Task Mathematics for Economic Applications 12/11/2016 $\boxed{MfEA1}$

IM1) For $x=-3$: $-27+27-12+12=0 \Rightarrow$ RUFFINI:

$$\begin{array}{c|ccc|c} & 1 & 3 & 4 & 12 \\ -3 & & -3 & 0 & -12 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$\Rightarrow x^3+3x^2+4x+12=(x+3)(x^2+4)=0$. $x^2+4=0 \Rightarrow$

$\Rightarrow x^2=-4 \Rightarrow x=\pm 2i$. The three solutions are $x_1=-3$; $x_2=2i$; $x_3=-2i$.

IM2) $$A|Y=\left\|\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 2 & -1 & 0 & -1 & 2 \\ 1 & -1 & -1 & m & k \end{array}\right\| \xrightarrow[R_3-R_1]{R_2-2R_1} \left\|\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -2 & m+1 & k-1 \end{array}\right\| \xrightarrow[R_3-R_2]{} \left\|\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & m & k-1 \end{array}\right\| \Rightarrow$$

For $m\neq 0$ $\forall k\in\mathbb{R}$: $\text{RANK}(A)=3=\text{RANK}(A|Y) \Rightarrow \infty^{4-3}=\infty^1$ Solutions.
For $m=0$ and $k=1$: $\text{RANK}(A)=2=\text{RANK}(A|Y) \Rightarrow \infty^{4-2}=\infty^2$ Solutions.
For $m=0$ and $k\neq 1$: $\text{RANK}(A)=2<\text{RANK}(A|Y)=3 \Rightarrow$ No solutions.

IM3) $$X=\left\|\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{matrix}\right\|\cdot\left\|\begin{matrix} 1 \\ -1 \\ 2 \end{matrix}\right\|=\left\|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right\|\cdot\left\|\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\|=\left\|\begin{matrix} (1;0;1)\cdot(1;-1;2) \\ (0;1;1)\cdot(1;-1;2) \\ (1;1;0)\cdot(1;-1;2) \end{matrix}\right\|=\left\|\begin{matrix} 3 \\ 1 \\ 0 \end{matrix}\right\|=\left\|\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right\|=X_e.$$

IM4) $$\left\|\begin{matrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{matrix}\right\| \to \left|\begin{matrix} 2-\lambda & 2 & 1 \\ 0 & 1-\lambda & -1 \\ 1 & 2 & -\lambda \end{matrix}\right| = (2-\lambda)(\lambda^2-\lambda+2)+1\cdot(-2-1+\lambda)=$$

$=2\lambda^2-2\lambda+4-\lambda^3+\lambda^2-2\lambda+\lambda-3=0 \Rightarrow \lambda^3-3\lambda^2+3\lambda-1=(\lambda-1)^3=0 \Rightarrow$
$\Rightarrow \lambda_1=\lambda_2=\lambda_3=1$.

For $\lambda=1$: $\|A-1\cdot\mathbb{I}\|=\left\|\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & -1 \\ 1 & 2 & -1 \end{matrix}\right\|$. Since $\left|\begin{matrix} 2 & 1 \\ 0 & -1 \end{matrix}\right|=-2\neq 0$ $\text{RANK}(A-1\cdot\mathbb{I})=2 \Rightarrow$

$=m_1^g=3-2=1<3=m_1^a$. The matrix is not a diagonalizable one.

$|A|=\lambda_1\cdot\lambda_2\cdot\lambda_3=1$. $$\left\|\begin{matrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{matrix}\right\| \xrightarrow{Adj} \left\|\begin{matrix} 2 & -1 & -1 \\ 2 & -1 & -2 \\ -3 & 2 & 2 \end{matrix}\right\| \xrightarrow{T} \left\|\begin{matrix} 2 & 2 & -3 \\ -1 & -1 & 2 \\ -1 & -2 & 2 \end{matrix}\right\|=A^{-1}.$$

Since eigenvalues of A^{-1} are the reciprocal of A's eigenvalues: $\lambda_1=\lambda_2=\lambda_3=1$.

IIM1) $f(x;y)=xy-x^2+xy^2$. $\nabla f(x;y)=(0;0) \Rightarrow \begin{cases} f'_x=y-2x+y^2=0 \\ f'_y=x+2xy=x(1+2y)=0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x=0 \\ y+y^2=y(1+y)=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} x=0 \\ y=-1 \end{cases} \cup \begin{cases} -\frac{1}{2}-2x+\frac{1}{4}=0 \\ y=-\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x=-\frac{1}{8} \\ y=-\frac{1}{2} \end{cases}$: $(0;0)-(0;-1)-\left(-\frac{1}{8};-\frac{1}{2}\right)$.

$H(x;y)=\left\|\begin{matrix} -2 & 1+2y \\ 1+2y & 2x \end{matrix}\right\|$. $H(0;0)=\left\|\begin{matrix} -2 & 1 \\ 1 & 0 \end{matrix}\right\|$: $|H_2|=-1<0$: Saddle point. $H(0;-1)=\left\|\begin{matrix} -2 & -1 \\ -1 & 0 \end{matrix}\right\|$:

: $|H_2|=-1<0$ Saddle point. $H\left(-\frac{1}{8};-\frac{1}{2}\right)=\left\|\begin{matrix} -2 & 0 \\ 0 & -\frac{1}{4} \end{matrix}\right\|$: $\begin{cases} |H_1|=-2 \text{ or } |H_1|=-\frac{1}{4}<0 \\ |H_2|=\frac{1}{2}>0 \end{cases}$: Maximum Point.

