Compito di Analisi Matematica del 27/6/2016 $\boxed{\text{CAM1}}$

IM1) $\dfrac{(1+2i)^2-(1+i)^2}{1+Ki} = i \Rightarrow 1+4i-4-(1+2i-1) = i\cdot(1+Ki) \Rightarrow$

$\Rightarrow 4i-3-2i = 2i-3 = i-K \Rightarrow K = i-2i+3 = 3-i.$

IM2) $f(x;y)$ due volte differenziabile; $e_1=(1;0)$; $e_2=(0;1) \Rightarrow$

$$\mathcal{D}^2_{e_1;-e_2} f(x;y) = e_1\cdot H\cdot(-e_2)^T = \|1\ 0\|\cdot\begin{Vmatrix} f''_{xx} & f''_{xy}\\ f''_{xy} & f''_{yy}\end{Vmatrix}\cdot\begin{Vmatrix}0\\-1\end{Vmatrix} = \|1\ 0\|\cdot\begin{Vmatrix}-f''_{xy}\\-f''_{yy}\end{Vmatrix} = -f''_{xy}=0 \Rightarrow f''_{xy}=0;$$

$$\mathcal{D}^2_{-e_1;e_1} f(x;y) = -e_1\cdot H\cdot(e_1)^T = \|-1\ 0\|\cdot\begin{Vmatrix} f''_{xx} & f''_{xy}\\ f''_{xy} & f''_{yy}\end{Vmatrix}\cdot\begin{Vmatrix}1\\0\end{Vmatrix} = \|-1\ 0\|\cdot\begin{Vmatrix}f''_{xx}\\f''_{xy}\end{Vmatrix} = -f''_{xx}=-1 \Rightarrow f''_{xx}=1;$$

$$\mathcal{D}^2_{e_2;e_2} f(x;y) = e_2\cdot H\cdot(e_2)^T = \|0\ 1\|\cdot\begin{Vmatrix} f''_{xx} & f''_{xy}\\ f''_{xy} & f''_{yy}\end{Vmatrix}\cdot\begin{Vmatrix}0\\1\end{Vmatrix} = \|0\ 1\|\cdot\begin{Vmatrix}f''_{xy}\\f''_{yy}\end{Vmatrix} = f''_{yy}=2. \quad \Rightarrow$$

$$H(P_0) = \begin{Vmatrix} f''_{xx} & f''_{xy}\\ f''_{xy} & f''_{yy}\end{Vmatrix} = \begin{Vmatrix} 1 & 0\\ 0 & 2\end{Vmatrix} \Rightarrow \begin{cases} |H_1|: 1>0;\ 2>0 \\ |H_2|: 2\cdot 1-0>0 \end{cases} \Rightarrow \text{Punto di minimo.}$$

IM3) $\begin{cases} f(x;y;z) = xe^{y-x}-ye^{x-y}=0 \\ g(x;y;z) = x-y+ze^z=0 \end{cases} \Rightarrow \begin{cases} f(1;1;0) = 1\cdot1-1\cdot1=0 \\ g(1;1;0)=1-1+0=0 \end{cases}.$

$$\frac{\partial(f;g)}{\partial(x;y;z)} = \begin{Vmatrix} e^{y-x}-xe^{y-x}-ye^{x-y} & xe^{y-x}-e^{x-y}+ye^{x-y} & 0 \\ 1 & -1 & e^z+ze^z \end{Vmatrix}.$$

$$\frac{\partial(f;g)}{\partial(x;y;z)}(1;1;0) = \begin{Vmatrix} 1-1-1 & 1-1+1 & 0 \\ 1 & -1 & 1+0\end{Vmatrix} = \begin{Vmatrix} -1 & 1 & 0\\ 1 & -1 & 1\end{Vmatrix}. \quad \text{f. implicita}: \mathbb{R}\to\mathbb{R}^2.$$

$\begin{vmatrix}-1 & 1\\ 1 & -1\end{vmatrix}=0$ quindi non si può definire $z\to(x(z);y(z))$.

