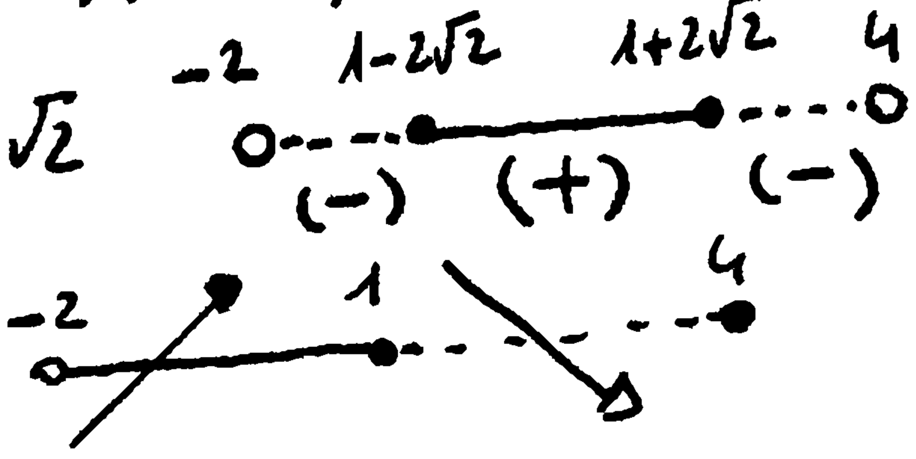
Compito di Matematica Generale del 27/1/14 Compito A $\boxed{\text{MGA1}}$

1) $f(x)=\log(x+2)+\log(4-x)$. C.E.: $\begin{cases} x+2>0: & x>-2 \\ 4-x>0: & x<4 \end{cases} \Rightarrow$ C.E. $=]-2;4[$.

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to 4^-} f(x) = -\infty$. $f(x)>0$: $\log\big((x+2)(4-x)\big)>0 \Rightarrow (x+2)(4-x)>1 \Leftrightarrow$

$\Rightarrow -x^2+2x+7>0 \Rightarrow x^2-2x-7<0$: $x=1\pm\sqrt{1+7}=1\pm2\sqrt{2}$

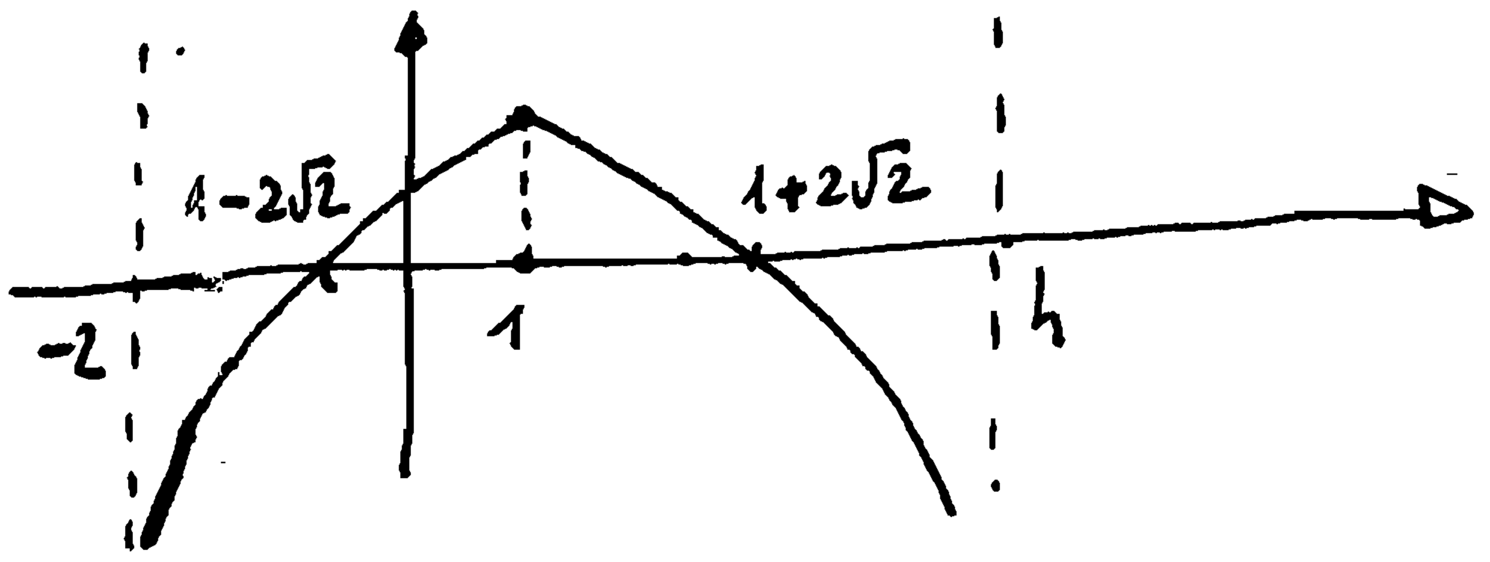


$f'(x)=\frac{1}{x+2}-\frac{1}{4-x}=\frac{4-x-x-2}{(x+2)(4-x)}=\frac{2-2x}{(x+2)(4-x)}\geq 0 \Rightarrow x\leq 1$

$f''(x)=-\frac{1}{(x+2)^2}-\left(-\frac{1}{(4-x)^2}\cdot(-1)\right)<0\ \forall x$: funzione sempre concava

$f(1)=\log 3+\log 3=2\log 3=\log 9$

Grafico:



2) $\lim\limits_{x\to 0}\frac{1-\cos 3x}{\operatorname{sen} 4x^2}=\lim\limits_{x\to 0}\frac{1-\cos 3x}{9x^2}\cdot 9\cdot\frac{1}{4}\cdot\frac{4x^2}{\operatorname{sen} 4x^2}=\frac{1}{2}\cdot\frac{9}{4}=\frac{9}{8}$.

$\lim\limits_{x\to +\infty}\left(\frac{1+x-\operatorname{sen}x}{3+2x}\right)^{1-x}=\lim\limits_{x\to +\infty}\left(\frac{1+x}{3+2x}\right)^{1-x}=\left(\to\frac{1}{2}\right)^{\to(-\infty)}=+\infty$.

3) Rette tangenti parallele se $f'(x_0)=g'(x_0) \Rightarrow$

$\Rightarrow 6x^2-2x=6x+2 \Rightarrow 6x^2-8x-2=2(3x^2-4x-1)=0 \Rightarrow x=\frac{2\pm\sqrt{4+3}}{3}=\begin{cases}\frac{2+\sqrt{7}}{3}\\ \frac{2-\sqrt{7}}{3}\end{cases}$.

