Prova Intermedia di Matematica Generale del 11/11/2013 Compito A2

1) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-2^x}{x} = \lim\limits_{x\to 0} \dfrac{(1+x)^{\frac12}-1}{x} - \dfrac{2^x-1}{x} = \dfrac12 - \log 2.$

$\lim\limits_{x\to +\infty} \left(\dfrac{1+x^2}{1+2x}\right)^{\left(\frac{1+x}{1-x}\right)} = (\to +\infty)^{(\to -1)} = 0^+.$

2) $f(x) = \log_2 x;\ g(x) = 3+2x;\ h(x) = 3^x.$

$f(g(h(x))) = f(g(3^x)) = f(3+2\cdot 3^x) = \log_2(3+2\cdot 3^x) = y \Rightarrow 3+2\cdot 3^x = 2^y \Rightarrow 2\cdot 3^x = 2^y - 3 \Rightarrow$

$\Rightarrow 3^x = \frac12 (2^y-3) \Rightarrow x = \log_3\left(\frac12(2^y-3)\right) \Rightarrow$ Inversa: $y = \log_3\left(\frac12(2^x-3)\right).$

3) $f(x) = 3^{2x-1} - 2^{3x+1} > 0 \Rightarrow 3^{2x-1} > 2^{3x+1} \Rightarrow 3^{2x}\cdot\frac13 > 2^{3x}\cdot 2 \Rightarrow$

$\Rightarrow \frac13\cdot 9^x > 2\cdot 8^x \Rightarrow \frac{9^x}{8^x} > 6 \Rightarrow \left(\frac98\right)^x > 6 \Rightarrow x > \log_{\frac98} 6.$

4) $(3-x)\cdot\log(x^2-6x+9) = (3-x)\log(x-3)^2.$ C.E.: $x \neq 3$

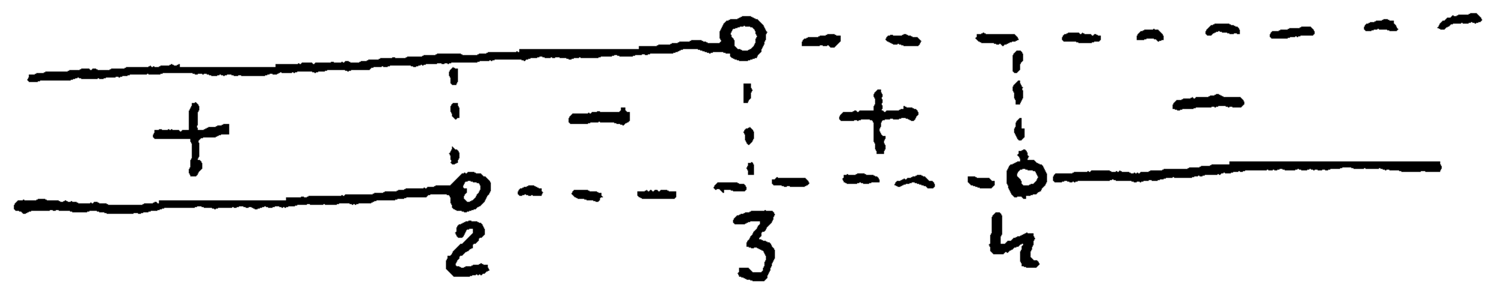
$(3-x)\log(x^2-6x+9) > 0 <$ $3-x>0:\ x<3$

$\log(x^2-6x+9)>0 \Rightarrow x^2-6x+9>1 \Rightarrow x^2-6x+8>0 \Rightarrow (x-2)(x-4)>0$

$x<2$ oppure $x>4$.

$3-x>0$

$\log(x^2-6x+9)>0$:



Soluzione: $]-\infty;2[\ \cup\ ]3;4[.$

5)

ABC	$(B \Leftrightarrow C)$	$[A \Rightarrow (B \Leftrightarrow C)]$	non A	(non A e B e C)	P o Q
~~111~~	~~1~~	~~1~~	~~0~~	~~0~~	~~1~~
~~110~~	~~0~~	0	0	0	0
* 101	0	0	0	0	0
* 100	1	1	0	0	1
* 011	1	1	1	1	1
* 010	0	1	1	~~0~~	~~1~~
~~001~~	~~0~~	~~1~~	~~1~~	0	~~1~~
~~000~~	~~1~~	~~1~~	~~1~~	0	1

Valgono solo le righe * dove A e B sono una vera e l'altra falsa o viceversa.