Essendo $\begin{vmatrix}1 & 0\\ -1 & 1\end{vmatrix}=1\neq 0$ si può definire $x\to(y(x);z(x))$ con derivate:

$$\frac{dy}{dx} = -\frac{\begin{vmatrix}-1 & 0\\ 1 & 1\end{vmatrix}}{\begin{vmatrix}1 & 0\\ -1 & 1\end{vmatrix}} = -\frac{-1}{1} = 1; \quad \frac{dz}{dx} = -\frac{\begin{vmatrix}1 & -1\\ -1 & 1\end{vmatrix}}{\begin{vmatrix}1 & 0\\ -1 & 1\end{vmatrix}} = -\frac{0}{1} = 0.$$

IM4) $f(x;y;z) = 8x^3+24xz+z^3-y^3+3y^2.\ \nabla f(x;y;z) = \mathbb{O} \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = 24x^2+24z = 24(x^2+z)=0 \\ f'_y = 6y-3y^2 = 3y(2-y)=0 \\ f'_z = 24x+3z^2 = 3(8x+z^2)=0 \end{cases} \Rightarrow \begin{cases} z=-x^2 \\ y=0 \cup y=2 \\ 8x+x^4 = x(x^3+8)=0 \end{cases} \Rightarrow$$

$\boxed{\text{CAM2}}$

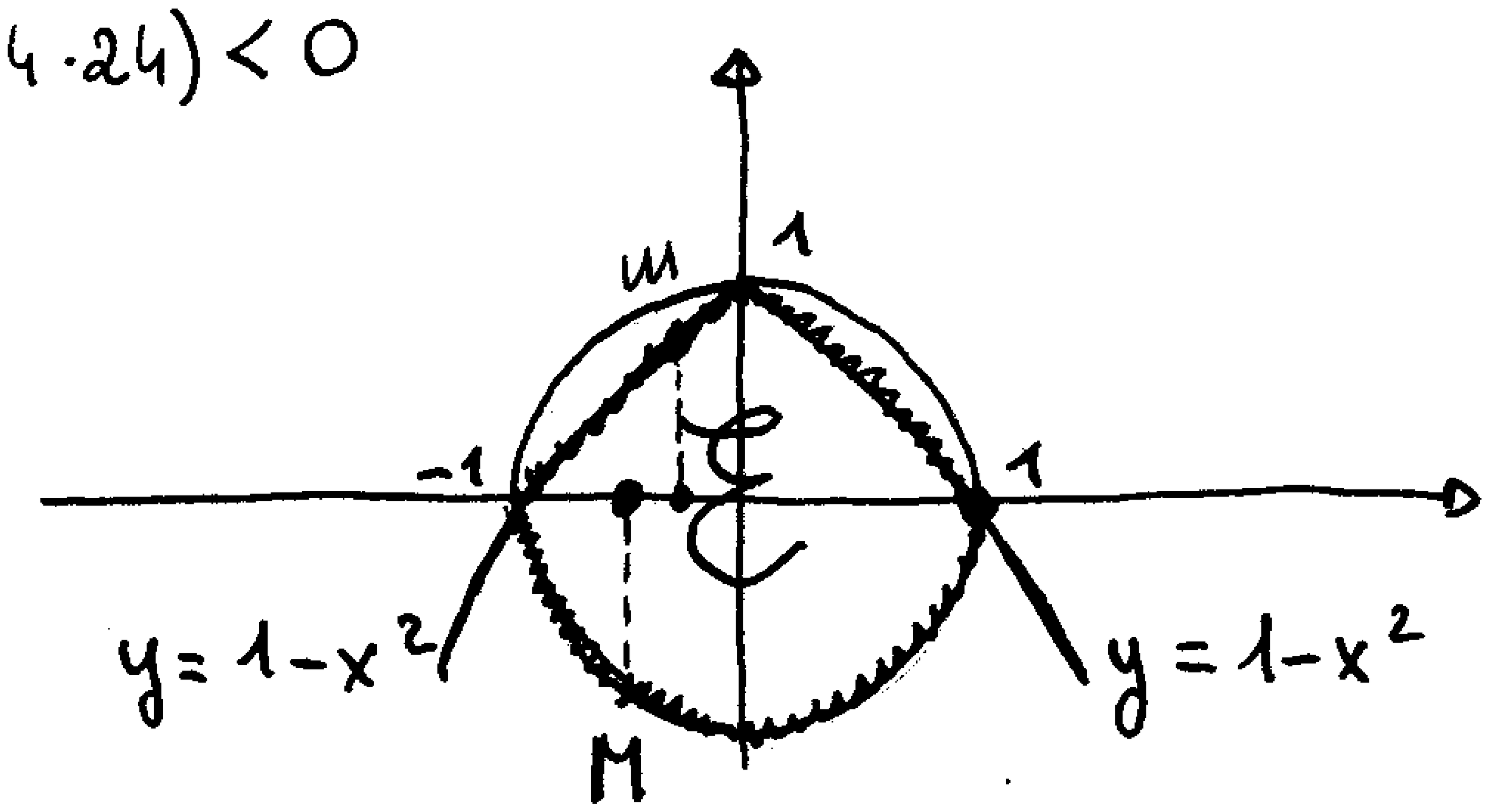
$$\Rightarrow \begin{cases} x=0\\ y=0\\ z=0\end{cases} \cup \begin{cases}x=0\\ y=2\\ z=0\end{cases} \cup \begin{cases}x^3=-8\\ y=0 \cup y=2\\ z=-x^2\end{cases} \Rightarrow \begin{cases}x=-2\\ y=0\\ z=-4\end{cases} \cup \begin{cases}x=-2\\ y=2\\ z=-4\end{cases}$$. Ci sono 4 punti stazionari.