Ci sono due punti in cui le tangenti sono parallele.

4) $\lim\limits_{x\to 0}\frac{K^{2x}-1}{3x}=\lim\limits_{x\to 0}\frac{K^{2x}-1}{2x}\cdot\frac{2}{3}=\frac{2}{3}\log K=5 \Rightarrow \log K=\frac{15}{2} \Rightarrow K=e^{\frac{15}{2}}=\sqrt{e^{15}}=e^7\cdot\sqrt{e}$.

5) $f(x)=e^{3x}-ke^{2x}$; $f'(x)=3e^{3x}-2ke^{2x}=e^{2x}(3e^x-2k)\geq 0$ se $3e^x\geq 2k \Rightarrow$

$\Rightarrow e^x\geq\frac{2}{3}k \Rightarrow x\geq\log\frac{2}{3}k=1 \Rightarrow \frac{2}{3}k=e \Rightarrow k=\frac{3}{2}e$.

6)

ABC	non A	(B e C)	(non A ⇒ (B e C))	[(non A ⇒ (B e C)] e non B	{[non A ⇒ (B e C)] e non B} ⇒ A
111	0	1	1	0	1
110	0	0	1	0	1
101	0	0	1	1	1
100	0	0	1	1	1
011	1	1	1	0	1
010	1	0	0	0	1
~~001~~	~~1~~	~~0~~	~~0~~	~~0~~	~~1~~
~~000~~	~~1~~	~~0~~	~~0~~	~~0~~	~~1~~

Vengono tolte la $\overline{\text{VII}}$ e $\overline{\text{VIII}}$ riga dove la proposizione (A o B) è falsa, contro l'ipotesi.

$$7)\ A\cdot X = \begin{Vmatrix} 1 & 2 \\ 3 & 4 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ x \end{Vmatrix} = \begin{Vmatrix} 1+2x \\ 3+4x \end{Vmatrix}.$$

MGA2

a) $A\cdot X \parallel (2;3) \Rightarrow \dfrac{1+2x}{2} = \dfrac{3+4x}{3} \Rightarrow 3+6x = 6+8x \Rightarrow 2x=-3 \Rightarrow x = -\dfrac{3}{2}.$

b) $A\cdot X \perp (3;2) \Rightarrow (1+2x)\cdot 3 + (3+4x)\cdot 2 = 3+6x+6+8x = 14x+9 = 0 \Rightarrow x = -\dfrac{9}{14}.$

c) Da $X\cdot Y = \|X\|\cdot\|Y\|\cdot\cos\alpha \Rightarrow$ $\left(\cos 45^\circ = \dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow (1+2x;\ 3+4x)\cdot(1;1) = \sqrt{(1+2x)^2+(3+4x)^2}\cdot\sqrt{1+1}\cdot\dfrac{1}{\sqrt{2}} \Rightarrow$

$\Rightarrow 4+6x = \sqrt{1+4x^2+4x+9+16x^2+24x} \Rightarrow (4+6x)^2 = 20x^2+28x+10 \Rightarrow$

$\Rightarrow 36x^2+48x+16 = 20x^2+28x+10 \Rightarrow 16x^2+20x+6 = 2(8x^2+10x+3) = 0 \Rightarrow$

$\Rightarrow x = \dfrac{-5\pm\sqrt{25-24}}{8} = \dfrac{-5\pm 1}{8} = \begin{cases} -\frac{3}{4} \\ -\frac{1}{2} \end{cases} \Rightarrow x = -\dfrac{3}{4}$ oppure $x = -\dfrac{1}{2}.$

8) $\displaystyle\int_0^1 (x-2)e^x + \sqrt{x+1}\,dx \Rightarrow \int (x-2)e^x dx + \int\sqrt{x+1}\,dx = (x-2)e^x - \int e^x dx + \frac{1}{1+\frac{1}{2}}(1+x)^{1+\frac{1}{2}} \Rightarrow$

$\Rightarrow \displaystyle (x-2)e^x - e^x + \frac{2}{3}\sqrt{(1+x)^3}\Big|_0^1 = \left(-e-e+\frac{2}{3}\cdot 2\sqrt{2}\right) - \left(-2-1+\frac{2}{3}\right) = -2e+\frac{4}{3}\sqrt{2}+\frac{7}{3}.$

9) $f(x;y) = 3x^2 - x^3 + 3xy + y^2 - y.$

$\begin{cases} f'_x = 6x-3x^2+3y = 3(2x-x^2+y) = 0 \\ f'_y = 3x+2y-1 = 0 \end{cases} \Rightarrow \begin{cases} 6x-3x^2+\frac{3}{2}-\frac{9}{2}x = 0 \\ y = \frac{1}{2}-\frac{3}{2}x \end{cases} \Rightarrow \begin{cases} 6x^2-3x-3 = 3(2x^2-x-1) = 0 \\ y = \frac{1}{2}-\frac{3}{2}x \end{cases}$

$2x^2-x-1 = 0 \Rightarrow x = \dfrac{1\pm\sqrt{1+8}}{4} = \dfrac{1\pm 3}{4} \Rightarrow \begin{cases} x = 1 \\ y = -1 \end{cases}$ e $\begin{cases} x = -\frac{1}{2} \\ y = \frac{5}{4} \end{cases}.$

$H(x;y) = \begin{Vmatrix} 6-6x & 3 \\ 3 & 2 \end{Vmatrix};\ H(1;-1) = \begin{Vmatrix} 0 & 3 \\ 3 & 2 \end{Vmatrix}$: Sella ; $H\left(-\frac{1}{2};\frac{5}{4}\right) = \begin{Vmatrix} 9 & 3 \\ 3 & 2 \end{Vmatrix}$: Minimo.

10) $f(x;y) = x^2\cdot 2^y - 3x\cdot \operatorname{sen} 2y + x.$

$\nabla f(x;y) = (f'_x;f'_y) = (2x\cdot 2^y - 3\operatorname{sen}2y + 1;\ x^2\cdot 2^y\log 2 - 6x\cos 2y)$

$\nabla f(0;0) = (0\cdot 1 - 3\cdot 0 + 1;\ 0\cdot 1\cdot\log 2 - 0\cdot 1) = (1;0)$

