Task Mathematics for Economic Applications 3/6/2015 $\boxed{\text{MfEA1}}$

I M1) $x^4-3x^2-4=0 \Rightarrow x^2=\frac{3\pm\sqrt{9+16}}{2}=\frac{3\pm 5}{2}=\begin{cases}4\\-1\end{cases}$.

$x^2=4 \Rightarrow x=\pm 2$; $2=2(\cos 0+i\,\mathrm{sen}\,0)$; $-2=2(\cos\pi+i\,\mathrm{sen}\,\pi)$

$x^2=-1 \Rightarrow x=\pm i$; $i=1(\cos\frac{\pi}{2}+i\,\mathrm{sen}\frac{\pi}{2})$; $-i=1\cdot(\cos\frac{3}{2}\pi+i\,\mathrm{sen}\frac{3}{2}\pi)$.

I M2) $A=\begin{Vmatrix}1&0&0\\1&1&0\\0&1&K\end{Vmatrix}$. Since A is a triangular matrix : $\lambda_1=\lambda_2=1$; $\lambda_3=K$.

If $K=1$: $A=\begin{Vmatrix}1&0&0\\1&1&0\\0&1&1\end{Vmatrix} \Rightarrow \|A-1\cdot\mathbb{I}\|=\begin{Vmatrix}0&0&0\\1&0&1\\0&1&0\end{Vmatrix}$: $\mathrm{RANK}(A-1\cdot\mathbb{I})=2 \Rightarrow m_1^g=1<m_1^a=3$;

If $K\neq 1$: $A=\begin{Vmatrix}1&0&0\\1&1&0\\0&1&K\end{Vmatrix} \Rightarrow \|A-1\cdot\mathbb{I}\|=\begin{Vmatrix}0&0&0\\1&0&0\\0&1&K-1\end{Vmatrix}$: $\mathrm{RANK}(A-1\cdot\mathbb{I})=2 \Rightarrow m_1^g=1<m_1^a=2$.

The matrix is not diagonalizable $\forall K\in\mathbb{R}$.

I M3) $\begin{cases}x_1+2x_2-x_3=1\\2x_1+x_3=1\\4x_1-4x_2+Kx_3=m\end{cases} \Rightarrow \left\|\begin{array}{ccc:c}1&2&-1&1\\2&0&1&1\\4&-4&K&m\end{array}\right\| \to \left\|\begin{array}{ccc:c}1&2&-1&1\\0&-4&3&-1\\0&-12&K+4&m-4\end{array}\right\|$ $\begin{matrix}(R_2\leftarrow R_2-2R_1)\\(R_3\leftarrow R_3-4R_1)\end{matrix}$ $\to$

$\to \left\|\begin{array}{ccc:c}1&2&-1&1\\0&-4&3&-1\\0&0&K-5&m-1\end{array}\right\| (R_3\leftarrow R_3-3R_2) \Rightarrow \begin{cases}K\neq 5; \forall m : 1 \text{ solution}\\K=5; m\neq 1 : 0 \text{ solution}.\\K=5; m=1 : \infty^1 \text{ solutions}\end{cases}$

I M4) $\begin{Vmatrix}1&2&-1\\2&0&1\\4&-4&K\end{Vmatrix}\cdot\begin{Vmatrix}1\\1\\1\end{Vmatrix}=\begin{Vmatrix}2\\3\\4\end{Vmatrix} \Rightarrow \begin{cases}1+2-1=2\\2+0+1=3\\4-4+K=4\end{cases} \Rightarrow \begin{cases}2=2\\3=3\\K=4\end{cases} \Rightarrow K=4$.

II M1) $f(x;y)=xy-x^2+y^2-x^3=0$. $f(-1;1)=-1-1+1+1=0$.

$\nabla f(x;y)=(y-2x-3x^2; x+2y)$; $\nabla f(-1;1)=(1+2-3;-1+2)=(0;1)$.

$y'(-1)=-\frac{0}{1}=0$: Stationary point.

$H(f(x;y))=\begin{Vmatrix}-2-6x&1\\1&2\end{Vmatrix}$; $H(f(-1;1))=\begin{Vmatrix}4&1\\1&2\end{Vmatrix}$.

$y''(-1)=-\frac{4+2\cdot1\cdot0+2\cdot(0)^2}{1}=-4<0 \Rightarrow x=-1$ is a Maximum point.

II M2) $f(x;y)=x^2+y^2 \Rightarrow \nabla f(x;y)=(2x;2y) \Rightarrow H=\begin{Vmatrix}2&0\\0&2\end{Vmatrix}$.

$\mathcal{D}^2_{u,u}f(P_0)=\begin{Vmatrix}\cos\alpha & \mathrm{sen}\,\alpha\end{Vmatrix}\cdot\begin{Vmatrix}2&0\\0&2\end{Vmatrix}\cdot\begin{Vmatrix}\cos\alpha\\\mathrm{sen}\,\alpha\end{Vmatrix}=2\cos^2\alpha+2\,\mathrm{sen}^2\alpha=2 \quad \forall P_0 \ \forall\alpha$.