Prova Intermedia di Matematica Generale del 11/11/2013 Compito B2

1) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\cos x}{x}=\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{2}}-1}{x}+\dfrac{1-\cos x}{x}=\dfrac{1}{2}+0=\dfrac{1}{2}.$

$\lim\limits_{x\to +\infty}\left(\dfrac{2x+x^2}{1+x}\right)^{\left(\frac{1+x^2}{1-x}\right)}=(\to +\infty)^{(\to -\infty)}=0^+.$

2) $f(x)=2^x;\ g(x)=2+3x;\ h(x)=\log_3 x.$

$f(g(h(x)))=f(g(\log_3 x))=f(2+3\cdot\log_3 x)=2^{2+3\cdot\log_3 x}=y \Rightarrow 2+3\cdot\log_3 x=\log_2 y \Rightarrow$

$\Rightarrow 3\log_3 x=\log_2 y-2 \Rightarrow \log_3 x=\frac{1}{3}(\log_2 y-2)\Rightarrow x=3^{\frac{1}{3}(\log_2 y-2)}.$

Inversa: $y=3^{\frac{1}{3}(\log_2 x-2)}.$

3) $f(x)=4^{x-2}-3^{2x+1}>0 \Rightarrow \dfrac{4^x}{4^2}>3^{2x}\cdot 3 \Rightarrow \dfrac{4^x}{9^x}>3\cdot 16 \Rightarrow \left(\dfrac{4}{9}\right)^x>48 \Rightarrow$

$\Rightarrow x>\log_{\frac{4}{9}}48.$

4) $(x-2)\cdot\log(x^2-4x+4)=(x-2)\cdot\log(x-2)^2.$ C.E.: $x\neq 2.$

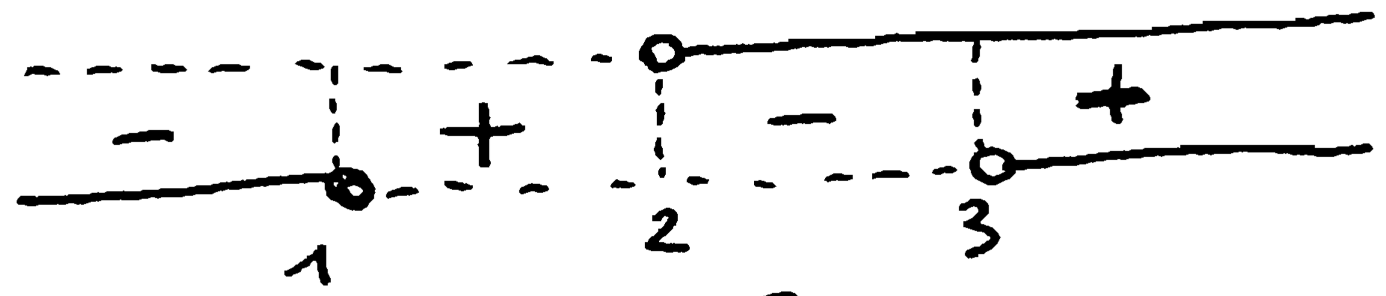
$(x-2)\cdot\log(x^2-4x+4)>0 <$ $x-2>0:\ x>2$

$\log(x^2-4x+4)>0 \Rightarrow x^2-4x+4>1 \Rightarrow x^2-4x+3>0 \Rightarrow (x-1)(x-3)>0.$

$x<1$ oppure $x>3$

$x-2>0:$

$\log(x^2-4x+4)>0:$



Soluzione: $]1;2[\ \cup\ ]3;+\infty[.$

5)

A B C	non B	(A e non B e C)	(B ⇒ C)	[A ⇔ (B ⇒ C)]	P o Q
~~1 1 1~~	0	0	1	1	1
* 1 1 0	0	0	0	0	0
* 1 0 1	1	1	1	1	1
~~1 0 0~~	1	0	1	1	1
~~0 1 1~~	0	0	1	0	0
* 0 1 0	0	0	0	1	1
* 0 0 1	1	0	1	0	0
~~0 0 0~~	1	0	1	0	0

Valgono solo le righe * dove B e C sono una vera e l'altra falsa o viceversa.