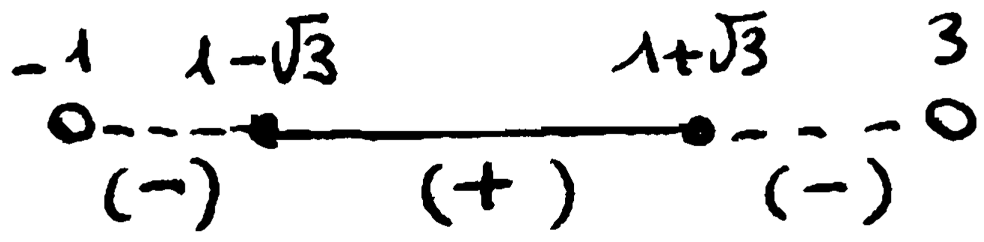
$$H(x;y) = \begin{Vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{Vmatrix} = \begin{Vmatrix} 2\cdot 2^y + 0 + 0 & 2x\cdot 2^y\cdot\log 2 - 6\cos 2y \\ 2x\cdot 2^y\cdot\log 2 - 6\cos 2y & x^2\cdot 2^y\cdot\log^2 2 + 12x\operatorname{sen}2y \end{Vmatrix}$$

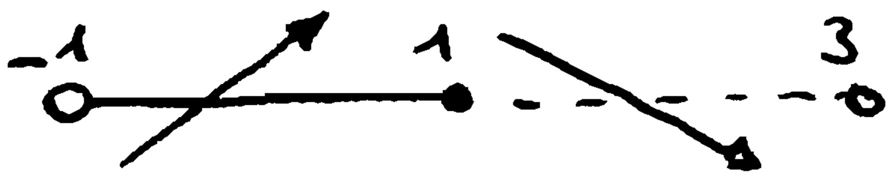
$$H(0;0) = \begin{Vmatrix} 2 & -6 \\ -6 & 0 \end{Vmatrix}.$$

Compito di Matematica Generale del 27/1/14 Compito B $\boxed{\text{MGB1}}$

1) $f(x) = \log(x+1) + \log(3-x)$. C.E.: $\begin{cases} x+1>0 : x>-1 \\ 3-x>0 : x<3 \end{cases} \Rightarrow$ C.E. $=]-1;3[$.

$\lim_{x \to -1^+} f(x) = \lim_{x \to 3^-} f(x) = -\infty$. $f(x)>0$: $\log((x+1)(3-x))>0 \Rightarrow (x+1)(3-x)>1 \Rightarrow$

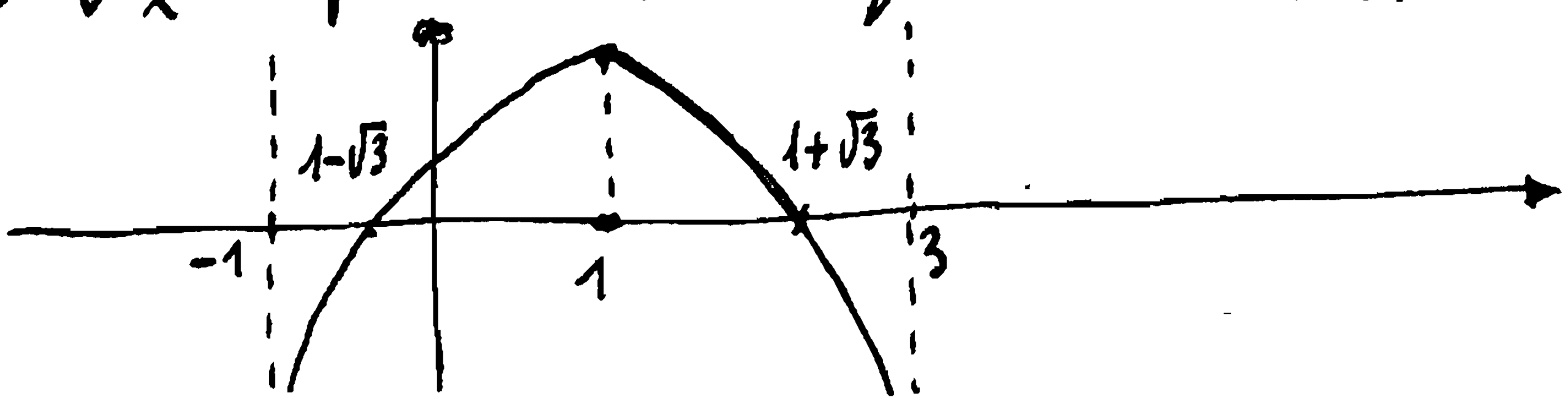
$\Rightarrow -x^2+2x+2>0 \Rightarrow x^2-2x-2<0$: $x = 1 \pm \sqrt{1+2} = 1 \pm \sqrt{3}$ 

$f'(x) = \frac{1}{x+1} - \frac{1}{3-x} = \frac{3-x-x-1}{(x+1)(3-x)} = \frac{2-2x}{(x+1)(3-x)} \geq 0 \Rightarrow x \leq 1$ 

$f''(x) = -\frac{1}{(x+1)^2} - \left(-\frac{1}{(3-x)^2}\right)\cdot(-1) < 0 \ \forall x$: funzione sempre concava.

$f(1) = \log 2 + \log 2 = 2\log 2 = \log 4$.

Grafico:



2) $\lim_{x \to 0} \frac{\log(1+3x^2)}{1-\cos 3x} = \lim_{x \to 0} \frac{\log(1+3x^2)}{3x^2} \cdot \frac{3x^2}{9x^2} \cdot \frac{9x^2}{1-\cos 3x} = 1 \cdot \frac{3}{9} \cdot \frac{1}{\frac{1}{2}} = \frac{2}{3}$.

$\lim_{x \to +\infty} \left(\frac{1+2x-\log x}{2+x}\right)^{x-5} = \lim_{x \to +\infty} \left(\frac{1+2x}{2+x}\right)^{x-5} = (\to 2)^{\to(+\infty)} = +\infty$.

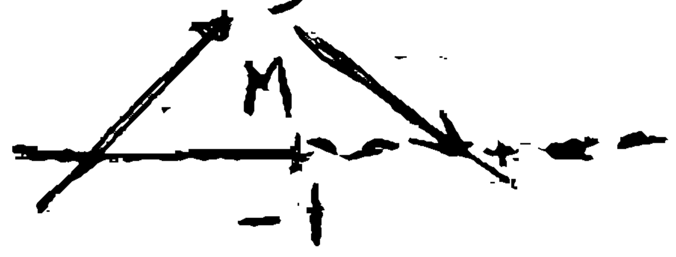
3) Rette tangenti parallele se $f'(x_0) = g'(x_0) \Rightarrow$

$\Rightarrow 3x^2 - 6x = 4x+3 \Rightarrow 3x^2 - 10x - 3 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25+9}}{3} = \frac{5 \pm \sqrt{34}}{3} < \begin{matrix} \frac{5+\sqrt{34}}{3} \\ \frac{5-\sqrt{34}}{3} \end{matrix}$.

Ci sono due punti in cui le tangenti sono parallele.

