; O41

1 Introduction

Recent and less recent contributions to the macro-dynamics literature of demand-led growth (Freitas and Serrano, 2015; Allain, 2015; Lavoie, 2016;

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Serrano, 1995A,B) have revived the idea expressed long ago by Hicks (1950) that Harrodian instability may be tamed by a source of autonomous expenditure in the economy. Incidentally, this gave rise to a welcome convergence between different strands of thought in macrodynamics, of Sraffian and Kaleckian inspiration (Cesaratto, 2015; Trezzini and Palumbo, 2016; Serrano and Freitas, 2017; Lavoie, 2017). In these contributions, autonomous expenditure is mostly identified with an exogenously growing flow of either consumption or non-capacity creating government expenditure.

In this paper, we draw a sharp distinction between the terms autonomous and exogenous. What defines the autonomous character of expenditure is that it is not determined by (but may have a causal influence on) short-run output. In what follows, autonomous expenditure occurs in a market economy without government intervention and is supplied by two sources: (i) a flow of endogenous modernization expenditures carried out by firms producing final output, with the aim of introducing best practice knowledge into production; (ii) a flow of autonomous consumption expenditure E_t, that is endogenously growing through time with labour productivity. Firms, wishing to stay in the market, are forced by competition to carry out modernization expenditures, that are increasing with the rate of technological progress. In the aggregate, these expenditures are also increasing with the size of the capital stock. In this way, technological progress is introduced in an aggregate model with fixed capital, thus avoiding the complications of vintage models or of joint production. It may also be worth observing that, since technology in the final output sector is Leontiev, modernization expenditures are not capacity creating, in that the full capacity output at time t is proportional to the capital stock K_t, hence it is independent of labour productivity. To facilitate comparison with contributions (Freitas and Serrano, 2015; Allain, 2015; Lavoie, 2016) in which autonomous demand is exogenous, we provide, first, a preliminary version of the model in which modernization expenditures grow through time as a result of exogenous innovation.

In the more complex, endogenous-growth version of the model, modernization 'software' is supplied by a monopolist, holding a property right on the best practice technology, that results from his profit seeking R&D expenditure. The existence and stability of the growth path requires in this case that the flow of autonomous consumption expenditure E_t is not too small, compared to productivity.

In the present framework, the link between innovation and firms' expenditure is married with a second link between innovation and labour demand. The overall effect on aggregate demand dynamics will crucially depend on the way in which the productivity gains are distributed between wages and profits. At the present stage of our work, the state of distribution, hence the wage share, is exogenously fixed. The model gives rise to a macro-dynamics in a two dimensional state space, that may converge to, or give rise to limit cycles around, an endogenous growth path. Long-run growth is wage led, in that the growth rate is a decreasing function of the profit share. At the same time, persistent growth of aggregate demand comes from rising labour productivity, hence from labour-saving technological progress. In such conditions, a failure of institutions in preserving a constant wage share would most likely produce self-reinforcing effects, because it exerts a downward pressure on the absolute level of employment. Thus the model provides insights into the inter-relations between labour-saving technological progress, distribution and growth. These relations, together with the changing nature of policy action (that lies outside the scope of the present analysis) contribute to explaining the post-1970s phase of slow growth in Europe and other OECD countries.

The organization of the paper is as follows. Section 2 provides an outline of the main arguments and relates them to the literature on demand-led growth. Section 3 presents the exogenous growth framework. The endogenous growth model is spelled out and discussed in section 4. Section 5 concludes.

2 Relation with the literature

Since the publications of Serrano (1995A, 1995bB), the growth literature of Kaleckian and classical-Marxian inspiration has shown a revived interest in the role of aggregate-expenditure components that are autonomous, in that (i) they are not explained by short-run output, but (ii) have a causal influence on it. Exports and government expenditure are two obvious examples, but residential construction, the Duesenberry (1949) ratchet effect and other forms of consumption are also in the list. The hypothesis received recent empirical corroboration in Girardi and Pariboni (2015) (see also, for further discussion and evidence, Lavoie, 2016, section 5).

This paper builds on the premise that there are flows of expenditure that may be broadly related to innovation and that meet the two conditions (i) and (ii) above. This was also the view often expressed by the late Richard Goodwin, in the footsteps of his master J. Schumpeter. First, R&D is more persistent, compared to other components of firms' expenditure, because firing and re-hiring highly specialized R&D personnel implies a substantial loss of firm-specific human capital (Falk, 2006) that cannot be easily transferred to other activities (Harhoff, 1998). Also, innovation causes the anticipated scrapping and substitution of machinery, modernization and re-organization expenditures and the building of new plants to satisfy newly created needs. Autonomous demand related to innovation is rarely, if ever, mentioned in the discussion on the role of autonomous expenditure in the explanation of demand-led growth. The main objective of this paper is to consider this hypothesis and to study its implications.

We are also partly motivated by the diffusion of automation and other labour saving techniques in recent decades. On these grounds, we shall assume that technological progress is labour augmenting. Notice that, to the extent that innovation is the only source of long-run growth in the model, this will also guarantee that the long-term growth path is coherent with the labour supply constraint in the economy.

The role assigned to innovation should not be misleading. As will turn out, short-run output is caused by demand (non vice-versa) and the bulk of investment demand is induced by demand expectations. Thus the model is demand-led and to emphasize this point, we shall first consider the simplified case in which R&D expenditure grows exogenously, much as autonomous expenditure is the exogenous driver of growth in Freitas and Serrano (2015) and Lavoie (2016). In this respect, the similarity of our exogenous-growth framework and theirs (especially Lavoie, 2016) is intentional and is meant to underline the qualitative correspondence of many results. In particular, the stability of the positive steady state is local and is conditional upon a sufficiently slow adaptation of long-term expectations, according to a simple Harrodian rule. On the steady-state path, capacity utilization is at its normal (desired) rate and the growth rate is obviously unaffected by distribution. This is parametrized by the value of the profit share, which is exogenous. Drawing a comparative dynamics across steady states, the profit share has only level effects: a lower profit share is associated with higher levels of employment and higher values of the (productivity adjusted) capital stock and output.