$$H(x;y;z)=\begin{Vmatrix} 48x & 0 & 24\\ 0 & 6-6y & 0\\ 24 & 0 & 6z\end{Vmatrix}. \quad H(0;0;0)=\begin{Vmatrix} 0 & 0 & 24\\ 0 & 6 & 0\\ 24 & 0 & 0\end{Vmatrix} : \begin{cases}|H_1|=6>0\\ |H_3|=24\cdot(-6\cdot 24)<0\end{cases} \Rightarrow \text{Sella}.$$

$$H(0;2;0)=\begin{Vmatrix} 0 & 0 & 24\\ 0 & -6 & 0\\ 24 & 0 & 0\end{Vmatrix} : \begin{cases}|H_1|=-6<0\\ |H_3|=24\cdot(6\cdot 24)>0\end{cases} : \text{Sella}. \quad H(-2;0;-4)=\begin{Vmatrix} -96 & 0 & 24\\ 0 & 6 & 0\\ 24 & 0 & -24\end{Vmatrix} : \begin{cases}|H_1|=-96<0\\ |H_1|=6>0\end{cases} : \text{Sella}.$$

$$H(-2;2;-4)=\begin{Vmatrix} -96 & 0 & 24\\ 0 & -6 & 0\\ 24 & 0 & -24\end{Vmatrix} : \begin{cases}|H_1| : -96<0;\ -6<0;\ -24<0\\ |H_2| = 6\cdot 96>0;\ 6\cdot 24>0\\ |H_3| = -6\cdot(96\cdot 24-24\cdot 24)<0\end{cases} : (-;+;-) : \text{Punto di Massimo}.$$

II M1) $\begin{cases}\text{Max}|\text{min } f(x;y) = xy-y\\ \text{s.v.} \begin{cases}x^2+y^2\le 1\\ y\le 1-x^2\end{cases} \Rightarrow \begin{cases}x^2+y^2-1\le 0\\ y+x^2-1\le 0\end{cases}\end{cases}$.



Funzione differenziabile, $\mathcal{E}$ insieme limitato e chiuso, vincoli qualificati.

$$\Lambda = xy-y-\lambda_1(x^2+y^2-1)-\lambda_2(y+x^2-1).$$

<u>Caso $\lambda_1=\lambda_2=0$</u>

$$\begin{cases}\Lambda'_x = y = 0\\ \Lambda'_y = x-1=0\\ x^2+y^2\le 1\\ y\le 1-x^2\end{cases} \Rightarrow \begin{cases}x=1\\ y=0\\ 0+0\le 1\\ 0\le 1-0\end{cases} ;\ H(x;y)=H(1;0)=\begin{Vmatrix}0 & 1\\ 1 & 0\end{Vmatrix} \Rightarrow (0;0) \text{ punto di Sella.}$$

