TASKS of MATHEMATICS
for Economic Applications AA. 2015/16

Intermediate Test January 2016

I M 1) Find the three numbers x for which $(i-1)^4 = (1-x)^3$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 3 & -1 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & k \end{Vmatrix}$, determine, on varying the parameter k, if it is a diagonalizable matrix when it admits multiple eigenvalues and if it may admit complex eigenvalues.

I M 3) Using the columns of the matrix $\mathbb{A} = \begin{Vmatrix} k_1 & 1 & 1 \\ -1 & k_2 & 2 \\ 1 & -1 & k_3 \end{Vmatrix}$, determine appropriate values for the parameters k_1, k_2, k_3 in order to obtain, from these columns, an orthogonal matrix.

I M 4) Given the linear map $f : \mathbb{R}^3 \to \mathbb{R}^4, f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 1 \\ 2 & -1 & 3 \\ 1 & -2 & k \\ k & 1 & 1 \end{Vmatrix}$ determine the value of the parameter k so that the dimension of the Kernel is the maximum possible, and then, for such value, find a basis for the Kernel and a basis for the Image of such linear map.

I M 5) Check if there are values of m and k for which the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 2 \\ 3 & -1 \end{Vmatrix}$ is similar to the matrix $\mathbb{B} = \begin{Vmatrix} 0 & 1 \\ 7 & 0 \end{Vmatrix}$ if the matrix of the similarity transformation is $\mathbb{P} = \begin{Vmatrix} m & 1 \\ 2 & k \end{Vmatrix}$.

I Winter Exam Session 2016

I M 1) Calculating $\dfrac{1+i}{\sqrt{3}+i}$ determine the sine and cosine of $\dfrac{\pi}{12}$.

I M 2) Given $\mathbb{X}_1 = (1, -1, 2)$ and $\mathbb{X}_2 = (-2, m, 2)$ find the relation between m and k for which the vector $\mathbb{Y} = (0, 1, k)$ is a linear combination of $\mathbb{X}_1$ and $\mathbb{X}_2$.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 2 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & -3 & -4 \end{Vmatrix}$, check if such matrix is diagonalizable.

I M 4) For the linear map $f : \mathbb{R}^5 \to \mathbb{R}^3, f(x_1, x_2, x_3, x_4, x_5) = (y_1, y_2, y_3)$ such that: $\begin{cases} y_1 = x_1 + x_2 + x_3 + x_4 + x_5 \\ y_2 = x_1 + x_2 + x_3 + x_4 - x_5 \\ y_3 = x_1 + x_2 + x_3 - x_4 + x_5 \end{cases}$, find a basis for the Kernel and a basis for the Image of such linear map.

II M 1) The equation $f(x, y) = \log\left(x^2 + y^2\right) - \dfrac{y}{x} = 0$ defines an implicit function $y = y(x)$ in a neighboroud of the point $(1, 0)$. Calculate $y'(1)$.

II M 2) Given $f(x, y) = e^{x-y}$ determine at least one direction $v = (\cos\alpha, \operatorname{sen}\alpha)$ such that $\mathcal{D}_v f(0,0) = \mathcal{D}^2_{v,-v} f(0,0)$.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = xy - y^2 \\ \text{u.c.} \begin{cases} x \le 1 - y^2 \\ 0 \le x \end{cases} \end{cases}$.

II M 4) Verify that the function $f(x,y,z) = x^2 + y^2 + z^2 - xy + yz$ has a second order total differential that is everywhere a positive definite quadratic form.

II Winter Exam Session 2016

I M 1) Compute $i^{15} \cdot \dfrac{(1+i)^5}{(1-i)^6}$.

I M 2) Given the linear map $f : \mathbb{R}^3 \to \mathbb{R}^4, f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 0 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{Vmatrix}$, find a matrix $\mathbb{A}$ for which $f(1,0,1) = (1,0,1,0)$, $f(1,-1,0) = (2,1,0,1)$, such that the Kernel of the map has dimension equal to 1 and vector $(1,1,1)$ belongs to the Kernel. Then find a basis for the Kernel of such linear map.

I M 3) Verify that the matrix $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 0 \\ -1 & k & 1 \\ 0 & 1 & 1 \end{Vmatrix}$ cannot have multiple eigenvalues for every value of the real parameter k.

I M 4) Determine an orthogonal matrix which diagonalizes $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{Vmatrix}$.

II M 1) With the system $\begin{cases} f(x,y,z) = e^{x^4-y^4} + e^{z^4-y^4} - 2xyz = 0 \\ g(x,y,z) = x^3 - y^3 + z^3 + 3xz = 0 \end{cases}$ we define at the point $(-1,1,-1)$ an implicit function $x \to (y(x), z(x))$. Calculate $(y'(-1), z'(-1))$.