In the more general version of the model, R&D expenditure is explained by profit-seeking behaviour, to the effect that, in the long-run equilibrium, the different components of autonomous expenditure are endogenously growing through time. The local stability of the positive steady state requires, in this case too, a slow adaptation of long-term expectations. The persistent level effects of a change in distribution are likewise consistent with those of the exogenous-growth framework. But there are also persistent growth effects. A lower profit share is now causing a higher rate of growth.

Since steady-state capacity utilization is at its normal level, these persistent growth effects of distribution do not act through long run changes in capacity utilization. This property differentiates the present framework from the class of models, closely associated with the seminal contributions by Marglin and Bhaduri (1990) and Bhaduri and Marglin (1990), where the opposite holds true. Moreover, there is no labour hoarding in the model and no direct feedback of output on labour productivity, as is characteristic of the Keynesian growth models adopting some version of Verdoorn's law (see Rezai, 2012 and the references quoted therein).

A crucial implication of the present framework is that output growth is divorced from the growth of employment. Employment levels are preserved, in the long run, only if the real wage grows at least in line with productivity. A failure of institutions in preventing a fall of the wage share would likely exert self-reinforcing effects on employment and the wage share itself.

3 Exogenous technological progress

In this paper, the main source of autonomous demand is expenditure related, directly or indirectly, to technological progress. To clarify exposition, and stress the analogies with similar results in the literature, we shall consider exogenous technological progress first.

Let us consider a standard aggregate model with gross output Y_t that is either used for consumption C_t, gross investment I_t, capital modernization expenditure Z_t, or R&D expenditure R_t. Net investment is defined by:

$$\dot{K}_t = I_t - \delta K_t \tag{1}$$

The aggregate production function is

$$Y_t = \min(\frac{1}{v}K_t, A_t L_t) \tag{2}$$

where L is labour employment and A is labour productivity. Throughout this paper we shall consider trajectories such that output Y_t is constrained by demand, not by capacity $(1/v)\,K_t$, and the adaptation of output to demand occurs though changes in employment. The actual rate of capacity utilization is $u_t = Y_t/Y_{K,t}$, where $Y_{K,t}$ is full capacity output $(1/v)\,K_t$. The need of promptly meeting unexpected peaks in demand, that may result from accidental shocks or endogenous fluctuations, requires that the desired rate of capacity utilization u_n is less than one. Empirical work suggests that firms may regard as 'normal' a rate of utilization u_n that may be as low as 75%, or 80%.[1]

With output never constrained by capacity, we can write $Y_t = A_t L_t$, hence $L_t = a_t Y_t$, where $a_t = 1/A_t$ is labour input per unit of output.

[1]See Trezzini (2017, f. 33) and the surveys cited therein.

Best practice labour productivity grows as a result of R&D expenditure performed by firms and within bounds that are fixed by historically contingent technological opportunities g_T:

$$\frac{\dot{A}_t}{A_t} = g_T \Psi(r_{A,t}) \tag{3}$$

where $r_{A,t} = R_t/A_t$ is productivity-adjusted R&D and the function $\Psi(r_A)$ has the properties $\Psi' > 0$, $\lim_{r_A \to 0} \Psi(r_{A,t}) = 0$ and $\lim_{r_A \to \infty} \Psi(r_{A,t}) = 1$. Here, $g_T > 0$ is the maximum productivity growth offered by historical technological opportunities and $\Psi(r_{A,t})$ is the fraction of these opportunities that is captured by R&D effort $r_{A,t}$. According to this hypothesis, greater knowledge A_t makes R&D activity more complex and demanding. As a prototype formulation, we take:

$$\Psi(r_{A,t}) = \left(1 - \frac{1}{1 + r_{A,t}}\right) \tag{4}$$

In this section we assume an exogenously fixed and constant $r_{A,t} = r_A > 0$. This amounts to assuming a dynamics of R&D expenditure such that

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{A}_t}{A_t} \tag{5}$$

with initial condition $R_0 = r_A A_0$, where A_0 is pre-determined by history.

For the sake of later reference, we define $r_t = R_t/K_t$ and we observe that

$$r_t = r_A k_t^{-1} \tag{6}$$

where $k_t = K_t/A_t$.

To introduce best practice knowledge into production at time $t+dt$, firms carry out modernization expenditures Z_t that are proportional to the rate of technological progress and to the size of their capital stock:

$$Z_t = p_z \beta \left(\frac{\dot{A}_t}{A_t}\right) K_t \tag{7}$$

where p_z is the price of one update.

The situation we have in mind is that of a technology improvement step, or update, consisting of an innovation routine produced by R&D. For the sake of simplicity, we assume that the routine is embodied in an intermediate good produced with one unit of output.[2] As in the case of the computer, a unit of

[2] A nearly equivalent assumption is that updating is carried out by skilled workers, that assist firms in the installation and running of the routine. This assumption does not change the quality of our results, provided that the ratio between the wage rates earned by skilled and unskilled workers is fixed.

the capital stock is indivisible with respect to the possibility of being updated by new routines. The total cost of updating increases with the price p_z, with the number K_t/A_t of efficiency units of capital that require updating and with the number $\dot{A}_t$ of updates. It is worth observing that modernization expenditures are not capacity creating, in that the full capacity output from capital stock K_t, is K_t/v, no matter how high labour productivity A_t may be. This is the simplest way in which non-embodied technological progress is introduced into an aggregate model with fixed capital, thus avoiding the complications of vintage models, or of joint production. For the sake of later reference we define

$$z_t = \frac{Z_t}{K_t} = p_z \beta g_T \Psi(r_A) \qquad \beta > 0 \tag{8}$$

Taking into account the alternative uses of gross output Y_t, market clearing in the good market requires:

$$Y_t = Z_t + R_t + C_t + I_t \tag{9}$$

Consumption comes entirely from the expenditure of the wage bill and we assume for simplicity that workers do not save, while consumption out of profit is zero:

$$C_t = w_t L_t = w_t a_t Y_t \tag{10}$$

where w is the real wage, and the money price of output is normalized to 1. As is customary in Keynesian models, any deviation of demand from current output is corrected through a short-run adaptation of output.