<u>Caso $\lambda_1\ne 0; \lambda_2=0$</u>

$$\begin{cases}\Lambda'_x = y-2\lambda_1 x=0\\ \Lambda'_y = x-1-2\lambda_1 y=0\\ x^2+y^2=1\\ y\le 1-x^2\end{cases} \Rightarrow \begin{cases}y=2\lambda_1 x\\ x-1-4\lambda_1^2 x=0\\ x^2+y^2=1\\ y\le 1-x^2\end{cases} \Rightarrow \begin{cases}y=2\lambda_1 x\\ x=\dfrac{1}{1-4\lambda_1^2}\\ x^2+y^2=1\\ y\le 1-x^2\end{cases} \Rightarrow y=\frac{2\lambda_1}{1-4\lambda_1^2} \Rightarrow$$

$$\Rightarrow x^2+y^2=1 \Rightarrow 1+4\lambda_1^2=(1-4\lambda_1^2)^2 \Rightarrow 16\lambda_1^4-12\lambda_1^2 = 4\lambda_1^2\cdot(4\lambda_1^2-3)=0 \Rightarrow \begin{cases}\lambda_1=0\\ \lambda_1=\pm\frac{\sqrt{3}}{2}\end{cases}.$$

$$\Rightarrow \begin{cases}\lambda_1=0\\ x=1\\ y=0\\ 0\le 1-1\end{cases} \cup \begin{cases}\lambda_1=\frac{\sqrt{3}}{2}\\ x=-\frac{1}{2}\\ y=-\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2}\le 1-\frac{1}{4} \text{ (vera)}\end{cases} \text{Max?} \cup \begin{cases}\lambda=-\frac{\sqrt{3}}{2}\\ x=-\frac{1}{2}\\ y=\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2}\le 1-\frac{1}{4}\ \boxed{\text{NO}}\end{cases} \notin \text{C.}\mathcal{E}..$$

Caso $\lambda_1 = 0;\ \lambda_2 \neq 0$

$$\begin{cases} \Lambda'_x = y - 2\lambda_2 x = 0 \\ \Lambda'_y = x - 1 - \lambda_2 = 0 \\ y = 1 - x^2 \\ x^2 + y^2 \leq 1 \end{cases} \Rightarrow \begin{cases} x = 1 + \lambda_2 \\ y = 2\lambda_2(1+\lambda_2) \\ 2\lambda_2(1+\lambda_2) = 1 - (1+\lambda_2)^2 \\ x^2 + y^2 \leq 1 \end{cases} \Rightarrow \begin{cases} x = 1 + \lambda_2 \\ y = 2\lambda_2(1+\lambda_2) \\ 3\lambda_2^2 + 4\lambda_2 = \lambda_2 \cdot (3\lambda_2 + 4) = 0 \\ x^2 + y^2 \leq 1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \lambda_2 = 0 \\ x = 1 \\ y = 0 \\ 1 + 0 \leq 1 \end{cases} \cup \begin{cases} \lambda_2 = -\frac{4}{3} \\ x = -\frac{1}{3} \\ y = \frac{8}{9} \\ \frac{1}{9} + \frac{64}{81} \leq 1 : \text{Vera} \end{cases}$$ Min ? .

Caso $\lambda_1 \neq 0;\ \lambda_2 \neq 0$

$$\begin{cases} \Lambda'_x = y - 2\lambda_1 x - 2\lambda_2 x \doteq 0 \\ \Lambda'_y = x - 1 - 2\lambda_1 y - \lambda_2 = 0 \\ \begin{cases} x^2 + y^2 = 1 \\ y = 1 - x^2 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 0 \end{cases} \cup \begin{cases} x = -1 \\ y = 0 \end{cases} \cup \begin{cases} x = 0 \\ y = 1 \end{cases}. \end{cases}$$

$$\begin{cases} x = 1 \\ y = 0 \\ -2\lambda_1 - 2\lambda_2 = 0 \\ -\lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 0 \\ \lambda_1 = 0 \\ \lambda_2 = 0 \end{cases} \text{già visto};\quad \begin{cases} x = -1 \\ y = 0 \\ 2\lambda_1 + 2\lambda_2 = 0 \\ -2 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = 0 \\ \lambda_2 = -2 \\ \lambda_1 = 2 \end{cases} : \underline{\text{Nulla}};\quad \begin{cases} x = 0 \\ y = 1 \\ 1 = 0 \\ \text{----} \end{cases} \text{Impossibile.}$$

Per il Teorema di Weierstrass, il punto $\left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right)$ con $f\left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{4}$ è il punto di massimo; il punto $\left(-\frac{1}{3}; \frac{8}{9}\right)$ con $f\left(-\frac{1}{3}; \frac{8}{9}\right) = -\frac{32}{27}$ è il punto di minimo.