MFEA2

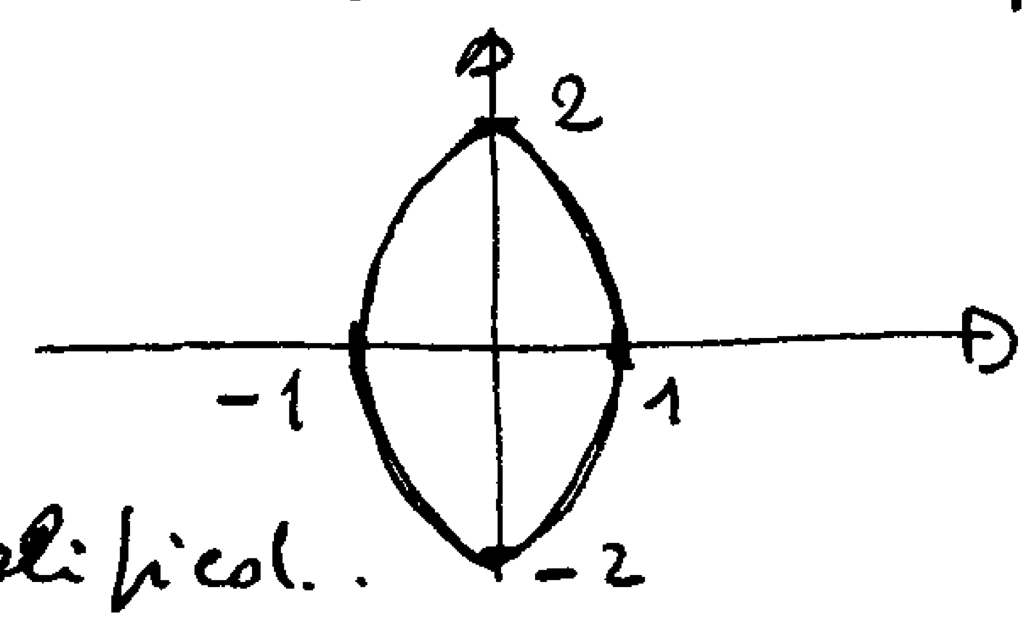
$$\mathcal{D}^2_{u,v} f(P_0) = \left\| \cos\alpha \ \text{sen}\,\alpha \right\| \left\| \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix} \right\| \cdot \left\| \begin{matrix} \cos\beta \\ \text{sen}\,\beta \end{matrix} \right\| = \left\| \cos\alpha \ \text{sen}\,\alpha \right\| \cdot \left\| \begin{matrix} 2\cos\beta \\ 2\,\text{sen}\,\beta \end{matrix} \right\| =$$

$$= 2\cos\alpha\cos\beta + 2\,\text{sen}\,\alpha\,\text{sen}\,\beta = 2\cdot\cos(\alpha-\beta).$$

If $\mathcal{D}^2_{u,v} f(P_0) = 2 \Rightarrow 2\cos(\alpha-\beta) = 2 \Rightarrow \cos(\alpha-\beta) = 1 \Rightarrow \alpha-\beta = 0 \Rightarrow \alpha = \beta.$

If $\mathcal{D}^2_{u,v} f(P_0) = \sqrt{2} \Rightarrow 2\cos(\alpha-\beta) = \sqrt{2} \Rightarrow \cos(\alpha-\beta) = \frac{\sqrt{2}}{2} \Rightarrow \alpha-\beta = \frac{\pi}{4} \Rightarrow \alpha = \beta + \frac{\pi}{4}.$

II M3) $\begin{cases} \text{Max/min } f(x;y) = x-y \\ \text{u.e.}: 4x^2+y^2 \le 4 \Rightarrow x^2 + \frac{y^2}{4} = 1 \end{cases}$



f is a continuous function; $\mathcal{E}$ is a bounded and closed set, constraints are qualified.

$\Lambda = x-y-\lambda(4x^2+y^2-4)$. For $\lambda=0$: $\begin{cases} \Lambda'_x = 1 \ne 0 \\ \Lambda'_y = -1 \ne 0 \end{cases}$: no solutions.

For $\lambda \ne 0$: $\begin{cases} \Lambda'_x = 1-8\lambda x = 0 \\ \Lambda'_y = -1-2\lambda y = 0 \\ 4x^2+y^2 = 4 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{8\lambda} \\ y = -\frac{1}{2\lambda} \\ 4x^2+y^2 = 4 \Rightarrow \frac{1}{16\lambda^2} + \frac{1}{4\lambda^2} = 4 \Rightarrow \frac{1+4}{16\lambda^2} = 4 \Rightarrow \lambda^2 = \frac{5}{64}. \end{cases}$

For $\lambda = \frac{\sqrt{5}}{8} > 0 \Rightarrow \begin{cases} x = \frac{1}{\sqrt{5}} \\ y = -\frac{4}{\sqrt{5}} \end{cases}$: Maximum Point; For $\lambda = -\frac{\sqrt{5}}{8} < 0 \Rightarrow \begin{cases} x = -\frac{1}{\sqrt{5}} \\ y = \frac{4}{\sqrt{5}} \end{cases}$: Minimum Point.