4) $\lim_{x \to 0} \frac{k^{3x}-1}{2x} = \lim_{x \to 0} \frac{k^{3x}-1}{3x} \cdot \frac{3}{2} = \frac{3}{2}\log k = 3 \Rightarrow \log k = 2 \Rightarrow k = e^2$.

5) $f(x) = e^x - ke^{3x}$; $f'(x) = e^x - 3ke^{3x} = e^x(1-3ke^{2x}) \geq 0$ per $3ke^{2x} \leq 1 \Rightarrow e^{2x} \leq \frac{1}{3k} \Rightarrow$

$\Rightarrow 2x \leq \log\frac{1}{3k} \Rightarrow x \leq \frac{1}{2}\log\frac{1}{3k} = -1 \Rightarrow \log\frac{1}{3k} = -2 \Rightarrow \frac{1}{3k} = e^{-2} \Rightarrow 3k = e^2 \Rightarrow k = \frac{e^2}{3}$ 

6)

A B C	non B	(A e C)	(non B ⇒ (A e C))	[(non B ⇒ (A e C)] e non C	{[(non B ⇒ (A e C)] e non C} ⇒ B
~~1 1 1~~	~~0~~	~~1~~	~~1~~	~~0~~	~~1~~
1 1 0	0	0	1	1	1
1 0 1	1	1	1	0	1
1 0 0	1	0	0	0	1
~~0 1 1~~	~~0~~	~~0~~	~~1~~	~~0~~	~~1~~
0 1 0	0	0	1	1	1
0 0 1	1	0	0	0	1
0 0 0	1	0	0	0	1

Vengono tolte la I e V riga dove la proposizione (B e C) è vera, contro l'ipotesi.

MGB2

7) $A \cdot X = \begin{Vmatrix} 1 & 2 \\ 1 & 3 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ x \end{Vmatrix} = \begin{Vmatrix} 1+2x \\ 1+3x \end{Vmatrix}$.

a) $A \cdot X \parallel (1;3) \Rightarrow \frac{1+2x}{1} = \frac{1+3x}{3} \Rightarrow 3+6x = 1+3x \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$.

b) $A \cdot X \perp (2;2) \Rightarrow (1+2x)\cdot 2 + (1+3x)\cdot 2 = 2+4x+2+6x = 10x+4 = 0 \Rightarrow x = -\frac{2}{5}$.

c) Da $X \cdot Y = \|X\| \cdot \|Y\| \cdot \cos\alpha \Rightarrow \quad (\cos 45° = \frac{1}{\sqrt{2}})$

$\Rightarrow (1+2x; 1+3x)\cdot(1;1) = \sqrt{(1+2x)^2+(1+3x)^2} \cdot \sqrt{1+1} \cdot \frac{1}{\sqrt{2}} \Rightarrow$

$\Rightarrow 2+5x = \sqrt{1+4x^2+4x+1+9x^2+6x} \Rightarrow (2+5x)^2 = 13x^2+10x+2 \Rightarrow 25x^2+20x+4 = 13x^2+10x+2 \Rightarrow$

$\Rightarrow 12x^2+10x+2 = 2(6x^2+5x+1) = 0 \Rightarrow x = \frac{-5 \pm \sqrt{25-24}}{12} = \frac{-5 \pm 1}{12} < \begin{matrix} -\frac{1}{2} \\ -\frac{1}{3} \end{matrix} \Rightarrow x = -\frac{1}{2}$ oppure $x = -\frac{1}{3}$.

8) $\int_0^1 (x+2)e^x - \frac{1}{x+1}\,dx \Rightarrow \int (x+2)e^x dx - \int \frac{1}{x+1} dx = (x+2)e^x - \int e^x dx - \log(x+1) \Rightarrow$

$\Rightarrow \left((x+2)e^x - e^x - \log(x+1)\right|_0^1 = (3\cdot e - e - \log 2) - (2-1-0) = 2e - \log 2 - 1$.

9) $f(x;y) = 3y^2 - y^3 + 3xy + x^2 - x$.

$\begin{cases} f'_x = 3y + 2x - 1 = 0 \\ f'_y = 6y - 3y^2 + 3x = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} - \frac{3}{2}y \\ 6y - 3y^2 + \frac{3}{2} - \frac{9}{2}y = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} - \frac{3}{2}y \\ 6y^2 - 3y - 3 = 3(2y^2 - y - 1) = 0 \end{cases} \Rightarrow$

$\Rightarrow 2y^2 - y - 1 = 0 \Rightarrow y = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \Rightarrow \begin{cases} x = -1 \\ y = 1 \end{cases}$ e $\begin{cases} x = \frac{5}{4} \\ y = -\frac{1}{2} \end{cases}$.

$H(x;y) = \begin{Vmatrix} 2 & 3 \\ 3 & 6-6y \end{Vmatrix}$; $H(-1;1) = \begin{Vmatrix} 2 & 3 \\ 3 & 0 \end{Vmatrix}$: Sella ; $H(\frac{5}{4}; -\frac{1}{2}) = \begin{Vmatrix} 2 & 3 \\ 3 & 9 \end{Vmatrix}$: Minimo.

10) $f(x;y) = 2y \cdot \cos 3x + 3^x \cdot y^3 - y$.

$\nabla f(x;y) = (f'_x; f'_y) = (-6y \operatorname{sen} 3x + 3^x \cdot \log 3 \cdot y^3 - 0 ; 2\cos 3x + 3 \cdot 3^x \cdot y^2 - 1)$.

$\nabla f(0;0) = (0 + 1 \cdot \log 3 \cdot 0 ; 2 \cdot 1 + 3 \cdot 1 \cdot 0 - 1) = (0;1)$.

$H(x;y) = \begin{Vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{Vmatrix} = \begin{Vmatrix} -18y\cos 3x + 3^x \cdot \log^2 3 \cdot y^3 & -6 \operatorname{sen} 3x + 3^x \cdot \log 3 \cdot 3y^2 \\ -6 \operatorname{sen} 3x + 3^x \log 3 \cdot 3y^2 & 0 + 3 \cdot 3^x \cdot 2y + 0 \end{Vmatrix}$

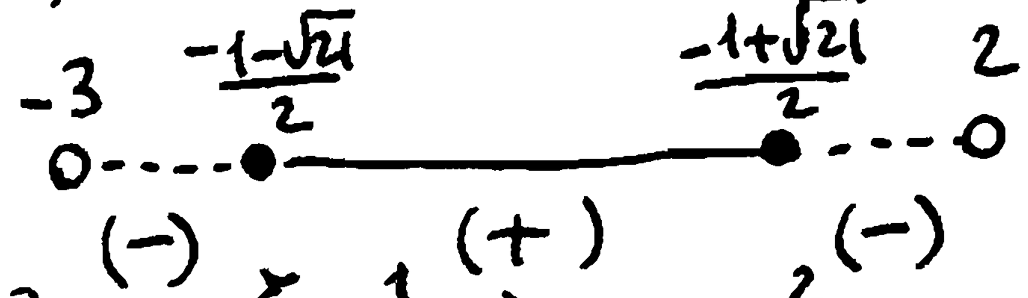
$H(0;0) = \begin{Vmatrix} 0 & 0 \\ 0 & 0 \end{Vmatrix}$.