II M 2) $\begin{cases} y' = \dfrac{1+y^2}{x^2} \\ y\left(\frac{4}{\pi}\right) = 1 \end{cases} \Rightarrow \dfrac{1}{1+y^2} \cdot y' = \dfrac{1}{x^2} \Rightarrow \displaystyle\int \frac{1}{1+y^2}\,dy = \int \frac{1}{x^2}\,dx + K \Rightarrow \operatorname{arctg} y = K - \frac{1}{x} \Rightarrow$

$\Rightarrow y = \operatorname{tg}\left(K - \frac{1}{x}\right)$. $y\left(\frac{4}{\pi}\right) = 1 \Rightarrow 1 = \operatorname{tg}\left(K - \frac{\pi}{4}\right) \Rightarrow K - \frac{\pi}{4} = \operatorname{arctg} 1 = \frac{\pi}{4} \Rightarrow K = \frac{\pi}{2}$.

Soluzione del problema: $y = \operatorname{tg}\left(\frac{\pi}{2} - \frac{1}{x}\right)$.

II M 3) $\begin{cases} x' = x + 2y + e^t \\ y' = -y - 2e^t \end{cases} \Rightarrow \begin{cases} x' - x - 2y = e^t \\ 0 + y' + y = -2e^t \end{cases} \Rightarrow \begin{Vmatrix} D-1 & -2 \\ 0 & D+1 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \end{Vmatrix} = \begin{Vmatrix} e^t \\ -2e^t \end{Vmatrix} \Rightarrow$

CAM4

$$\Rightarrow \begin{vmatrix} D-1 & -2 \\ 0 & D+1 \end{vmatrix}(x) = \begin{vmatrix} e^t & -2 \\ -2e^t & D+1 \end{vmatrix} \Rightarrow (D^2-1)(x) = (D+1)(e^t) - 4e^t \Rightarrow x'' - x = -2e^t .$$

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow$ Soluzione omogenea: $x(t) = c_1 e^t + c_2 e^{-t}$.

Usando gli annichilatori, occorre $x_0(t) = a e^t + b t e^t \Rightarrow$

$\Rightarrow x_0' = a e^t + b e^t + b t e^t$; $x_0'' = a e^t + b e^t + b e^t + b t e^t \Rightarrow x'' - x = -2e^t \Rightarrow$