II M4) $f(x;y) = xy^2 - x^2 - ky$.

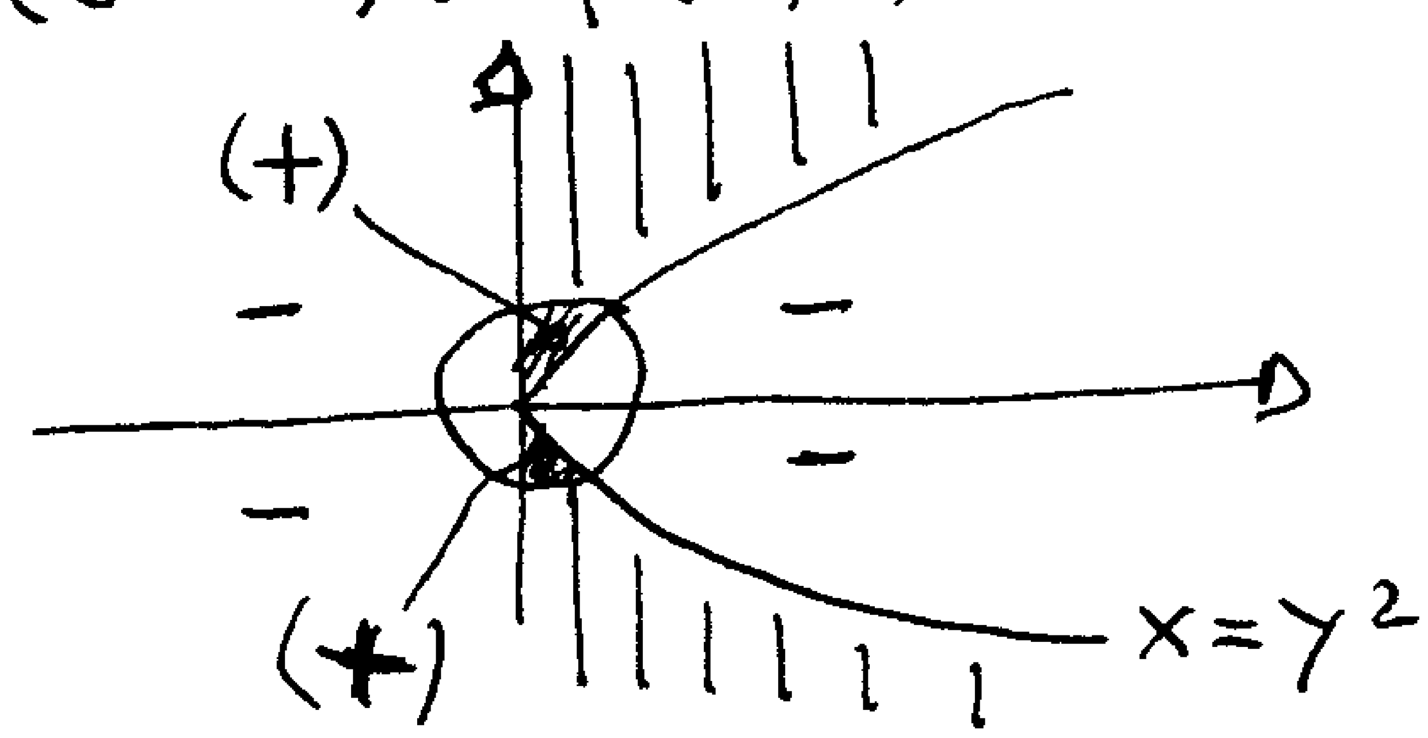
$\begin{cases} f'_x = y^2-2x = 0 \\ f'_y = 2xy - k = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{y^2}{2} \\ 2\cdot\frac{y^2}{2}\cdot y - k = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{y^2}{2} \\ y^3 = k \end{cases} \Rightarrow \begin{cases} x = \frac{\sqrt[3]{k^2}}{2} \\ y = \sqrt[3]{k} \end{cases}.$

$H = \left\| \begin{matrix} -2 & 2y \\ 2y & 2x \end{matrix} \right\|$; $H\left(\frac{\sqrt[3]{k^2}}{2}; \sqrt[3]{k}\right) = \left\| \begin{matrix} -2 & 2\sqrt[3]{k} \\ 2\sqrt[3]{k} & \sqrt[3]{k^2} \end{matrix} \right\|.$

Since $-2 < 0$ and $\sqrt[3]{k^2} > 0 \ \forall k \ne 0$: $\left(\frac{\sqrt[3]{k^2}}{2}; \sqrt[3]{k}\right)$ is a Saddle point.

For $k = 0 \Rightarrow f(x;y) = xy^2 - x^2 = x(y^2-x)$. $f(0;0) = 0$.

$f(x;y) \ge 0 = \underset{(1)}{\begin{cases} x > 0 \\ y^2 > x \end{cases}} \cup \underset{(2)}{\begin{cases} x < 0 \\ y^2 < x \end{cases}} \Rightarrow$



||| : (1)

(2): no solutions

For $k = 0 \Rightarrow (0;0)$ is a Saddle Point.