II M 2) Given the function $f(x,y) = e^{\alpha(x-y)}$, determine the value of the parameter α knowing that $D_v f(0,0) + D^2_{v,-v} f(0,0) = \dfrac{1}{4}$, where v is the unit vector of $(1,-1)$.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x,y,z) = x + y + z \\ \text{u.c. } x^2 + 2y^2 + 3z^2 = 66 \end{cases}$.

II M 4) Verify that the second order total differential of the function $f(x,y) = e^{3x-y}$ never is a negative definite quadratic form.

I Additional Exam Session 2016

I M 1) If $z = i^{19} - 3i^6 - i^{18} + 3i^3$, find which between the fourth roots of z have a positive imaginary part.

I M 2) Given the linear map $f : \mathbb{R}^4 \to \mathbb{R}^3, f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 2 & -2 \\ 1 & 2 & 1 & 1 \\ 3 & 2 & m & k \end{Vmatrix}$, since $\text{Dim(Ker)} = 2$, find a basis for the Kernel and a basis for the Image of such linear map.

I M 3) Since the matrix $\mathbb{A} = \begin{Vmatrix} 3 & 2 & 1 \\ 1 & 4 & k \\ 1 & 2 & -3 \end{Vmatrix}$ has the multiple eigenvalue $\lambda = 2$, find the value of the real parameter k and than check if the matrix is a diagonalizable one.

I M 4) Determine proper values for the real parameters k and m such that the matrix $\mathbb{A}=\begin{Vmatrix}1 & 2\\ 1 & 3\end{Vmatrix}$ may be similar to the matrix $\mathbb{B}=\begin{Vmatrix}0 & m\\ 1 & k\end{Vmatrix}$ and than find at least a square non-singular matrix $\mathbb{P}$ such that $\mathbb{A}\cdot\mathbb{P}=\mathbb{P}\cdot\mathbb{B}$.

II M 1) Given the function $f(x,y)=e^{x^2-y^2}$, determine all the points (x,y) of the real plan satisfying the condition $D^2_{v,v}f(x,y)+D^2_{-v,-v}f(x,y)=0$, where v is the versor of $(1,1)$.

II M 2) The equation $\log\left(x^2+y^2\right)-xy=0$ defines an implicit function $y=y(x)$ in a neighborhood of the point $P=(0,1)$. Determine $y'(0)$ and $y''(0)$.

II M 3) Given $g:\mathbb{R}^3\to\mathbb{R}^2,(t_1,t_2,t_3)\to(x_1,x_2)$ and $f:\mathbb{R}^2\to\mathbb{R},(x_1,x_2)\to y$ differentiable functions, if $\dfrac{\partial(y)}{\partial(t_1,t_2,t_3)}=\|1\quad 3\quad 4\|$ and $\dfrac{\partial(x_1,x_2)}{\partial(t_1,t_2,t_3)}=\begin{Vmatrix}1 & -1 & 2\\ 0 & 2 & 1\end{Vmatrix}$, determine $\dfrac{\partial(y)}{\partial(x_1,x_2)}$.

II M 4) Determine maximum and minimum points for $f(x,y)=x^3+y^3$ in the region of $\mathbb{R}^2$ consisting by the triangle of vertices $(0,0)$, $(1,0)$ and $(0,1)$.

I Summer Exam Session 2016

I M 1) Find the complex value of k for which $\dfrac{(2+i)^2-(1-i)^2}{k+2i}=i$.

I M 2) For the linear map $f:\mathbb{R}^4\to\mathbb{R}^3, f(x_1,x_2,x_3,x_4)=(x_1+x_2,x_2+x_3,x_3+x_4)$ find a basis for the Kernel and a basis for the Image of such linear map.

I M 3) In the basis $\mathbb{W}=\{(1,1,1),(1,-1,1),(1,1,-1)\}$ the vector $\mathbb{X}$ has coordinates $(1,-2,1)$. Determine its coordinates in the basis $\mathbb{V}=\{(1,0,1),(0,1,0),(0,0,1)\}$.

I M 4) Given the matrix $\mathbb{A}=\begin{Vmatrix}3 & 1 & 1\\ 2 & 4 & 2\\ 1 & m & k\end{Vmatrix}$, verify that it admits the eigenvalue $\lambda=2$ for every value of m and k and than check, on varying the parameters m and k, if such eigenvalue may be a multiple eigenvalue and if it is a diagonalizable matrix for such values of the parameters.

II M 1) Given a function $f(x,y)$, two times differentiable at a certain stationary point P_0, since $D^2_{e_1,-e_2}f(P_0)=2$, $D^2_{-e_1,e_1}f(P_0)=3$ and $D^2_{e_2,e_2}f(P_0)=-2$, determine the nature of the stationary point P_0. (e_1,e_2) is the standard basis.