Compito di Matematica Generale del 27/1/14 Compito C $\boxed{MGC1}$

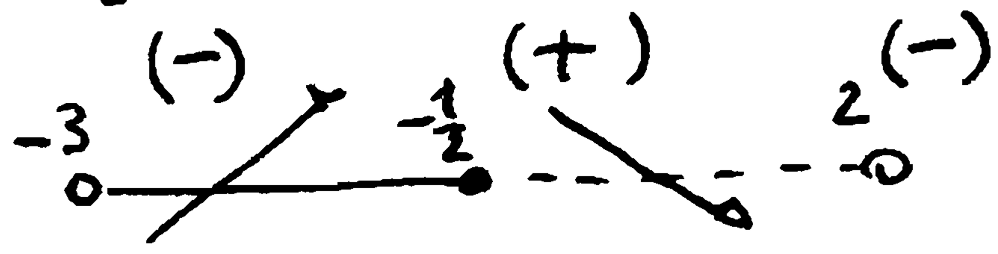
1) $f(x)=\log(x+3)+\log(2-x)$. C.E.: $\begin{cases} x+3>0 : x>-3 \\ 2-x>0 : x<2 \end{cases} \Rightarrow$ C.E. $=\,]-3;2[$.

$\lim_{x\to -3^+} f(x) = \lim_{x\to 2^-} f(x) = -\infty$. $f(x)>0$: $\log\left((x+3)(2-x)\right)>0 \Rightarrow (x+3)(2-x)>1 \Rightarrow$

$\Rightarrow -x^2-x+5>0 \Rightarrow x^2+x-5<0 : x=\dfrac{-1\pm\sqrt{1+20}}{2}=\dfrac{-1\pm\sqrt{21}}{2}$



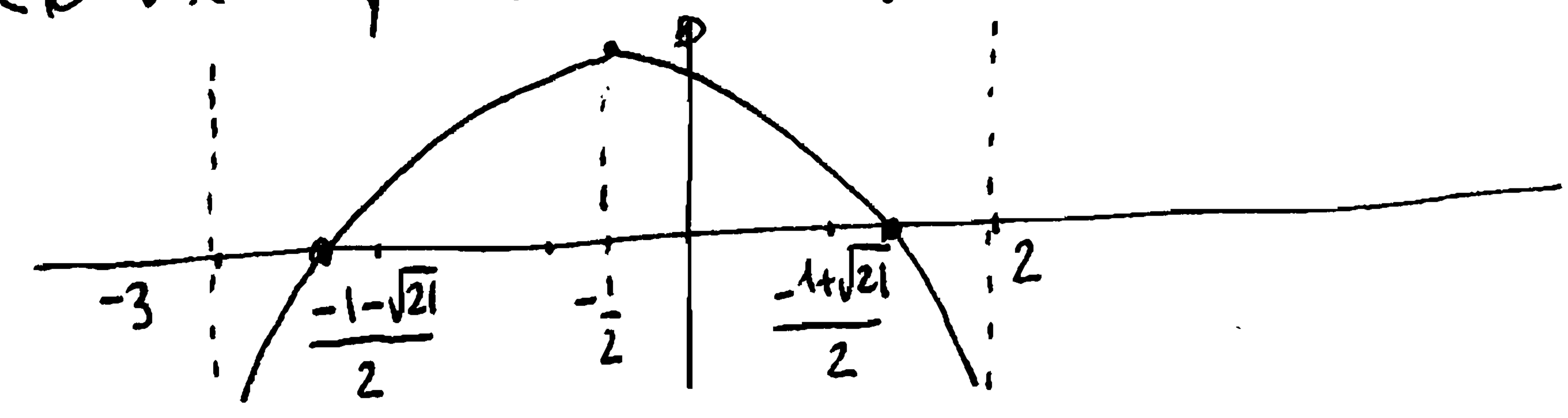
$f'(x)=\dfrac{1}{x+3}-\dfrac{1}{2-x}=\dfrac{2-x-x-3}{(x+3)(2-x)}=\dfrac{-2x-1}{(x+3)(2-x)}\geq 0 \Rightarrow x\leq -\dfrac{1}{2}$



$f''(x)=-\dfrac{1}{(x+3)^2}-\left(-\dfrac{1}{(2-x)^2}\right)\cdot(-1)<0\ \forall x$: funzione sempre concava

$f\left(-\frac{1}{2}\right)=\log\frac{5}{2}+\log\frac{5}{2}=\log\frac{25}{4}$.

Grafico:



2) $\lim_{x\to 0}\dfrac{1-\cos 4x}{\operatorname{tg}^2 2x}=\lim_{x\to 0}\dfrac{1-\cos 4x}{16x^2}\cdot\dfrac{16x^2}{4x^2}\cdot\dfrac{4x^2}{\operatorname{tg}^2 2x}=\dfrac{1}{2}\cdot 4\cdot 1=2.$

$\lim_{x\to +\infty}\left(\dfrac{x-5+\cos x}{1+3x}\right)^{1+x}=\lim_{x\to+\infty}\left(\dfrac{x-5}{3x+1}\right)^{1+x}=\left(\to\dfrac{1}{3}\right)^{\to+\infty}=0^+.$

3) Rette tangenti parallele se $f'(x_0)=g'(x_0) \Rightarrow$

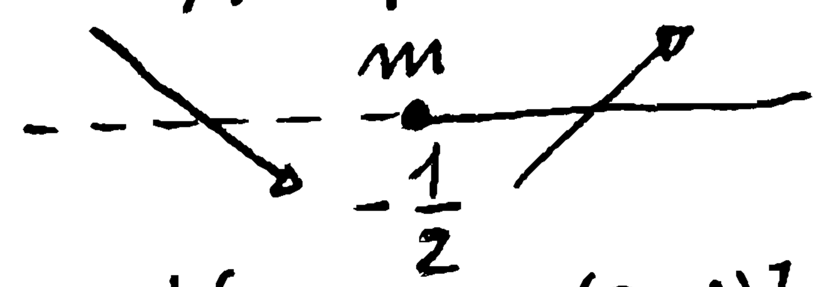
$\Rightarrow 2x+5=6x^2+6x \Rightarrow 6x^2+4x-5=0 \Rightarrow x=\dfrac{-2\pm\sqrt{4+30}}{6}=\dfrac{-2\pm\sqrt{34}}{6}=\begin{cases}\dfrac{-2+\sqrt{34}}{6}\\ \dfrac{-2-\sqrt{34}}{6}\end{cases}$.