Prova Intermedia di Matematica Generale del 11/11/2013 Compito C2

1) $\lim_{x\to 0} \dfrac{\frac{1}{1+x} - e^x}{x} = \lim_{x\to 0} \dfrac{(1+x)^{-1}-1}{x} - \dfrac{e^x-1}{x} = -1-1=-2.$

$\lim_{x\to+\infty}\left(\dfrac{x^2-2x+3}{x^3+1}\right)^{\left(\frac{1+x}{1-x}\right)} = \left(\to 0^+\right)^{(\to -1)} = +\infty.$

2) $f(x)=\log_2 x;\ g(x)=1+3^x;\ h(x)=2x+1.$

$f(g(h(x)))=f(g(2x+1))=f(1+3^{2x+1})=\log_2(1+3^{2x+1})=y \Rightarrow 1+3^{2x+1}=2^y \Rightarrow$

$\Rightarrow 3^{2x+1}=2^y-1 \Rightarrow 2x+1=\log_3(2^y-1) \Rightarrow 2x=\log_3(2^y-1)-1 \Rightarrow$

$\Rightarrow x=\frac{1}{2}\left(\log_3(2^y-1)-1\right)$. Inversa: $y=\frac{1}{2}\left(\log_3(2^x-1)-1\right).$

3) $f(x)=2^{3x-1}-3^{2x+2}>0 \Rightarrow 2^{3x}\cdot 2^{-1}>3^{2x}\cdot 3^2 \Rightarrow \frac{1}{2}\cdot 8^x>9\cdot 9^x \Rightarrow \frac{9^x}{8^x}<\frac{1}{18} \Rightarrow$

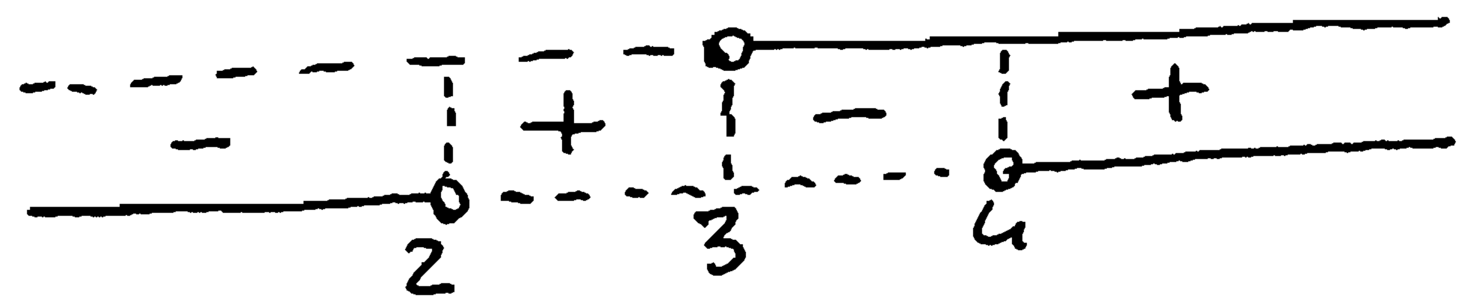
$\Rightarrow \left(\frac{9}{8}\right)^x<\frac{1}{18} \Rightarrow x<\log_{\frac{9}{8}}\frac{1}{18}=-\log_{\frac{9}{8}}18.$

4) $(x-3)\cdot\log(x^2-6x+9)=(x-3)\log(x-3)^2.$ C.E.: $x\neq 3.$

$(x-3)\log(x^2-6x+9)>0$
- $x-3>0:\ x>3$
- $\log(x^2-6x+9)>0 \Rightarrow x^2-6x+9>1 \Rightarrow x^2-6x+8>0 \Rightarrow (x-2)(x-4)>0.$
  $x<2$ oppure $x>4$

$x-3>0$:

$\log(x^2-6x+9)>0$:



Soluzione: $]2;3[\ \cup\ ]4;+\infty[$.