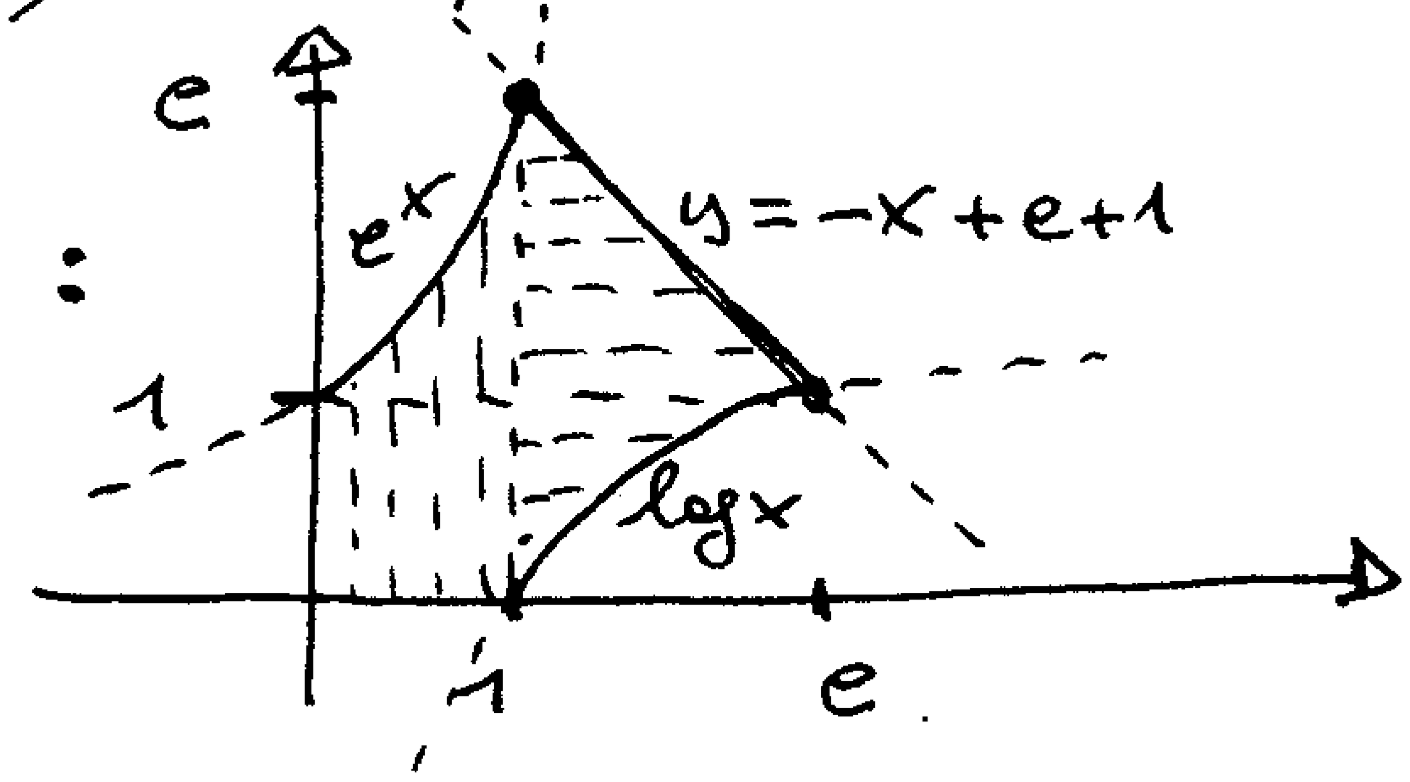
$\Rightarrow \cancel{a e^t} + 2 b e^t + \cancel{b t e^t} - \cancel{a e^t} - \cancel{b t e^t} = -2e^t \Rightarrow 2 b e^t = -2 e^t \Rightarrow b = -1$.

Soluzione della non omogenea: $x(t) = c_1 e^t + c_2 e^{-t} - t e^t$.

$y(t) = \frac{1}{2}\left(x' - x - e^t\right) = \frac{1}{2}\left(\cancel{c_1 e^t} - c_2 e^{-t} - e^t - \cancel{t e^t} - \cancel{c_1 e^t} - c_2 e^{-t} + \cancel{t e^t} - e^t\right) = -c_2 e^{-t} - e^t$.

II M4) $\iint_D f(x;y)\,dx\,dy = \iint_D 1 \cdot dx\,dy$.

$\mathcal{D}$:



Vista la regione D possiamo calcolare:

$$\iint_D 1\,dx\,dy = \int_0^1 \int_0^{e^x} (1\,dy)\,dx + \int_1^e \int_{\log x}^{e+1-x} (1\,dy)\,dx =$$

$$= \int_0^1 \left(y \Big|_0^{e^x} \right) dx + \int_1^e \left(y \Big|_{\log x}^{e+1-x} \right) dx = \int_0^1 (e^x - 0)\,dx + \int_1^e (e+1-x) - \log x \, dx =$$

$$= \left(e^x \Big|_0^1 \right) + \left((e+1)x - \frac{x^2}{2} \Big|_1^e \right) - \left(x \log x - \int x \cdot \frac{1}{x}\,dx \Big|_1^e \right) =$$

$$= e - 1 + \left((e+1)e - \frac{e^2}{2} \right) - \left(e + 1 - \frac{1}{2} \right) - \left(x \log x - x \Big|_1^e \right) =$$

$$= \cancel{e} - 1 + e^2 + e - \frac{e^2}{2} \cancel{- e} - 1 + \frac{1}{2} \cancel{- e} + \cancel{e} - 1 = \frac{e^2}{2} + e - \frac{5}{2} .$$