Ci sono due punti in cui le tangenti sono parallele.

4) $\lim_{x\to 0}\dfrac{k^{2x}-1}{x}=\lim_{x\to 0}\dfrac{k^{2x}-1}{2x}\cdot 2=2\log k=\dfrac{1}{3}\Rightarrow \log k=\dfrac{1}{6}\Rightarrow k=e^{\frac{1}{6}}=\sqrt[6]{e}.$

5) $f(x)=e^{2x}-ke^x$; $f'(x)=2e^{2x}-ke^x=e^x(2e^x-k)\geq 0$ per $2e^x\geq k \Rightarrow e^x\geq\dfrac{k}{2}\Rightarrow$

$\Rightarrow x\geq\log\dfrac{k}{2}=-\dfrac{1}{2}\Rightarrow \dfrac{k}{2}=e^{-\frac{1}{2}}=\dfrac{1}{\sqrt{e}}\Rightarrow k=\dfrac{2}{\sqrt{e}}$.



6)

A B C	non C	(B e A)	(non C ⇒ (B e A))	[non C ⇒ (B e A)] e non A	{[non C ⇒ (B e A)] e non A} ⇒ C
1 1 1	0	1	1	0	1
1 1 0	1	1	1	0	1
1 0 1	0	0	1	0	1
1 0 0	1	0	0	0	1
0 1 1	0	0	1	1	1
~~0 1 0~~	~~1~~	~~0~~	~~0~~	~~0~~	~~1~~
0 0 1	0	0	1	1	1
~~0 0 0~~	~~1~~	~~0~~	~~0~~	~~0~~	~~1~~

Vengono tolte le $\overline{\text{VI}}$ e $\overline{\text{VIII}}$ riga dove la proposizione (A o C) è falsa, contro l'ipotesi.

MGC2

7) $A \cdot X = \begin{Vmatrix} 4 & 3 \\ 2 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ 1 \end{Vmatrix} = \begin{Vmatrix} 4x+3 \\ 2x+1 \end{Vmatrix}$.

a) $A \cdot X \parallel (2;-1) \Rightarrow \frac{4x+3}{2} = \frac{2x+1}{-1} \Rightarrow -4x-3 = 4x+2 \Rightarrow 8x = -5 \Rightarrow x = -\frac{5}{8}$.

b) $A \cdot X \perp (2;2) \Rightarrow (4x+3)\cdot 2 + (2x+1)\cdot 2 = 8x+6+4x+2 = 12x+8 = 0 \Rightarrow x = -\frac{8}{12} = -\frac{2}{3}$.

c) Da $X \cdot Y = \|X\| \cdot \|Y\| \cdot \cos\alpha \Rightarrow (\cos 45° = \frac{1}{\sqrt{2}})$

$\Rightarrow (4x+3;\, 2x+1)\cdot(1;1) = \sqrt{(4x+3)^2 + (2x+1)^2} \cdot \sqrt{1+1} \cdot \frac{1}{\sqrt{2}} \Rightarrow$

$\Rightarrow 6x+4 = \sqrt{16x^2+9+24x+4x^2+1+4x} \Rightarrow (6x+4)^2 = 20x^2+28x+10 \Rightarrow$

$\Rightarrow 36x^2+16+48x = 20x^2+28x+10 \Rightarrow 16x^2+20x+6 = 2(8x^2+10x+3) = 0 \Rightarrow$

$\Rightarrow x = \frac{-5 \pm \sqrt{25-24}}{8} = \frac{-5 \pm 1}{8} = \left\langle \begin{matrix} -\frac{3}{4} \\ -\frac{1}{2} \end{matrix} \right. \Rightarrow x = -\frac{3}{4}$ oppure $x = -\frac{1}{2}$.

8) $\int_0^1 (x-1)e^x + \sqrt{x+2}\,dx \Rightarrow \int (x-1)e^x dx + \int (x+2)^{\frac{1}{2}} dx = (x-1)e^x - \int e^x dx + \frac{1}{1+\frac{1}{2}} (x+2)^{1+\frac{1}{2}} \Rightarrow$

$\Rightarrow \left((x-1)e^x - e^x + \frac{2}{3}\sqrt{(x+2)^3}\right|_0^1 = \left(0\cdot e - e + \frac{2}{3}\sqrt{27}\right) - \left(-1\cdot 1 - 1 + \frac{2}{3}\sqrt{8}\right) = -e + 2\sqrt{3} + 2 - \frac{4}{3}\sqrt{2}$.

9) $f(x;y) = x^3 - 3x^2 - 3xy - y^2 + y$.

$\begin{cases} f'_x = 3x^2 - 6x - 3y = 0 \\ f'_y = -3x - 2y + 1 = 0 \end{cases} \Rightarrow \begin{cases} 3x^2 - 6x + \frac{9}{2}x - \frac{3}{2} = 0 \\ y = -\frac{3}{2}x + \frac{1}{2} \end{cases} \Rightarrow \begin{cases} 6x^2 - 3x - 3 = 3(2x^2 - x - 1) = 0 \\ y = -\frac{3}{2}x + \frac{1}{2} \end{cases} \Rightarrow$

$\Rightarrow 2x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \Rightarrow \begin{cases} x = 1 \\ y = -1 \end{cases}$ e $\begin{cases} x = -\frac{1}{2} \\ y = \frac{5}{4} \end{cases}$.

$H(x;y) = \begin{Vmatrix} 6x-6 & -3 \\ -3 & -2 \end{Vmatrix}$; $H(1;-1) = \begin{Vmatrix} 0 & -3 \\ -3 & -2 \end{Vmatrix}$: Sella. $H(-\frac{1}{2};\frac{5}{4}) = \begin{Vmatrix} -9 & -3 \\ -3 & -2 \end{Vmatrix}$: Massimo.

10) $f(x;y) = x^3 \cdot 3^y - 2y \cdot \operatorname{sen} 3x + y$.

$\nabla f(x;y) = (f'_x ; f'_y) = (3x^2 \cdot 3^y - 6y \cos 3x + 0 ; x^3 \cdot 3^y \cdot \log 3 - 2 \cdot \operatorname{sen} 3x + 1)$

$\nabla f(0;0) = (0 - 0 + 0 ; 0 - 2\cdot 0 + 1) = (0 ; 1)$.