Gross investment demand I_t reflects (i) the need of performing maintenance expenditures δK_t, (ii) the state of long term expectations concerning the average future growth of demand γ_t, (iii) the short-term forecast regarding capacity utilization at time t, namely $u_t^e = vY_t^e/K_t$, together with the will to reduce the gap between actual and desired capacity utilization:

$$I_t = \left[\gamma_t + \gamma_u \left(\frac{vY_t^e}{K_t} - u_n\right) + \delta\right] K_t$$

Following in the footsteps of Keynes' 1937 lecture notes (Keynes, 1973, p. 181), we shall however adopt the standard convention of assuming that short-term expectations are fulfilled, to the effect that $Y_t^e = Y_t$. This leads to:

$$I_t = [\gamma_t + \gamma_u(u_t - u_n) + \delta]K_t \tag{11}$$

so that

$$g_{K,t} = \frac{I_t - \delta K_t}{K_t} = \gamma_t + \gamma_u(u_t - u_n) \tag{12}$$

Substituting for C_t in equation (9) from (10), and dividing throughout by K_t, we obtain the short-term-equilibrium rate of capacity utilization:

$$u_t = \frac{v(z_t + r_t + \delta + \gamma_t - \gamma_u u_n)}{\pi_t - v\gamma_u} \tag{13}$$

where $z_t = Z_t/K_t$, and $\pi_t = 1 - w_t a_t$ is the gross profit share in output. Throughout this paper, we assume the short-run stability condition $\pi - v\gamma_u > 0$, and $\delta > \gamma_u u_n$, with the implication that $u_t > 0$, if $r_t + z_t + \gamma_t > 0$.

We are concerned with the study of growth paths supported by an exogenously given state of distribution, that we identify with a given and constant profit share $\pi_t = \pi$. This amounts to introducing the working hypothesis that the real wage is growing at rate $\hat{w}_t = \hat{A}_t$. Any consideration about the plausibility of this working hypothesis, and the implications that may follow from different scenarios of real wage dynamics, are postponed to the final discussion in the concluding section.

Using (6), (8), and (13), we write

$$\gamma_u(u_t - u_n) = \Gamma(\gamma_t, k_t) = x\left(p_z\beta g_T\Psi(r_A) + \frac{r_A}{k_t} + \delta + \gamma_t - \frac{\pi u_n}{v}\right), \tag{14}$$

where

$$x = \frac{v\gamma_u}{\pi - v\gamma_u} > 0 \tag{15}$$

The short-term growth rate $g_{K,t}$ is then:

$$g_{K,t} = \gamma_t + \Gamma(\gamma_t, k_t) \tag{16}$$

Equations (13) and (16) define the short-run equilibrium of our economy, supported by the given state of long-term expectations γ_t and by the predetermined k_t. The full dynamic path of the economy is therefore defined by the growth paths of the state variables γ_t and k_t. If to obtain the growth rate of the latter is straightforward, the growth rate of the former depends on speculations about expectation formation. Harrod's firm belief that the dynamics of long term expectations is influenced by observations of the growth path of the economy may be expressed as (Lavoie, 2016; Allain, 2015):

$$\dot{\gamma}_t = \mu\left(g_{K,t} - \gamma_t\right)\gamma_t = \mu\Gamma(\gamma_t, k_t)\gamma_t \tag{17}$$

$$\dot{k}_t = \left(\gamma_t + \Gamma(\gamma_t, k_t) - g_T\Psi(r_A)\right)k_t \tag{18}$$

On the assumption that $\pi u_n/v - \delta - g_T\Psi(r_A)(1 + p_z\beta) > 0$, the dynamic system (17)-(18) admits two dynamic equilibria. One is the trivial stationary state $(\gamma_0^*, k_0^*) = (0,0)$, that results to be unstable,[3] and the other is the constant growth path (γ^*, k^*), such that

$$\begin{aligned} \gamma^* &= g_T\Psi(r_A) = g_K^* && (19) \\ k^* &= \frac{r_A}{\pi\frac{u_n}{v} - \delta - g_T\Psi(r_A)(1+p_z\beta)} && (20) \end{aligned}$$

The dynamic equilibrium (γ^*, k^*) is locally asymptotically stable, if the adjustment parameter μ is small enough. To see this, we write the Jacobian matrix of system (17)-(18), evaluated at $(\gamma^*,\ k^*)$

$$J(\gamma^*, k^*) = \begin{bmatrix} \gamma^*\mu x & -\gamma^*\mu x r_A(k^*)^{-2} \\ k^*(1+x) & -x r_A(k^*)^{-1} \end{bmatrix}$$

with the properties:

$$\begin{aligned} \det J(\gamma^*, k^*) &= \gamma^*(k^*)^{-1} r_A \mu x \\ \operatorname{tr} J(\gamma^*, k^*) &= x(\mu\gamma^* - r_A(k^*)^{-1}) \end{aligned}$$

The local asymptotic stability of the dynamic equilibrium (γ^*, k^*) relies on the condition $\det J(\gamma^*, k^*) > 0$ and $\operatorname{tr} J(\gamma^*, k^*) < 0$. Such condition is fulfilled, provided that the adjustment parameter μ is sufficiently close to zero. Stability is strictly local and, as shown in Fig. 1, for initial conditions outside the basin of attraction of $(\gamma^*,\ k^*)$, trajectories diverge to infinity.

In the parameter range in which local stability obtains, it is meaningful to consider the persistent effects of a change in distribution. Since long-term growth is exogenous, the profit share does not have steady-growth effects, but only level effects. A lower profit share causes higher productivity adjusted output y^* and capital stock k^*, hence higher steady-state employment.

It may be worth stressing that the qualitative dynamic properties of system (17)-(18) are in many respects similar to those of other demand-led growth models in which the engine of growth is provided by autonomous expenditure (Allain, 2015; Freitas and Serrano, 2015; Lavoie, 2016). The only, but somewhat crucial difference, is that in the present framework labour productivity is growing and, provided that the real wage is growing in line with productivity, labour employment would be constant on the steady-growth

[3] The Jacobian matrix of system (17)-(18) evaluated at $\Gamma(0,0) = (0,0)$ is:

$$\begin{bmatrix} \mu\Gamma(0,0) & 0 \\ 0 & \Gamma(0,0) \end{bmatrix}$$

The dynamic instability of the trivial stationary state follows from the fact that $\Gamma(0,0) > 0$.

