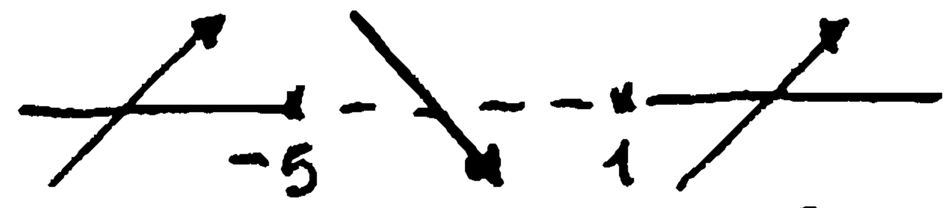
Compito di Matematica Generale del 17/2/2014 Compito A $\boxed{MGA1}$

1) $f(x)=\dfrac{x^2+5}{x+2}$. C.E.: $x\neq-2$. $\lim_{x\to-\infty} f(x)=-\infty$; $\lim_{x\to-2^-} f(x)=-\infty$; $\lim_{x\to 2^+} f(x)=+\infty$, $\lim_{x\to+\infty} f(x)=+\infty$.

$f(x)>0: x+2>0 \Rightarrow x>-2$. $f(0)=\frac{5}{2}$.

$f'(x)=\dfrac{2x(x+2)-1\cdot(x^2+5)}{(x+2)^2}=\dfrac{x^2+4x-5}{(x+2)^2}=\dfrac{(x-1)(x+5)}{(x+2)^2}\geq 0$

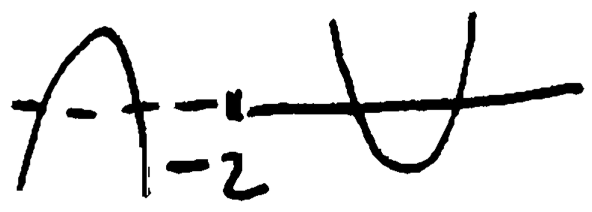
$\Rightarrow (x-1)(x+5)\geq 0 \Rightarrow x\leq -5 \cup x\geq 1$



$f''(x)=\dfrac{(2x+4)(x+2)^2-2(x+2)(x^2+4x-5)}{(x+2)^4}=$

$=\dfrac{2(x+2)}{(x+2)^4}(\cancel{x^2}+\cancel{4x}+4-\cancel{x^2}-\cancel{4x}+5)=\dfrac{18}{(x+2)^3}>0$