$H(x;y) = \begin{Vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{Vmatrix} = \begin{Vmatrix} 6x \cdot 3^y + 18y \operatorname{sen} 3x & 3x^2 \cdot 3^y \log 3 - 6\cos 3x \\ 3x^2 \cdot 3^y \cdot \log 3 - 6\cos 3x & x^3 \cdot 3^y \log^2 3 - 0 \end{Vmatrix}$

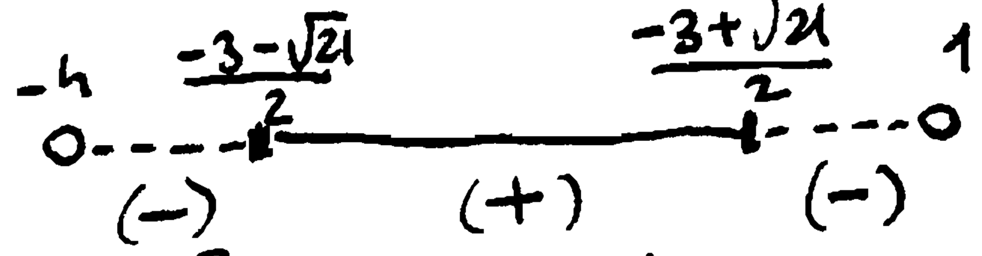
$H(0;0) = \begin{Vmatrix} 0 & -6 \\ -6 & 0 \end{Vmatrix}$.

Compito di Matematica Generale del 27/1/14 Compito D $\boxed{MGD1}$

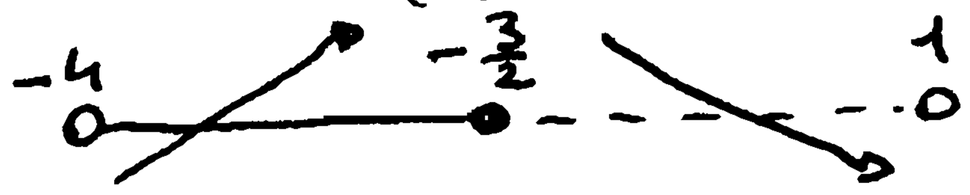
1) $f(x)=\log(x+4)+\log(1-x)$. C.E.: $\begin{cases} x+4>0 : x>-4 \\ 1-x>0 : x<1 \end{cases} \Rightarrow C.E. =]-4;1[$.

$\lim_{x\to -4^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$. $f(x)>0$: $\log\left((x+4)(1-x)\right)>0 \Rightarrow (x+4)(1-x)>1 \Rightarrow$

$\Rightarrow -x^2-3x+3>0 \Rightarrow x^2+3x-3<0 : x=\frac{-3\pm\sqrt{9+12}}{2}=\frac{-3\pm\sqrt{21}}{2}$



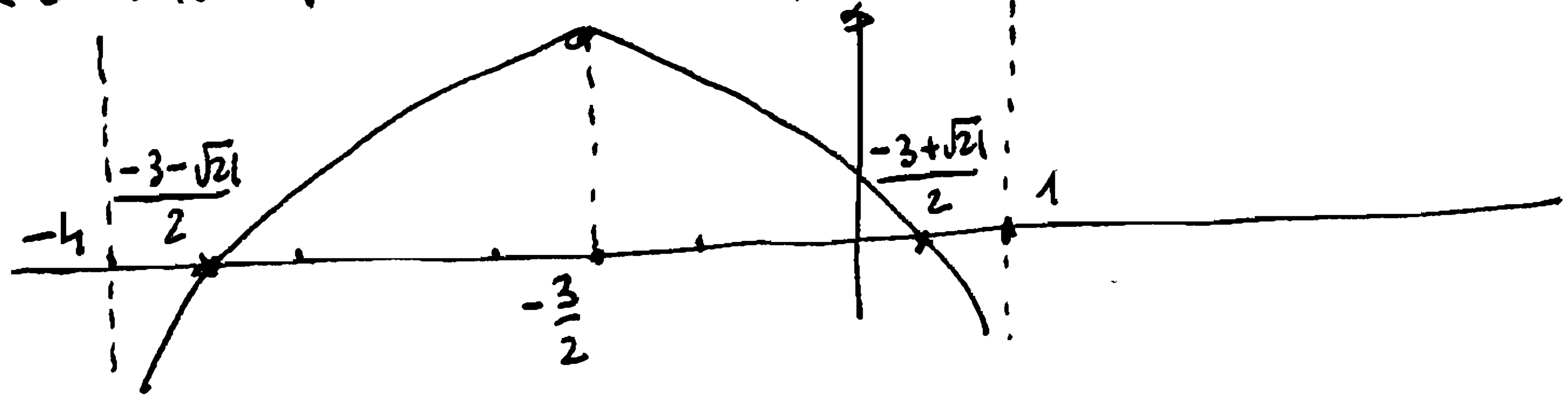
$f'(x)=\frac{1}{x+4}-\frac{1}{1-x}=\frac{1-x-x-4}{(x+4)(1-x)}=\frac{-2x-3}{(x+4)(1-x)}\geq 0 \Rightarrow x\leq -\frac{3}{2}$



$f''(x)=-\frac{1}{(x+4)^2}-\left(-\frac{1}{(1-x)^2}\right)\cdot(-1)<0 \ \forall x$: funzione sempre concava.

$f\left(-\frac{3}{2}\right)=\log\frac{5}{2}+\log\frac{5}{2}=\log\frac{25}{4}$

Grafico:



2) $\lim_{x\to 0}\frac{\log(1+4x^2)}{1-\cos 5x}=\lim_{x\to 0}\frac{\log(1+4x^2)}{4x^2}\cdot\frac{4x^2}{25x^2}\cdot\frac{25x^2}{1-\cos 5x}=1\cdot\frac{4}{25}\cdot\frac{1}{\frac{1}{2}}=\frac{8}{25}$.

$\lim_{x\to+\infty}\left(\frac{\log x+2x+1}{x-2}\right)^{5-x}=\lim_{x\to+\infty}\left(\frac{2x+1}{x-2}\right)^{5-x}=\left(\to 2\right)^{\to-\infty}=0^+$.

3) Rette tangenti parallele se $f'(x_0)=g'(x_0) \Rightarrow$

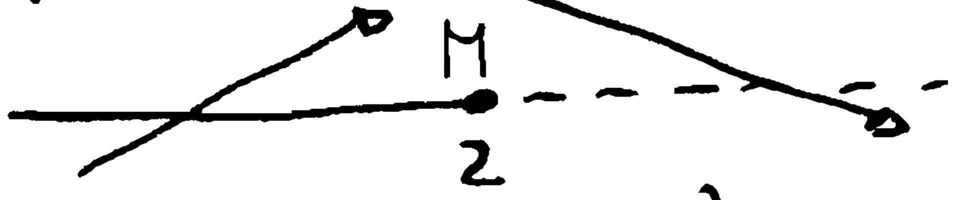
$\Rightarrow 6x+1=3x^2+4x \Rightarrow 3x^2-2x-1=0 \Rightarrow x=\frac{1\pm\sqrt{1+3}}{3}=\frac{1\pm 2}{3} \begin{cases} 1 \\ -\frac{1}{3} \end{cases}$.