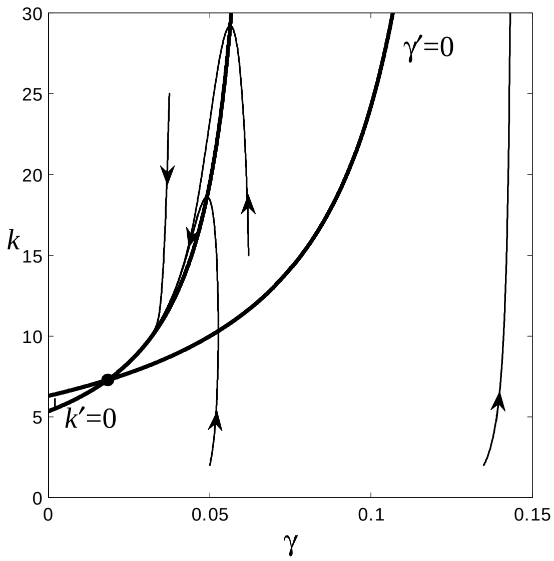

Figure 1: Trajectories in phase space for parameter settings $p_z = 1$, $\mu = 0.05$, $g_T = 0.04$, $\gamma_u = 0.10$, $\beta = 0.75$, $\pi = 0.3$, $\delta = 0.03$, $u_n = 0.775$, $v = 1.3$, $r_A = 0.55$ such that $\gamma^* \approx 0.0184$ and $k^* \approx 7.2846$. The trajectory on the right is diverging

path, while average employment would be mildly rising or falling on the transition path, depending on whether u_t happened to be lower or higher than u_n, at the initial date $t = 0$.

The scenario of rising labour productivity fits well with the assumption that output is never constrained by labour supply, but topics for debate are the plausibility of a rising real wage in the face of a steady level of employment, and the motivation behind the assumed R&D expenditure by firms. The second issue, together with the relation between the profit share and the rate of growth, is addressed in the next section.

4 Endogenous technological progress

In this section it is assumed that R&D activity is carried out by an independent firm, to the end of selling updating tool-kits to firms producing consumption and investment goods. A tool-kit is an intermediate good[4] produced with one unit of output and the routine embodied in it. The updating tool-kit has unit price $p_z > 1$ that comes from the intellectual property

[4] See however the footnote 2 above.

rights on the routine.[5] We shall abstract from free entry in R&D, for the sake of simplicity. With firms' updating expenditure Z_t specified as in (7) above, the profit from selling the updating tool-kits, net of the production and R&D cost, is

$$\Pi_{R,t} = (p_z - 1)\beta K_t g_T \left(1 - \frac{1}{1 + r_A}\right) - R_t \tag{21}$$

For any given $k_t = K_t/A_t$ fixed by past history, the maximization of profit $\Pi_{R,t}$, with respect to R_t, yields the productivity adjusted R&D expenditure as a function of k_t

$$r_A(k_t) = \begin{cases} 0 & \text{if } k_t \leq k_{\min} \\ [g_T(p_z - 1)\beta k]^{1/2} - 1 & \text{if } k_t > k_{\min} \end{cases} \tag{22}$$

where $k_{\min} = [g_T(p_z - 1)\beta]^{-1} > 0$. In the range $k > k_{\min}$, $r_A(k)$ is an increasing function of k; more precisely,

$$r'_A(k_t) = \begin{cases} 0 & \text{if } 0 < k_t \leq k_{\min} \\ \frac{1}{2}[g_T(p_z - 1)\beta]^{1/2} k_t^{-1/2} & \text{if } k_t > k_{\min} \end{cases} \tag{23}$$

Endogenous productivity growth is

$$\frac{\dot{A}_t}{A_t} = g_T \left(1 - \frac{1}{1 + r_A(k_t)}\right) \tag{24}$$

The ratios R_t/K_t and Z_t/K_t are:

$$r_t = r_A(k_t) k_t^{-1} \tag{25}$$

$$z_t = p_z \beta g_T \left(1 - \frac{1}{1 + r_A(k_t)}\right) \tag{26}$$

In this section we introduce a flow of autonomous consumption expenditure E_t that is influenced by the productivity level in the economy, according to $E_t = eA_t$. The term $e = E_t/A_t$ is labelled 'productivity adjusted autonomous consumption' and we assume $e > 1$. As before, market clearing in the good market requires

$$Y_t = Z_t + R_t + C_t + I_t + E_t \tag{27}$$

[5]The assumption that the price p_z is fixed and greater than one is justified by the hypothesis that monopoly price is constrained by the potential entry of imitators, who can produce the tool-kit at a constant unit cost $p_z > 1$. See Aghion and Howitt (2009).

whereas the short-term-equilibrium rate of capacity utilization is now:

$$u_t = \frac{v(z_t + r_t + \delta + \gamma_t + e k_t^{-1} - \gamma_u u_n)}{\pi_t - v\gamma_u} \tag{28}$$

By substituting for u_t in (12), and taking into account that $r_A = r_A(k_t)$, the growth rate of the capital stock is

$$g_{K,t} = \gamma_t + F(\gamma_t, k_t), \tag{29}$$

where $F(\gamma_t, k_t)$ is defined by

$$F(\gamma_t, k_t) = x\left[p_z \beta g_T \left(1 - \frac{1}{1 + r_A(k_t)}\right) + \frac{r_A(k_t)}{k_t} + \delta + \frac{e}{k_t} + \gamma_t - \frac{\pi u_n}{v}\right] \tag{30}$$

The Harrodian adjustment rule (17) for long-term expectations γ_t can now be expressed in compact form as

$$\dot{\gamma}_t = \mu F(\gamma_t, k_t)\gamma_t \tag{31}$$

while using (24) the law of motion (18) for k_t becomes:

$$\dot{k}_t = \left[\gamma_t + F(\gamma_t, k_t) - g_T \left(1 - \frac{1}{1 + r_A(k_t)}\right)\right] k_t \tag{32}$$