II M 2) With the system $\begin{cases}f(x,y,z)=x-z-y\,e^y=0\\ g(x,y,z)=x\,e^{z-x}-z\,e^{x-z}-y^2=0\end{cases}$ which type of implicit function can we define at the point $(1,0,1)$? Calculate its first order derivatives.

II M 3) Given $f(x,y,z)=x^3-8y^3-z^3+3x^2-24yz$ find its stationary points and study their nature.

II M 4) Solve the problem: $\begin{cases}\text{Max/min } f(x,y)=y-x\\ \text{u.c.}\begin{cases}x^2+y^2\le 1\\ y\le 1-x^2\end{cases}\end{cases}$.

I Autumn Exam Session 2016

I M 1) Find the two complex values of x for which $(x-i)^2=\dfrac{1+i}{1-i}$.

I M 2) For the linear system $\mathbb{A}\cdot\mathbb{X}=\mathbb{Y}$, $\mathbb{A}=\begin{Vmatrix}1 & 1 & 1 & 1 & 1\\ 1 & -1 & 0 & 1 & -1\\ 2 & 0 & 1 & m & k\end{Vmatrix}$ and $\mathbb{Y}=\begin{Vmatrix}1\\ 1\\ h\end{Vmatrix}$, find the values of the parameters m, k and h for which the system has ∞^3 solutions.

I M 3) Given the set of vectors $\mathbb{W}=\{(1,1,-2),(2,0,3),(-1,-1,k)\}$ find the value of the parameter k for which such set of vectors is not a basis for $\mathbb{R}^3$.

I M 4) Find an ortogonal matrix which diagonalizes $\mathbb{A}=\begin{Vmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 1\end{Vmatrix}$.

II M 1) Check for the nature of the stationary points of $f(x,y)=x^2\,y-k\,xy+x\,y^2$ on varying the parameter k.

II M 2) Given $f(x,y)=2xy-xy^2$ and $P_0=(1,-1)$, since $D_v f(P_0)=\dfrac{7}{\sqrt{2}}$, find at least a value for α in the unit vector $v=(\cos\alpha,\operatorname{sen}\alpha)$.

II M 3) With the equation $f(x,y,z)=xyz-x^2y+xz^2-yz=0$ which type of implicit function can we define at point $P_0=(1,1,1)$? Calculate its first order derivatives.

II M 4) Solve the problem: $\begin{cases}\text{Max/min } f(x,y)=x^2+y^2\\ \text{u.c. : } 4x^2+y^2=4\end{cases}$.

II Additional Exam Session 2016

I M 1) Find the three solutions of the equation $x^3+3x^2+4x+12=0$.

I M 2) Given $\mathbb{A}=\begin{Vmatrix}1 & 0 & 1 & -1\\ 2 & -1 & 0 & -1\\ 1 & -1 & -1 & m\end{Vmatrix}$ and $\mathbb{Y}=\begin{Vmatrix}1\\ 2\\ k\end{Vmatrix}$, check for the solvability of the linear system $\mathbb{A}\cdot\mathbb{X}=\mathbb{Y}$ on varying the parameters m and k.

I M 3) Consider the basis for $\mathbb{R}^3$: $\mathbb{V}=\{\mathbb{V}_1=(1,0,1),\mathbb{V}_2=(0,1,1),\mathbb{V}_3=(1,1,0)\}$; if the vector $\mathbb{X}$ has coordinates $\mathbb{X}_{\mathbb{V}}=(1,-1,2)$ in the basis $\mathbb{V}$, find its coordinates $\mathbb{X}_{\mathrm{e}}$ in the canonical basis.

I M 4) Given the matrix $\mathbb{A}=\begin{Vmatrix}2 & 2 & 1\\ 0 & 1 & -1\\ 1 & 2 & 0\end{Vmatrix}$, check if it is a diagonalizable matrix, and then find, if it exists, its inverse matrix and find the eigenvalues of such inverse.

II M 1) Check for the nature of the stationary points of $f(x,y)=xy-x^2+xy^2$.

II M 2) Given $f(x,y)=2x^3y-3xy^2$, $P_0=(1,-1)$ and v the unit vector of $(1,-2)$, calculate $D_v f(P_0)$.

II M 3) With the equation $e^{x^2-y^2}+x^2-y^2=1$ at point $(3,-3)$ we can define y as an implicit function of x. Calculate $y'(3)$.

II M 4) Solve the problem: $\begin{cases}\text{Max/min } f(x,y)=x^2-2y\\ \text{u.c. : } x^2+y^2\le 4\end{cases}$.