per $x+2>0 \Rightarrow x>-2$



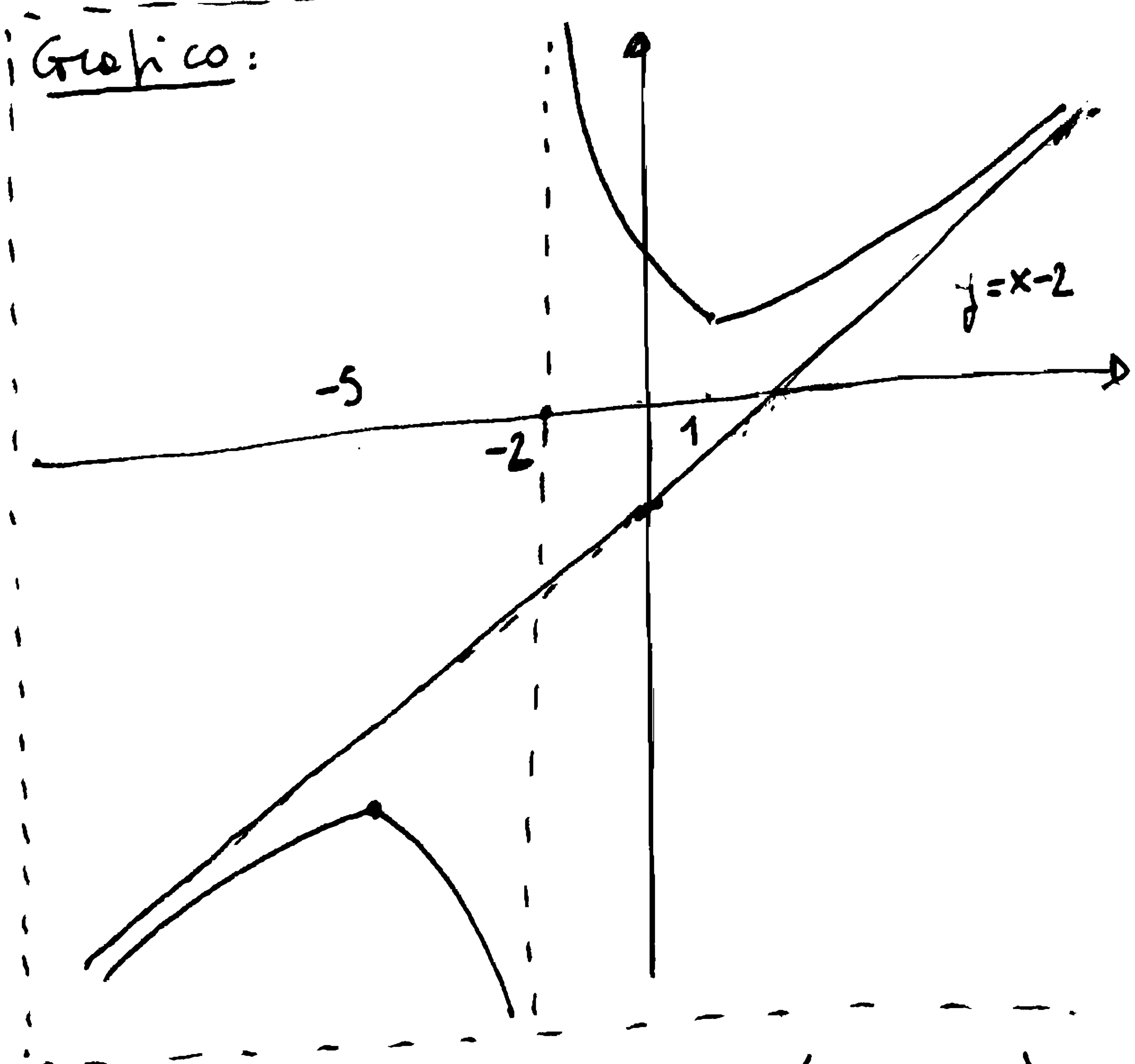
$f(-5)=-10$; $f(1)=2$.

$\lim_{x\to\infty}\dfrac{f(x)}{x}=\lim_{x\to\infty}\dfrac{x^2+5}{x^2+2x}=1=m$?

$\lim_{x\to\infty} f(x)-1\cdot x=\lim_{x\to\infty}\dfrac{5-2x}{x+2}=-2=q \Rightarrow$

$\Rightarrow y=x-2$ Asintoto Obliquo.

Grafico:



2) $\lim_{x\to0}\dfrac{\sqrt{(1+x)^3}-1}{\operatorname{sen}x}=\lim_{x\to0}\dfrac{(1+x)^{\frac{3}{2}}-1}{x}\cdot\dfrac{x}{\operatorname{sen}x}=\frac{3}{2}\cdot1=\frac{3}{2}$. $\left(\text{da } \lim_{x\to0}\dfrac{(1+x)^\alpha-1}{x}=\alpha\right)$.

$\lim_{x\to-\infty}\dfrac{3^x-\log(1-x)+x}{3x-\operatorname{sen}x}=\lim_{x\to-\infty}\dfrac{x}{3x}=\frac{1}{3}$. $\left(3^x\to0;\ \log(1-x)=o(x);\ \operatorname{sen}x=o(3x)\right)$.