As in the previous section, we have a dynamic system in the two state variables γ_t and k_t such that its dynamic equilibria satisfy $\dot{\gamma}_t = \dot{k}_t = 0$. One equilibrium is the positive steady state (γ^*, k^*), where $\gamma^* = \gamma(k^*) = g_T\left[1 - (1 + r_A(k^*))^{-1}\right]$ and k^* is the positive real solution to $F(\gamma^*(k^*), k^*) = 0$. The properties of the dynamic equilibrium (γ^*, k^*) are discussed below. To this end, let

$$h \equiv g_T^{1/2}\left\{(1 + p_z\beta)[(p_z - 1)\beta]^{-1/2} - [(p_z - 1)\beta]^{1/2}\right\} > 0 \tag{33}$$

$$s \equiv \pi u_n / v - \delta - g_T(1 + p_z\beta) \geq 0 \tag{34}$$

Notice that conditions (33) and (34) rely upon the plausible parameter restrictions $p_z - 1 < (1 + \beta)/\beta$ and $\pi \geq \bar{\pi} = (\delta + g_T(1 + \beta))v/u_n$. Appendix $A.1$ shows that, with such restrictions in place, we have:

$$k^* = \left[\frac{2(e - 1)}{h + \Delta^{1/2}}\right]^2 \tag{35}$$

where

$$\Delta = h^2 + 4(e - 1)s.$$

Thus, a necessary condition for the existence of a positive growth path is that productivity adjusted autonomous consumption e is larger than one. It may be also worth observing that k^* is negatively related to the value of the profit share, and because γ^* is an increasing function of k^*, we say that growth is wage led in the equilibrium (γ^*, k^*).[6]

To study the local stability of (γ^*, k^*), we write the Jacobian matrix of the first partial derivatives of system (31)-(32), evaluated at (γ^*, k^*), i.e.:

$$J(\gamma^*, k^*) = \begin{bmatrix} \mu x \gamma^* & \mu F_k(\gamma^*, k^*)\gamma^* \\ (1+x)k^* & k^* F_k(\gamma^*, k^*) - \frac{1}{2} g_T^{1/2}[(p_z - 1)\beta k^*]^{-1/2} \end{bmatrix}$$

This yields:

$$\begin{aligned} \det(J(\gamma^*, k^*)) &= -\mu\gamma^* \left[k^* F_k(\gamma^*, k^*) + x\frac{1}{2}\left(\frac{g_T}{k^*}\right)^{1/2} [(p_z - 1)\beta]^{-1/2} \right] \\ \operatorname{tr}(J(\gamma^*, k^*)) &= \mu x \gamma^* + k^* F_k(\gamma^*, k^*) - \frac{1}{2}\left(\frac{g_T}{k^*}\right)^{1/2} [(p_z - 1)\beta]^{-1/2} \end{aligned}$$

If technological opportunity g_T is small enough, then $sign[\det(J(\gamma^*, k^*))] = -sign[F_k(\gamma^*, k^*)]$, and if the adjustment parameter μ is sufficiently small, then $\operatorname{tr}(J(\gamma^*, k^*)) < 0$, if $F_k(\gamma^*, k^*) < 0$. It turns out that the local stability of the constant growth path (γ^*, k^*) hinges crucially upon the condition $F_k(\gamma^*, k^*) < 0$. Appendix $A.2$ shows that this restriction applies, thus yielding:

Proposition 1 *If $e > 1$, in the range of the profit share $\pi \geq \bar{\pi}$, there exists a positive steady state solution (γ^*, k^*) of the dynamic system (31)-(32). (γ^*, k^*) is locally asymptotically stable, if technological opportunity g_T and the adjustment parameter μ are small enough.*

An illustration of this case is shown in Fig. 2.

4.1 Comparative analysis

The transitional and steady state effects of a change in distribution on both output and employment are worth considering. In the parameter range in which the local stability of the positive dynamic equilibrium holds, let us contemplate an economy that at time t is fully adjusted to its steady-state position (γ_1^*, k_1^*), corresponding to $\pi = \pi_1$. Labor productivity is A_t and capacity utilization is $u_t = u_n$; thus, we can write $A_t L_t = u_n K_t$ and $L_t =$

[6] This borrows a terminology first introduced in Rowthorn (1982), Dutt (1984), Bhaduri and Marglin (1990) and that was revived by a recent symposium (cfr. Setterfield, 2016).

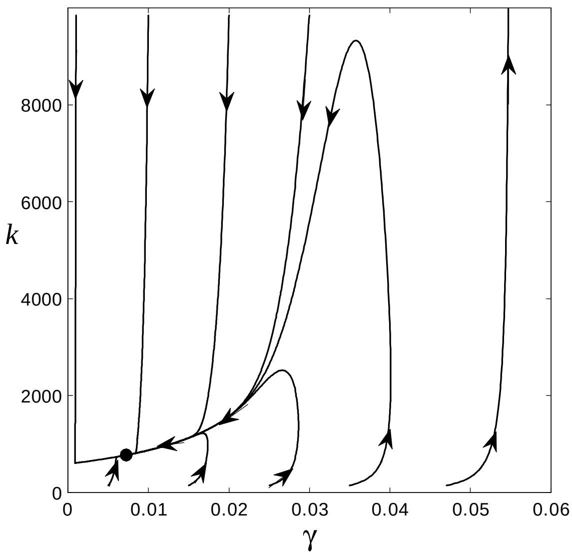

Figure 2: Trajectories in phase space for parameter settings: $\pi = 0.3$, $g_T = 0.04$, $\mu = 0.15$, $v = 3$, $\beta = 0.08$, $p_z = 1.6$, $\delta = 0.02$, $e = 40$, $u_n = 0.8$, $\gamma_u = 0.025$ such that $\gamma^* \approx 0.0072$ and $k^* \approx 775.6656$ The trajectory on the right is diverging

$L_1 = u_n k_1^*$. At time $t + \partial t$ a once and for all small parametric change of the profit share takes place, such that $\Delta\pi = \pi_2 - \pi_1 > 0$. Because k^* is a decreasing function of π, after convergence to the new steady state (γ_2^*, k_2^*), corresponding to π_2, productivity adjusted output is $y_2^* < y_1^*$. The new steady-state level of employment is $L_2^* = u_n k_2^* < L_1^*$. Thus, a once and for all rise of the profit share causes a persistent fall in steady-state employment. In the new steady state, output grows at the lower rate $\gamma_2^* < \gamma_1^*$. Conversely, a fall $\Delta\pi < 0$ of the profit share would cause a persistent increase of the growth rate and a persistent rise in employment, but no *persistent* effect on the rate of capacity utilization, that will eventually return to its steady-state normal level u_n. Still, as shown in Fig. 3, over any finite time interval, following the given fall of the profit share, average capacity utilization is higher than normal. This marks a sharp distinction between the time average of a variable, over a long interval of historical time, and its dynamic attractor.[7]

[7]Debates over the role and properties of capacity utilization in the analysis of demand-led growth have occasionally overlooked this distinction.

