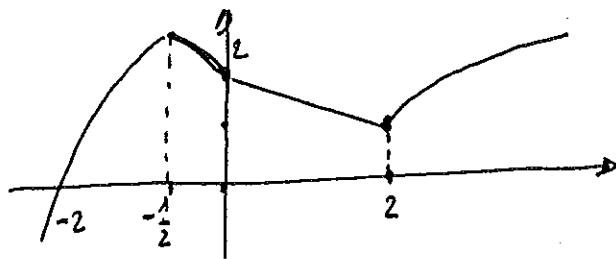


Piön Intermedia di Matematika Generale del 11/11/2013 Comp/ho A1

1) $F(x) = \frac{2x-1}{3x+3} = y \Rightarrow 2x-1 = 3xy+3y \Rightarrow x(2-3y) = 3y+1 \Rightarrow x = \frac{3y+1}{2-3y} \Rightarrow y = \frac{3x+1}{2-3x}$ è la funzione $f(g(x))$.

$f(g(x)) = 2^{g(x)} = \frac{3x+1}{2-3x} \Rightarrow g(x) = \log_2 \left(\frac{3x+1}{2-3x} \right)$.

2) $f(x) = \begin{cases} 2-x-x^2: x < 0 \\ \mu x + q: 0 \leq x \leq 2 \\ \log_3(x+1): 2 < x \end{cases}$; $\lim_{x \rightarrow 0^-} f(x) = 2$
 $\lim_{x \rightarrow 2^+} f(x) = 1$ \Rightarrow

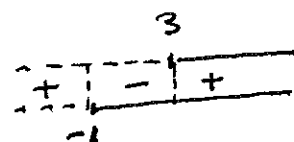


$\Rightarrow \begin{cases} x=0: q=2 \\ x=2: 2\mu+2=1 \Rightarrow \mu = -\frac{1}{2} \end{cases} \Rightarrow y = -\frac{1}{2}x+2$

3) $\lim_{x \rightarrow 0} \frac{1-\cos 3x}{x \cdot (2^x-1)} = \lim_{x \rightarrow 0} \frac{1-\cos 3x}{9x^2} \cdot 9 \cdot \frac{x}{2^x-1} = \frac{1}{2} \cdot 9 \cdot \frac{1}{\log 2} = \frac{9}{2} \log_2 e$.

$\lim_{x \rightarrow -\infty} \left(\frac{1+x^2}{2+x^4} \right)^{\left(\frac{1+x^2}{1-x} \right)} = \left(\rightarrow 0^+ \right)^{\left(\rightarrow +\infty \right)} = 0^+$.

4) $f(x) = \log \left(\frac{x-3}{x+1} \right)$. c.ε.: $\frac{x-3}{x+1} > 0 \begin{cases} x-3 > 0: x > 3 \\ x+1 > 0: x > -1 \end{cases}$



c.ε.: $]-\infty; -1[\cup]3; +\infty[$.

$\log \left(\frac{x-3}{x+1} \right) > 0 \Rightarrow \frac{x-3}{x+1} > 1 \Rightarrow \frac{x-3}{x+1} - 1 > 0 \Rightarrow \frac{x-3-x-1}{x+1} > 0 \Rightarrow \frac{-4}{x+1} > 0 \Rightarrow x+1 < 0 \Rightarrow x < -1$.

c.ε. $\frac{-4}{x+1} > 0$ \Rightarrow $\frac{-4}{x+1} > 0 \Rightarrow x < -1$.
 $f > 0$ \Rightarrow $f(x) > 0$ per $x < -1$.

5) $A B C \mid (B \vee C) \mid (A \Rightarrow (B \vee C)) \mid (A \vee C) \mid (B \Leftrightarrow (A \vee C)) \mid [A \Rightarrow (B \vee C)] \vee [B \Leftrightarrow (A \vee C)]$

* 1 1 1	1	1	1	1	1
1 1 0	1	1	0	0	1
* 1 0 1	1	1	1	0	1
1 0 0	0	0	0	1	1
0 1 1	1	1	0	0	1
* 0 1 0	1	1	0	0	1
0 0 1	1	1	0	1	1
* 0 0 0	0	1	0	1	1

Valgono solo le righe * dove A e C sono entrambe vere o/ma entrambe false.