II M2) $f(x;y) = 2x^3y - 3xy^2$ is a polynomial $\Rightarrow$ it is differentiable $\Rightarrow D_v f(P_0) = \nabla f(P_0) \cdot v$.

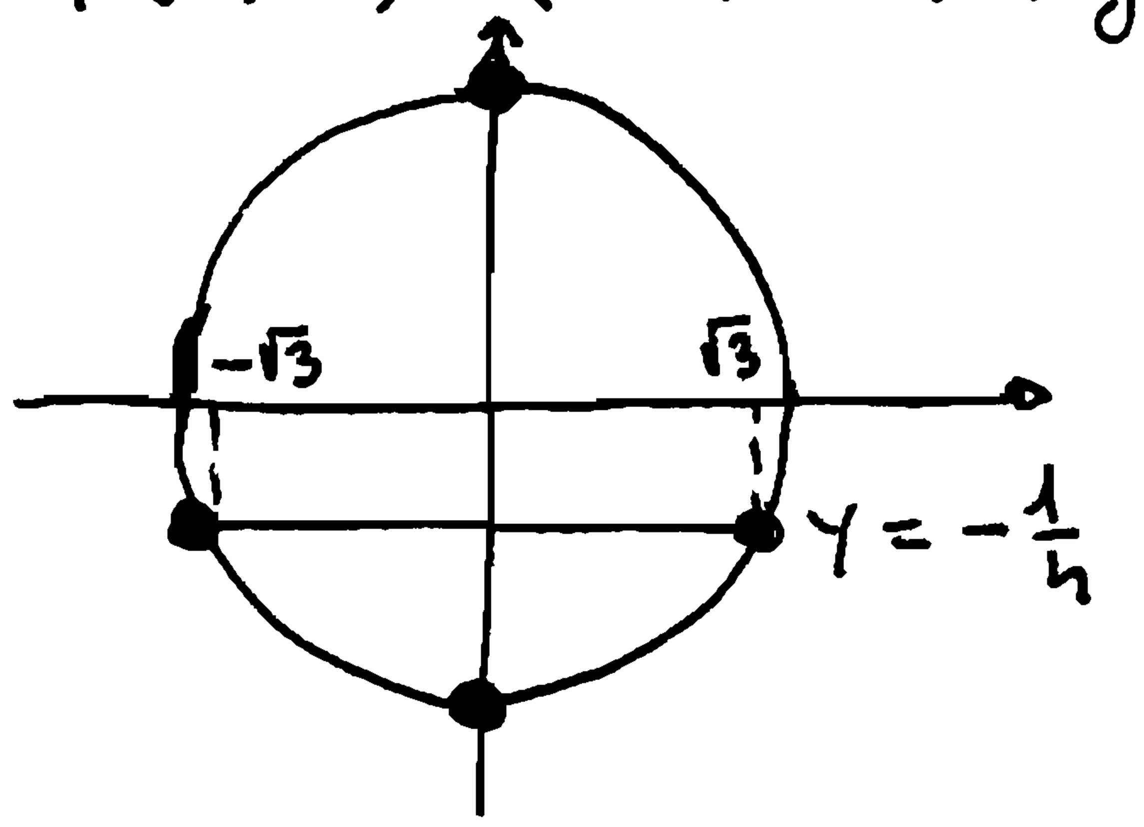
$v = \left(\frac{1}{\sqrt5}; -\frac{2}{\sqrt5}\right)$. $\nabla f(x;y) = \left(6x^2y - 3y^2; 2x^3 - 6xy\right)$; $\nabla f(1;-1) = (-9; 8)$.

$D_v f(1;-1) = (-9;8)\cdot\left(\frac{1}{\sqrt5}; -\frac{2}{\sqrt5}\right) = -\frac{9}{\sqrt5} - \frac{16}{\sqrt5} = -\frac{25}{\sqrt5} = -5\sqrt5$.

II M3) $f(x;y) = e^{x^2-y^2} + x^2 - y^2$. $f(3;-3) = e^0 + 9 - 9 = 1$.

$\nabla f(x;y) = \left(2xe^{x^2-y^2} + 2x; -2ye^{x^2-y^2} - 2y\right)$. $\nabla f(3;-3) = (12;12) \Rightarrow y'(3) = -\frac{12}{12} = -1$.

II M4) $\begin{cases} \text{Max/min } f(x;y) = x^2 - 2y \\ \text{u.e.: } x^2+y^2-4 \leq 0 \end{cases}$



$\Lambda = x^2 - 2y - \lambda(x^2+y^2-4)$.

For $\lambda = 0$: $\begin{cases} \Lambda'_x = 2x = 0 \\ \Lambda'_y = -2 \neq 0 \end{cases}$: NO solutions.