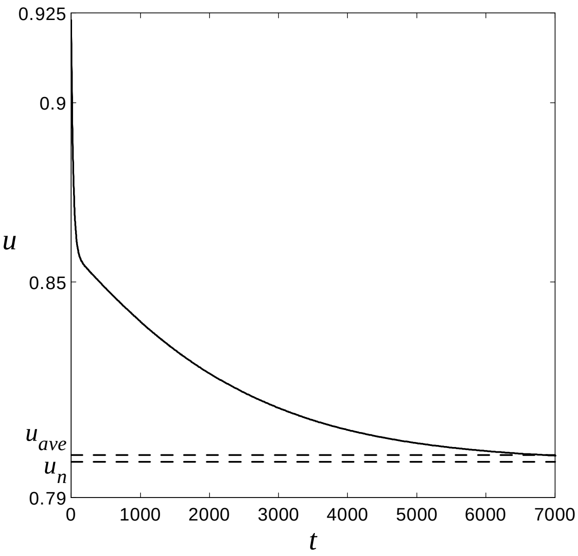

Figure 3: Behaviour in time of the rate of capacity utilization after an exogenous, once and for all change of the profit share $\Delta\pi = -0.03$, with all other parameters as in Fig. 2 and initial condition at the equilibrium (γ^*, k^*)

4.2 Limit cycles

Appendix $A.3$ shows that there are two other equilibria of the dynamic system (31)-(32). One is the unstable trivial solution $(0, 0)$. The other equilibrium is the saddle point $(0, k^{**})$. The existence of such equilibria derives exclusively from the multiplicative terms γ_t and k_t, that appear on the right-hand of (31) and of (32), respectively. Still, the grounds for introducing such terms are not the same. The multiplicative term k_t in the right-hand side of (32) is imposed by formal and logical consistency, including the necessary restriction $k_t \geq 0$. On the contrary, the multiplicative term γ_t in the right-hand side of (31) cannot be justified on similar grounds. While the form (31) requires $\gamma_t \geq 0$, such non-negativity restriction, far from being a logical requirement, is objectionable outside a strictly-local domain of analysis.

In our attempt to proceed in this direction, we eliminate the multiplicative term γ_t in (31) and, borrowing insights from the non-linear adjustment literature (Goodwin, 1951), we further impose that as the gap between the long-term expectation γ_t and the ex-post observation $g_{K,t}$ tends to increase, the adjustment rule of γ_t becomes increasingly conservative. Thus, using (29) we replace (31) with:

$$\dot{\gamma}_t = \mu F(\gamma_t, k_t) - \phi F^3(\gamma_t, k_t) \tag{36}$$

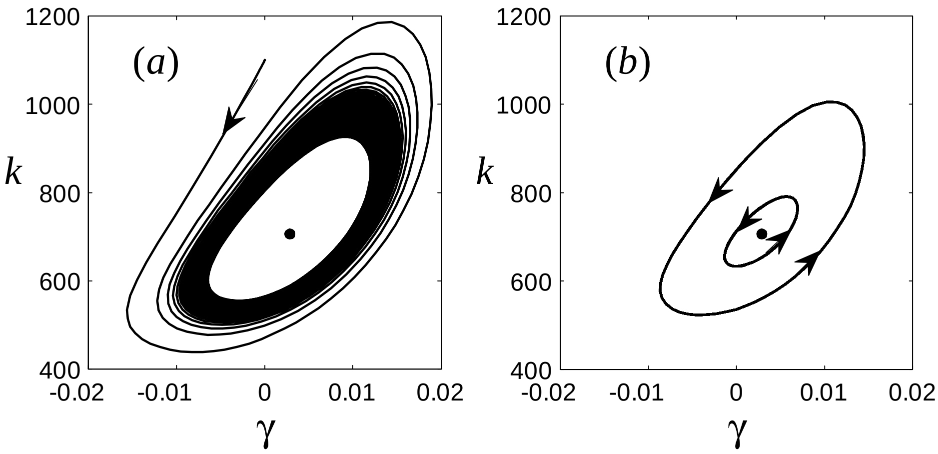

Figure 4: (a) Convergence to the external limit cycle in phase space for parameter settings $\pi = 0.3$, $g_T = 0.035$, $\mu \approx 0.1031$, $v = 3$, $\beta = 0.08$, $p_z = 1.6$, $\delta = 0.02$, $e = 40$, $u_n = 0.8$, $\gamma_u = 0.025$, $\phi = 660$ and initial conditions $(\gamma_{01}, k_{01}) = (0, 565)$, $(\gamma_{02}, k_{02}) = (0, 1100)$; (b) coexistence of two limit cycles

As it can be readily observed, the two equilibria $(0, 0)$ and $(0, k^{**})$ vanish, but the equilibrium (γ^*, k^*) does not. Appendix A.4 proves that the local stability properties of the equilibrium (γ^*, k^*) are qualitatively unchanged: namely, there exists a value $\hat{\mu} > 0$, such that (γ^*, k^*) is locally asymptotically stable if $0 < \mu < \hat{\mu}$. In this parameter range of μ, the temporary and persistent qualitative effects of a small change in distribution are those described in paragraph 4.1. For any $\mu > \hat{\mu}$ the dynamic equilibrium (γ^*, k^*) is unstable, and growth trajectories with initial conditions in a neighbourhood of the steady state, converge to a limit cycle around (γ^*, k^*). This is proved as follows (see Appendix A.4).