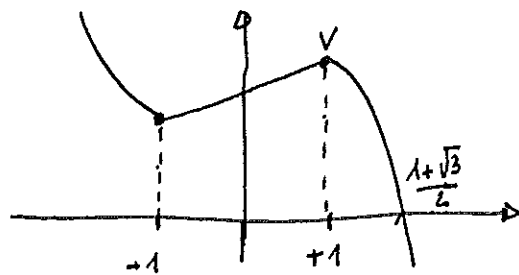
Prova Intermedia di Matematica Generale del 11/11/2013 Compito B1

1) $f(x) = \frac{x+2}{3x+4} = y \Rightarrow x+2 = 3xy+4y \Rightarrow x(1-3y) = 4y-2 \Rightarrow x = \frac{4y-2}{1-3y} \Rightarrow y = \frac{4x-2}{1-3x}$ è la funzione $f(g(x))$.

$f(g(x)) = 3^{g(x)} = \frac{4x-2}{1-3x} \Rightarrow g(x) = \log_3 \left(\frac{4x-2}{1-3x} \right)$.

2) $f(x) = \begin{cases} (\frac{1}{3})^x - 1 & : x < -1 \\ mx + q & : -1 \leq x \leq 1 \\ 2 + 2x - x^2 & : 1 < x \end{cases}$ $\lim_{x \rightarrow -1^-} f(x) = 3 - 1 = 2$
 $\lim_{x \rightarrow 1^+} f(x) = 4 - 1 = 3 \Rightarrow$

$\Rightarrow \begin{cases} x = -1: -m + q = 2 \\ x = 1: m + q = 3 \end{cases} \Rightarrow \begin{cases} m = \frac{1}{2} \\ q = \frac{5}{2} \end{cases} \Rightarrow y = \frac{1}{2}x + \frac{5}{2}$



3) $\lim_{x \rightarrow 0} \frac{\sin x \cdot (e^x - 1)}{1 - \cos 2x} = \lim_{x \rightarrow 0} \frac{4x^2}{1 - \cos 2x} \cdot \frac{1}{4} \cdot \frac{\sin x}{x} \cdot \frac{e^x - 1}{x} = 2 \cdot \frac{1}{4} \cdot 1 \cdot 1 = \frac{1}{2}$.

$\lim_{x \rightarrow -\infty} \left(\frac{1+x+x^3}{1+x} \right)^{\left(\frac{1+x}{1-x} \right)} = (-\infty + \infty)^{(-\infty - 1)} = 0^+$.

4) $f(x) = \log \left(\frac{x-2}{1-x} \right)$. C.E.: $\frac{x-2}{1-x} > 0 \begin{cases} x-2 > 0: x > 2 \\ 1-x > 0: x < 1 \end{cases}$

C.E.: $]1; 2[$.

$\log \left(\frac{x-2}{1-x} \right) > 0 \Rightarrow \frac{x-2}{1-x} > 1 \Rightarrow \frac{x-2}{1-x} - 1 > 0 \Rightarrow \frac{x-2-1+x}{1-x} = \frac{2x-3}{1-x} > 0 \begin{cases} 2x-3 > 0: x > \frac{3}{2} \\ 1-x > 0: x < 1 \end{cases}$

C.E. $\frac{1}{0} \frac{3}{2} \frac{2}{0}$
 $f > 0$ $0 \quad + \quad -$

$f(x) > 0$ per $1 < x < \frac{3}{2}$.

5) $ABC \mid (B \Rightarrow C) \mid [A \Rightarrow (B \Rightarrow C)] \mid (A \Rightarrow C) \mid [B \Leftrightarrow (A \Rightarrow C)] \mid [A \Rightarrow (B \Rightarrow C)] \mid e \mid [B \Leftrightarrow (A \Rightarrow C)]$

* 111	1	1	1	1	1	1
* 110	0	0	1	1	0	0
101	0	0	1	0	0	0
100	0	0	1	1	0	0
011	1	1	1	1	1	1
010	0	1	0	0	0	0
* 001	0	1	1	0	0	0
* 000	0	1	0	1	1	1

Valgono solo le righe * dove A e B sono entrambe vere oppure entrambe false.