3) $f(x)=3^{1-x}$. Coefficiente angolare retta perpendicolare: $-\frac{1}{m}=-\frac{1}{2}=f'(x_0)\Rightarrow$

$\Rightarrow -3^{1-x}\cdot\log3=-\frac{1}{2}\Rightarrow 3^{1-x}=\dfrac{1}{2\log3}\Rightarrow 1-x=\log_3\left(\dfrac{1}{2\log3}\right)\Rightarrow x=1-\log_3\left(\dfrac{1}{2\log3}\right)=1+\log_3(2\log3)$.

4) $\int_0^1 e^{2x}-ke^{3x}dx=\left(\frac{1}{2}e^{2x}-\frac{k}{3}e^{3x}\right|_0^1=\left(\frac{1}{2}e^2-\frac{k}{3}e^3\right)-\left(\frac{1}{2}-\frac{k}{3}\right)=1\Rightarrow$

$\Rightarrow \frac{k}{3}-\frac{k}{3}e^3=1+\frac{1}{2}-\frac{1}{2}e^2\Rightarrow\frac{k}{3}(1-e^3)=\frac{1}{2}(3-e^2)\Rightarrow k=\frac{3}{2}\cdot\dfrac{3-e^2}{1-e^3}$.

5) $f(x)=x^3-kx^2+3x-2$. $f'(x)=3x^2-2kx+3\geq0$.

$x=\dfrac{k\pm\sqrt{k^2-9}}{3}$. Se $\Delta=k^2-9\leq0\Rightarrow k^2\leq9\Rightarrow -3\leq k\leq3$ risulta

$f'(x)\geq0\ \forall x\in\mathbb{R}$ e quindi la funzione è strettamente monotòna.

Se $k=\pm3$ risulta $f'(x)=0$ per $x=\pm1$ e questi saranno punti di flesso a tangente orizzontale.

$\boxed{MGA2}$

6) $f(x) = e^{3-x} - e^x$. $f'(x) = -e^{3-x} - e^x = -(e^{3-x} + e^x) < 0 \; \forall x \in \mathbb{R}$.

$f(x)$ è invertibile su tutto $\mathbb{R}$. $\lim_{x \to -\infty} f(x) = +\infty$; $\lim_{x \to +\infty} f(x) = -\infty$.

$$y = e^3 \cdot \frac{1}{e^x} - e^x = \frac{e^3 - e^{2x}}{e^x} \Rightarrow e^{2x} + y e^x - e^3 = 0 \; (e^x = t) \Rightarrow t^2 + yt - e^3 = 0 \Rightarrow$$

$$\Rightarrow t = e^x = \frac{-y \pm \sqrt{y^2 + 4e^3}}{2} \Rightarrow x = \log\left(\frac{\sqrt{y^2+4e^3} - y}{2}\right). \left(\frac{-y - \sqrt{y^2+4e^3}}{2} < 0 \; \forall y \text{ va scartata}\right).$$

Inversa: $y = \log\left(\frac{\sqrt{x^2+4e^3} - x}{2}\right)$.

7) $x - 1 = o(e^x - x) \Rightarrow \lim_{x \to x_0} \frac{x-1}{e^x - x} = 0$.

Tale relazione è soddisfatta se $x \to 1$ e se $x \to +\infty$.

8) $f(x;y) = xy^2 - 2xy - x^2 - x$.

$$\begin{cases} f'_x = y^2 - 2y - 2x - 1 = 0 \\ f'_y = 2xy - 2x = 2x(y-1) = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y^2 - 2y - 1 = 0 \Rightarrow y = 1 \pm \sqrt{2} \end{cases} \cup \begin{cases} y = 1 \\ 2x = -2 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = 1 \end{cases}.$$

$H = \begin{Vmatrix} -2 & 2y-2 \\ 2y-2 & 2x \end{Vmatrix}$. $H(-1;1) = \begin{Vmatrix} -2 & 0 \\ 0 & -2 \end{Vmatrix}$: $\begin{cases} -2 < 0 \\ 4 > 0 \end{cases}$: punto di massimo.

$H(0;1-\sqrt{2}) = \begin{Vmatrix} -2 & -2\sqrt{2} \\ -2\sqrt{2} & 0 \end{Vmatrix}$: $-8 < 0$: Sella; $H(0;1+\sqrt{2}) = \begin{Vmatrix} -2 & 2\sqrt{2} \\ 2\sqrt{2} & 0 \end{Vmatrix}$: $-8 < 0$: Sella.