If μ/ϕ is small enough, there exists a compact positively invariant region D in the state space such that $(\gamma^*, k^*) \in D$ is the unique stationary point of (36)-(32) in D. In a right-neighbourhood of $\hat{\mu}$, the equilibrium (γ^*, k^*) is unstable, and by the Poincaré-Bendixon theorem, the region D contains a stable limit cycle as shown in Fig. 4(a). In addition, numerical simulation uncovers the existence of a multiplicity of limit cycles around the locally unstable (γ^*, k^*) (see, for an example, Fig 4(b)).

The persistent fluctuations around the positive steady state are such that the average rate of capacity utilization over the cycles does not coincide with the steady-state normal value u_n, but is higher (see Fig. 5). This extends the

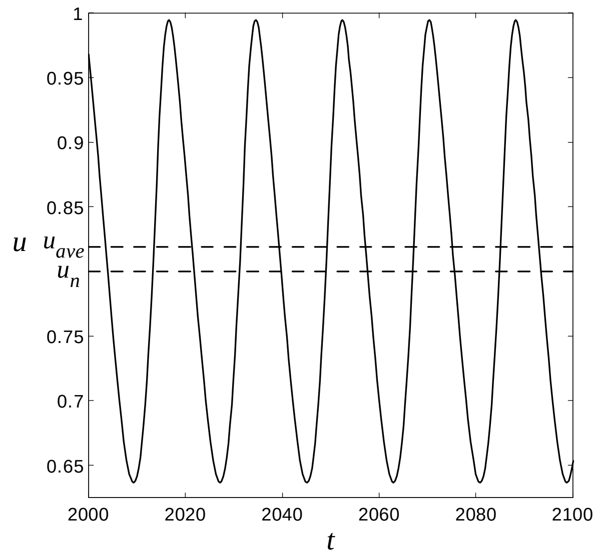

Figure 5: The cyclical behaviour of the rate of capacity utilization over the external limit cycle of Fig. $4(b)$

distinction between the long-term time average of a variable and its dynamic equilibrium to situations in which the economy is on its asymptotic attractor.

5 Conclusions

This paper builds on the hypothesis that R&D and various forms of expenditure triggered by innovation are autonomous, in that they are relatively unaffected by short-run output. Moreover, if and to the extent that innovations are primarily aimed at reducing the use of the human-labour input in production, while the use of capital inputs per unit of output is fixed, such expenditures do not create new capacity. Thus, they do not interfere with expansion investment, as determined by the state of long-term expectations on output growth and by the wish to bring capacity utilization into line with its desired level. We explore some implications of these hypotheses in the light of a demand-led endogenous-growth model.[8] R&D is carried out to maximize monopoly rents and is an increasing function of the capital stock and of the historically given technological opportunities. For the sake of simplicity,

[8]Results of the exogenous-growth framework are skipped for simplicity, because they are similar, except for the fact that distribution does not affect the long-run growth of output.

it is assumed that the marginal propensity to save out of wages is one and the marginal propensity to save out of profits is zero. In the short-run equilibrium, the average propensity to save depends on the level of autonomous expenditure. This includes not only R&D and modernization expenditures by firms, that are both a function of the capital stock. The existence and local stability of the positive growth path requires a flow of autonomous expenditure, that grows through time with labour productivity, but bears no strong direct relation with the size of the capital stock.[9] This flow is here interpreted as autonomous consumption financed by profit income.

The main results are as follows. A sufficiently slow adjustment of long term expectations, as parametrized by μ, ensures the local asymptotic stability of the positive growth path. At higher values of μ, the instability of the dynamic equilibrium requires replacing a strictly local expectation-formation rule, with one that may hold on a wider domain. In this case, the growth trajectories starting in a neighbourhood of the dynamic equilibrium remain bounded and converge to limit cycles, provided that the revision of long-term expectations is ever more conservative, as the gap between prediction γ_t and ex-post realization $g_{K,t}$ increases. On the steady-growth path, capacity utilization is at its desired level. Growth is wage led, both in the sense that long term output growth is inversely related to the profit share, and in the sense that a lower profit share raises the steady state level of productivity adjusted output and employment. Employment is constant on a steady-growth path and the output dynamics tends to be divorced from the employment dynamics. A higher profit share causing a slower long-run growth of output will in fact produce a persistent fall in employment. In this framework, any fall in the wage share, whether caused by market forces, or by changes in institutions, tends to produce self-reinforcing effects. In this way, the model may contribute to the task of interpreting the association between a falling manufacturing employment and a falling wage share, that are a characteristic of the present era in many western countries.

A Appendix

A.1 Computation of k^*

$$F(\gamma, k) = \gamma_u \left(\frac{v[p_z \beta g_T \left(1 - (1 + r_A(k))^{-1}\right) + r_A(k)k^{-1} + \delta + ek^{-1} + \gamma] - \pi u_n}{\pi - v\gamma_u} \right)$$

[9] In the exogenous-growth model in section 3, this expenditure component is identified by R&D itself.

Imposing $\gamma = g_T\left(1 - (1 + r_A(k))^{-1}\right)$, the equilibrium restriction $F(\gamma, k) = 0$ yields

$$(1 + p_z\beta)g_T\left(1 - (1 + r_A(k))^{-1}\right) + r_A(k)k^{-1} + ek^{-1} = \frac{\pi u_n}{v} - \delta$$

Substitute for $r_A(k)$ from (22) at $k > k_{\min}$ and rearrange, to obtain

$$k^{-1}(e-1)+k^{-1/2}g_T^{1/2}[[(p_z-1)\beta]^{1/2}-[(1+p_z\beta)[(p_z-1)\beta]^{-1/2}] = \frac{\pi u_n}{v}-\delta-(1+p_z\beta)g_T$$

that can be written in compact form as

$$(e-1)y^2 - hy - s = 0$$

where $y = k^{-1/2}$ and $h > 0$, $s \geq 0$ are defined (respectively) by (33) and (34) in the text and by the restrictions spelled out therein.