5)

	A B C	(B⇒C)	[A⟺(B⇒C)]	non B	(A o non B o C)	P e Q
	~~1 1 1~~	1	1	0	1	1
	~~1 1 0~~	0	0	0	1	0
*	1 0 1	1	1	1	1	1
*	1 0 0	1	1	1	1	1
*	0 1 1	1	0	0	1	0
*	0 1 0	0	1	0	0	0
	~~0 0 1~~	1	0	1	1	0
	~~0 0 0~~	1	0	1	1	0

Valgono solo le righe * dove A e B sono una vera e l'altra falsa o viceversa.

Prova Intermedia di Matematica Generale del 11/11/2013 Compito D2

1) $\lim_{x\to 0}\dfrac{\frac{1}{1+x}-\cos x}{x}=\lim_{x\to 0}\dfrac{(1+x)^{-1}-1}{x}+\dfrac{1-\cos x}{x}=-1+0=-1.$

$\lim_{x\to +\infty}\left(\dfrac{x+x^2}{x^3+1}\right)^{\left(\frac{1+x^2}{1+x}\right)}=\left(\to 0^+\right)^{(\to+\infty)}=0^+$

2) $f(x)=3^x$; $g(x)=1-\log_2 x$; $h(x)=2x-3.$

$f(g(h(x)))=f(g(2x-3))=f(1-\log_2(2x-3))=3^{1-\log_2(2x-3)}=y\Rightarrow$

$\Rightarrow 1-\log_2(2x-3)=\log_3 y\Rightarrow \log_2(2x-3)=1-\log_3 y\Rightarrow 2x-3=2^{1-\log_3 y}\Rightarrow 2x=2^{1-\log_3 y}+3\Rightarrow$

$\Rightarrow x=\frac{1}{2}\left(2^{1-\log_3 y}+3\right)$. Inversa : $y=\frac{1}{2}\left(2^{1-\log_3 x}+3\right).$

3) $f(x)=8^{x-1}-3^{2x+1}>0\Rightarrow 8^x\cdot 8^{-1}>3^{2x}\cdot 3\Rightarrow \dfrac{8^x}{8}>3\cdot 9^x\Rightarrow$

$\Rightarrow \dfrac{9^x}{8^x}<\dfrac{1}{24}\Rightarrow\left(\dfrac{9}{8}\right)^x<\dfrac{1}{24}\Rightarrow x<\log_{\frac{9}{8}}\dfrac{1}{24}=-\log_{\frac{9}{8}}24.$

4) $(2-x)\log(x^2-4x+4)=(2-x)\cdot\log(x-2)^2$. C.E. : $x\neq 2$

$(2-x)\cdot\log(x^2-4x+4)>0$
- $2-x>0$: $x<2$
- $\log(x^2-4x+4)>0\Rightarrow x^2-4x+4>1\Rightarrow x^2-4x+3>0\Rightarrow(x-1)(x-3)>0.$ $x<1$ oppure $x>3$

	1		2		3	
$2-x>0$						
$\log(x^2-4x+4)>0$						
segno	$+$		$-$	$+$		$-$

Soluzione: $]-\infty;1[\,\cup\,]2;3[$.

5)

A B C	(non C)	(A o B o non C)	(B<=>C)	[A=>(B<=>C)]	P e Q
~~1 1 1~~	0	1	1	1	1
* 1 1 0	1	1	0	0	0
~~1 0 1~~	0	1	0	0	0
* 1 0 0	1	1	1	1	1
* 0 1 1	0	1	1	1	1
~~0 1 0~~	1	1	0	1	1
* 0 0 1	0	0	0	1	0
~~0 0 0~~	1	1	1	1	1

Valgono solo le righe * dove A e C sono una vera e l'altra falsa o viceversa.