Ci sono due punti in cui le tangenti sono parallele.

4) $\lim_{x\to 0}\frac{k^{3x}-1}{4x}=\lim_{x\to 0}\frac{k^{3x}-1}{3x}\cdot\frac{3}{4}=\frac{3}{4}\log k=\frac{1}{3} \Rightarrow \log k=\frac{4}{9} \Rightarrow k=e^{\frac{4}{9}}=\sqrt[9]{e^4}$.

5) $f(x)=ke^x-e^{3x}$; $f'(x)=ke^x-3e^{3x}=e^x(k-3e^{2x})\geq 0 \Rightarrow 3e^{2x}\leq k \Rightarrow 2x\leq\log\frac{k}{3} \Rightarrow$

$\Rightarrow x\leq\frac{1}{2}\log\frac{k}{3}=2 \Rightarrow \log\frac{k}{3}=4 \Rightarrow \frac{k}{3}=e^4 \Rightarrow k=3\cdot e^4$.



6)

A B C	(non C)	(non C e B)	(A ⇒ (non C e B))	[A ⇒ (non C e B)] e C	{[A ⇒ (non C e B)] e C} ⇒ non A
~~1 1 1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~
1 1 0	1	1	1	0	1
~~1 0 1~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~
1 0 0	1	0	0	0	1
0 1 1	0	0	1	1	1
0 1 0	1	1	1	0	1
0 0 1	0	0	1	1	1
0 0 0	1	0	1	0	1

Vengono tolte la I e III riga dove la proposizione (A e C) è vera, contro l'ipotesi.

MGD2

7) $A\cdot X = \begin{Vmatrix} 2 & 1 \\ 3 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ 1 \end{Vmatrix} = \begin{Vmatrix} 2x+1 \\ 3x+1 \end{Vmatrix}$.

a) $A\cdot X \parallel (-2;3) \Rightarrow \frac{2x+1}{-2} = \frac{3x+1}{3} \Rightarrow 6x+3 = -6x-2 \Rightarrow 12x = -5 \Rightarrow x = -\frac{5}{12}$.

b) $A\cdot X \perp (1;-2) \Rightarrow (2x+1)(1)+(3x+1)(-2) = 2x+1-6x-2 = -4x-1 = 0 \Rightarrow x = -\frac{1}{4}$.

c) Da $X\cdot Y = \|X\|\cdot\|Y\|\cdot\cos\alpha \Rightarrow \quad (\cos 45° = \frac{1}{\sqrt{2}})$

$\Rightarrow (2x+1;3x+1)\cdot(1;1) = \sqrt{(2x+1)^2+(3x+1)^2}\cdot\sqrt{1+1}\cdot\frac{1}{\sqrt{2}} \Rightarrow$

$\Rightarrow 5x+2 = \sqrt{4x^2+1+4x+9x^2+1+6x} \Rightarrow (5x+2)^2 = 13x^2+10x+2 \Rightarrow$

$\Rightarrow 25x^2+20x+4 = 13x^2+10x+2 \Rightarrow 12x^2+10x+2 = 2(6x^2+5x+1) = 0 \Rightarrow$

$\Rightarrow x = \frac{-5\pm\sqrt{25-24}}{12} = \frac{-5\pm1}{12} = \begin{cases} -\frac{1}{2} \\ -\frac{1}{3} \end{cases} \Rightarrow x = -\frac{1}{2}$ oppure $x = -\frac{1}{3}$.

8) $\int_0^1 (x+1)e^x + \frac{1}{x+2}\,dx \Rightarrow \int (x+1)e^x\,dx + \int \frac{1}{x+2}\,dx = (x+1)e^x - \int e^x dx + \log(x+2) \Rightarrow$

$\Rightarrow \left((x+1)e^x - e^x + \log(x+2)\right)\Big|_0^1 = (2e - e + \log 3) - (1-1+\log 2) = e + \log 3 - \log 2 = e + \log\frac{3}{2}$.

9) $f(x;y) = y^3 - 3y^2 - 3xy - x^2 + x$.

$\begin{cases} f'_x = -3y-2x+1 = 0 \\ f'_y = 3y^2-6y-3x = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{3}{2}y+\frac{1}{2} \\ 3y^2-6y+\frac{9}{2}y-\frac{3}{2} = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{3}{2}y+\frac{1}{2} \\ 6y^2-3y-3 = 3(2y^2-y-1) = 0 \end{cases} \Rightarrow$

$\Rightarrow 2y^2-y-1 = 0 \Rightarrow y = \frac{1\pm\sqrt{1+8}}{4} = \frac{1\pm3}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=1 \end{cases}$ e $\begin{cases} x=\frac{5}{4} \\ y=-\frac{1}{2} \end{cases}$.

$H(x;y) = \begin{Vmatrix} -2 & -3 \\ -3 & 6y-6 \end{Vmatrix}$. $H(-1;1) = \begin{Vmatrix} -2 & -3 \\ -3 & 0 \end{Vmatrix}$: Sella. $H(\frac{5}{4};-\frac{1}{2}) = \begin{Vmatrix} -2 & -3 \\ -3 & -9 \end{Vmatrix}$: Massimo.

10) $f(x;y) = 3x\cdot\cos 2y + 2^x\cdot y^2 - x$

$\nabla f(x;y) = (f'_x; f'_y) = (3\cos 2y + 2^x\log 2\cdot y^2 - 1; -6x\,\text{sen}\,2y + 2\cdot 2^x\cdot y - 0)$

$\nabla f(0;0) = (3\cdot1 + 1\cdot\log 2\cdot 0 - 1; -0 + 2\cdot1\cdot0 - 0) = (2;0)$.

$H(x;y) = \begin{Vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{Vmatrix} = \begin{Vmatrix} 0 + 2^x\log^2 2\cdot y^2 & -6\,\text{sen}\,2y + 2^x\cdot\log 2\cdot 2y \\ -6\,\text{sen}\,2y + 2\cdot 2^x\log 2\cdot y & -12x\cos 2y + 2\cdot 2^x \end{Vmatrix}$

$H(0;0) = \begin{Vmatrix} 0 & 0 \\ 0 & 2 \end{Vmatrix}$