This leads to

$$y^* = \frac{h + \Delta^{1/2}}{2(e-1)}$$

where $\Delta = h^2 + 4s(e-1)$ and

$$k^* = \left[\frac{2(e-1)}{h + \Delta^{1/2}}\right]^2$$

A.2 Proof that $F_k(\gamma^*, k^*) < 0$

$$F_k(\gamma^*, k^*) = \frac{x}{(k^*)^2}\left[\frac{1}{2}\left(g_T k^*\right)^{1/2}\left[p_z\beta[(p_z-1)\beta]^{-1/2} - [(p_z-1)\beta]^{1/2}\right] + 1 - e\right]$$

Using (35) the term $\frac{1}{2}\left(g_T k^*\right)^{1/2}$ can be written as

$$\frac{1}{2}\left(g_T k^*\right)^{1/2} = \frac{e-1}{(h/g_T^{1/2}) + \left(\Delta/g_T\right)^{1/2}} \tag{37}$$

Substituting for h from (33)

$$\frac{1}{2}\left(g_T k^*\right)^{1/2} = \frac{e-1}{(1+p_z\beta)[(p_z-1)\beta]^{-1/2} - [(p_z-1)\beta]^{1/2} + \left(\Delta/g_T\right)^{1/2}}$$

Because $e > 1$ and $(p_z - 1)\beta < 1$, we have:

$$F_k(\gamma^*, k^*) = \frac{x}{(k^*)^2}\left[\frac{(e-1)\left[p_z\beta[(p_z-1)\beta]^{-1/2} - [(p_z-1)\beta]^{1/2}\right]}{(1+p_z\beta)[(p_z-1)\beta]^{-1/2} - [(p_z-1)\beta]^{1/2} + \left(\Delta/g_T\right)^{1/2}} + 1 - e\right] < 0$$

A.3 Properties of the equilibria $(0,0)$ and (γ_0, k^{**})

The Jacobian matrix of the dynamic system (31)-(32) evaluated at $(0,0)$ is:

$$J(0,0) = \begin{bmatrix} \mu \cdot F(0,0) & 0 \\ 0 & F(0,0) \end{bmatrix}$$

and because $F(0,0) > 0$, the trivial stationary equilibrium $(0,0)$ is locally unstable.

Using (31), the equilibrium $(0, k^{**})$ is defined by

$$F(0,k^{**}) = g_T[1 - [g_T(p_z - 1)\beta k^{**}]^{-1/2}] \tag{38}$$

In the interval $[0, k^*]$, $F(0,k)$ is a decreasing function of k, that satisfies: $F(0,0) = +\infty$, $F(0, k_{\min}) > 0$ and $F(0,k^*) < 0$. The function $\gamma(k)$ defined by $\gamma(k) \equiv g_T[1 - [g_T(p_z - 1)\beta k^{**}]^{-1/2}]$ is a non decreasing function of k and satisfies: $\gamma(k) = 0$ for $0 \leq k \leq k_{\min}$; $\gamma(k) > 0$ and $\gamma'(k) > 0$ at $k_{\min} < k \leq k^{**}$. By continuity, there exists $k^{**} > k_{\min}$ such that condition (38) holds and $F(0,k^{**}) > 0$.

The Jacobian matrix of (31), (32) evaluated at $(0, k^{**})$ is:

$$J(0,k^{**}) = \begin{bmatrix} \mu F(0,k^{**}) & 0 \\ (1+x)k^{**} & k^{**}F_k(0,k^{**}) - \frac{1}{2}g_T^{1/2}[(p_z - 1)\beta k^{**}]^{-1/2} \end{bmatrix}$$

Because $F(0,k^{**}) > 0$ and $F_k(0,k^{**}) < 0$, we have $\det J(0,k^{**}) < 0$; therefore, $(0,k^{**})$ is a saddle point.

A.4 Properties of the dynamic system (36)-(32)

The Jacobian matrix of (36)-(32) evaluated at (γ^*, k^*) is:

$$\hat{J}(\gamma^*, k^*) = \begin{bmatrix} \mu x & \mu F_k(\gamma^*, k^*) \\ (1+x)k^* & k^* F_k(\gamma^*, k^*) - \frac{1}{2}g_T^{1/2}[(p_z - 1)\beta k^*]^{-1/2} \end{bmatrix}$$

such that

$$\begin{aligned} \det(\hat{J}(\gamma^*, k^*)) &= -\mu \left[k^* F_k(\gamma^*, k^*) + x\frac{1}{2}\left(\frac{g_T}{k^*}\right)^{1/2} [(p_z - 1)\beta]^{-1/2} \right] \\ \mathrm{tr}(\hat{J}(\gamma^*, k^*)) &= \mu x + k^* F_k(\gamma^*, k^*) - \frac{1}{2}\left(\frac{g_T}{k^*}\right)^{1/2} [(p_z - 1)\beta]^{-1/2} \end{aligned}$$

Because $F_k(\gamma^*, k^*) < 0$ (Appendix A.2), there exists $\hat{\mu} > 0$, such that (γ^*, k^*) is locally asymptotically stable, if $0 < \mu < \hat{\mu}$.

Observe from (30), that if $s > 0$, there is a finite $\bar{k} > k^*$, such that for any $\gamma \in [0, g_T]$, $F(\gamma, k) < 0$, if $\bar{k} - k > 0$ and sufficiently small. Moreover, there is a strictly positive $\hat{k} < k^*$, such that, for any $\gamma \in [0, g_T]$, $F(\gamma, k) > 0$, if $k - \hat{k} > 0$ and sufficiently small.

Equation (36) implies that $\dot{\gamma}_t = 0$, if $F^2(\gamma, k) = \mu/\phi$. Moreover, because $F_\gamma(\gamma, k) > 0$, for each $k \in [\hat{k}, \bar{k}]$, we can define the correspondence $[\gamma_1(k), \gamma_2(k)]$ such that $F(\gamma_1(k), k) = -(\mu/\phi)^{1/2}$ and $F(\gamma_2(k), k) = (\mu/\phi)^{1/2}$.

Let

$$\hat{\gamma}_1(k) = \min_{k \in [\hat{k}, \bar{k}]} (\gamma_1(k));$$

$$\bar{\gamma}_2(k) = \max_{k \in [\hat{k}, \bar{k}]} (\gamma_2(k)).$$

Notice that $\hat{\gamma}_1(k) < \gamma^* < \bar{\gamma}_2(k)$ and that for μ/ϕ sufficiently small we have $0 \leq \hat{\gamma}_1(k) < \gamma^* < \bar{\gamma}_2(k) \leq g_T$. This proves that there is a positively invariant region D of (36), (32) around (γ^*, k^*).

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