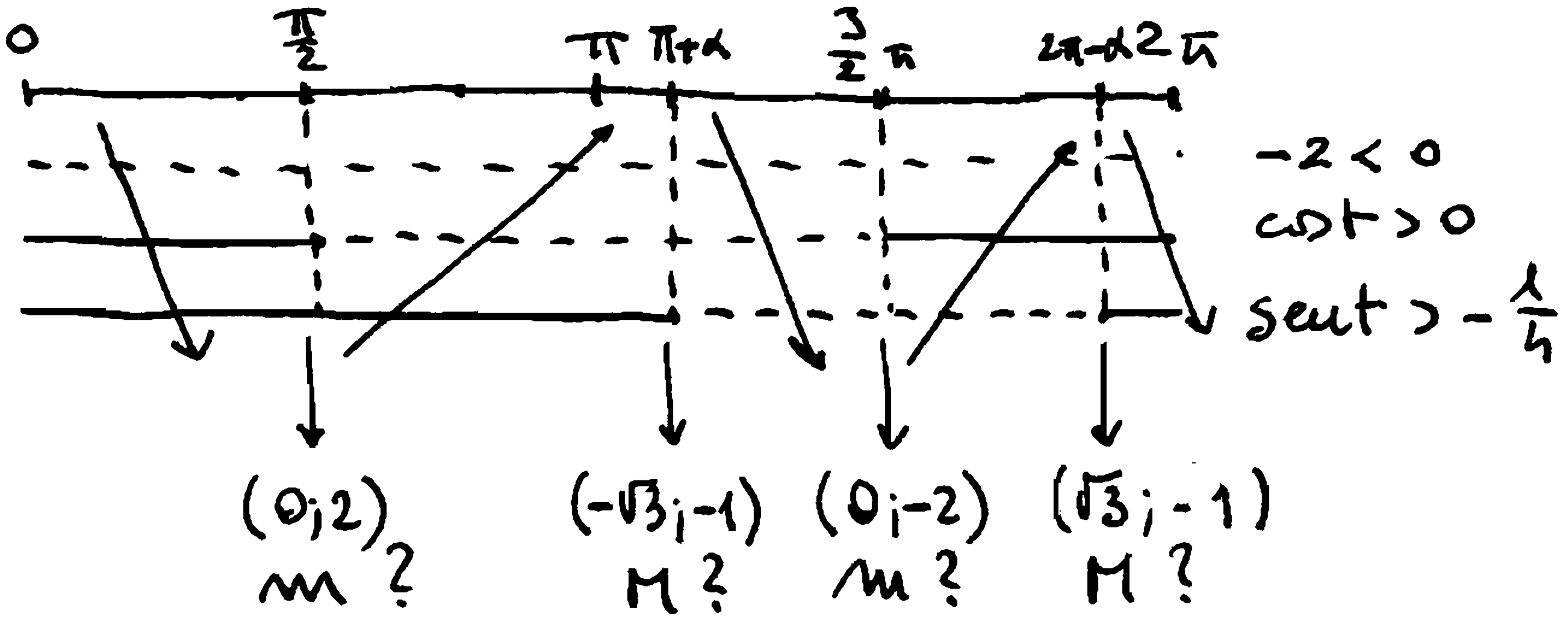
For $\lambda \neq 0$ $\begin{cases} \Lambda'_x = 2x - 2\lambda x = 2x(1-\lambda) = 0 \\ \Lambda'_y = -2 - 2\lambda y = -2(1+\lambda y) = 0 \\ x^2+y^2 = 4 \end{cases} \Rightarrow \begin{cases} x = 0 \\ \lambda = -\frac{1}{y} \\ y^2 = 4 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 2 \\ \lambda = -\frac12 \end{cases} \cup \begin{cases} x = 0 \\ y = -2 \\ \lambda = \frac12 \end{cases} \cup \begin{cases} \lambda = 1 \\ y = -1 \\ x^2 = 3 \end{cases} \Rightarrow \begin{cases} x = \sqrt3 \\ y = -1 \\ \lambda = 1 \end{cases} \cup \begin{cases} x = -\sqrt3 \\ y = -1 \\ \lambda = 1 \end{cases}$

min? Max? Max? Max?

We study the problem on the border $x^2+y^2 = 4$. If $\begin{cases} x = 2\cos t \\ y = 2\sen t \end{cases} \Rightarrow$

$\Rightarrow f(t) = 4\cos^2 t - 2\sen t \Rightarrow f'(t) = -8\sen t\cos t - 2\cos t = -2\cos t(4\sen t + 1) \geq 0$

$-2 < 0$; $\cos t > 0$ for $0 < t < \frac{\pi}{2}$ or $\frac32\pi < t < 2\pi$; $4\sen t + 1 > 0$ for $\sen t > -\frac14$.



So $(0;2)$ is the minimum point: $f(0;2) = -4$;

$(\sqrt3;-1)$ and $(-\sqrt3;-1)$ are two Maximum points: $f(\sqrt3;-1) = f(-\sqrt3;-1) = 5$;

$(0;-2)$ is nor a Maximum nor a minimum point.