9) $f(x;y) = x^2 - 2xy$. $\nabla f = (2x - 2y; -2x)$.

$\nabla f \parallel (2;1)$: $\frac{2x-2y}{2} = \frac{-2x}{1} \Rightarrow 2x - 2y + 4x = 0 \Rightarrow 2y = 6x \Rightarrow y = 3x$. $\nabla f = (-4x; -2x)$.

$\nabla f \perp (2;1)$: $(2x-2y; -2x) \cdot (2;1) = 4x - 4y - 2x = 2x - 4y = 0 \Rightarrow y = \frac{1}{2}x$. $\nabla f = (x; -2x)$.

10) $f(x) = \log(2 - \log_2(5-x))$.

$$\text{C.E.}: \begin{cases} 5 - x > 0 \\ 2 - \log_2(5-x) > 0 \end{cases} \Rightarrow \begin{cases} x < 5 \\ \log_2(5-x) < 2 \end{cases} \Rightarrow \begin{cases} x < 5 \\ 5 - x < 2^2 = 4 \end{cases} \Rightarrow \begin{cases} x < 5 \\ x > 1 \end{cases}.$$

C.E.: $1 < x < 5 \Rightarrow$ C.E. $=]1;5[$.

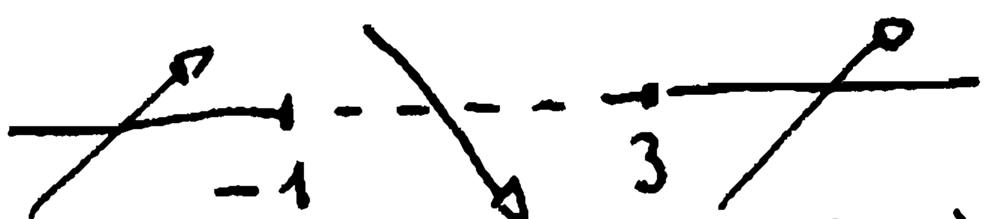
Compito di Matematica Generale del 17/2/2014 Compito B $\boxed{MGB1}$

1) $f(x)=\dfrac{x^2+3}{x-1}$. C.E.: $x\neq 1$. $\lim\limits_{x\to-\infty} f(x)=-\infty$; $\lim\limits_{x\to 1^-} f(x)=-\infty$; $\lim\limits_{x\to 1^+} f(x)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$f(x)>0: x-1>0 \Rightarrow x>1$. $f(0)=-3$.

$$f'(x)=\frac{2x(x-1)-1\cdot(x^2+3)}{(x-1)^2}=\frac{x^2-2x-3}{(x-1)^2}=\frac{(x+1)(x-3)}{(x-1)^2}\geq 0$$

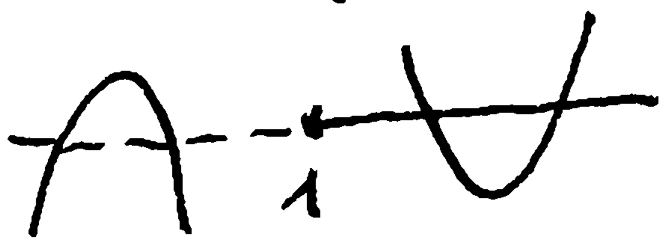
$\Rightarrow (x+1)(x-3)\geq 0 \Rightarrow x\leq -1 \cup x\geq 3$



$$f''(x)=\frac{(2x-2)(x-1)^2-2(x-1)(x^2-2x-3)}{(x-1)^4}=$$

$$=\frac{2(x-1)}{(x-1)^4}\cdot(\cancel{x^2}\cancel{-2x}+1\cancel{-x^2}\cancel{+2x}+3)=\frac{8}{(x-1)^3}>0$$

per $x-1>0 \Rightarrow x>1$



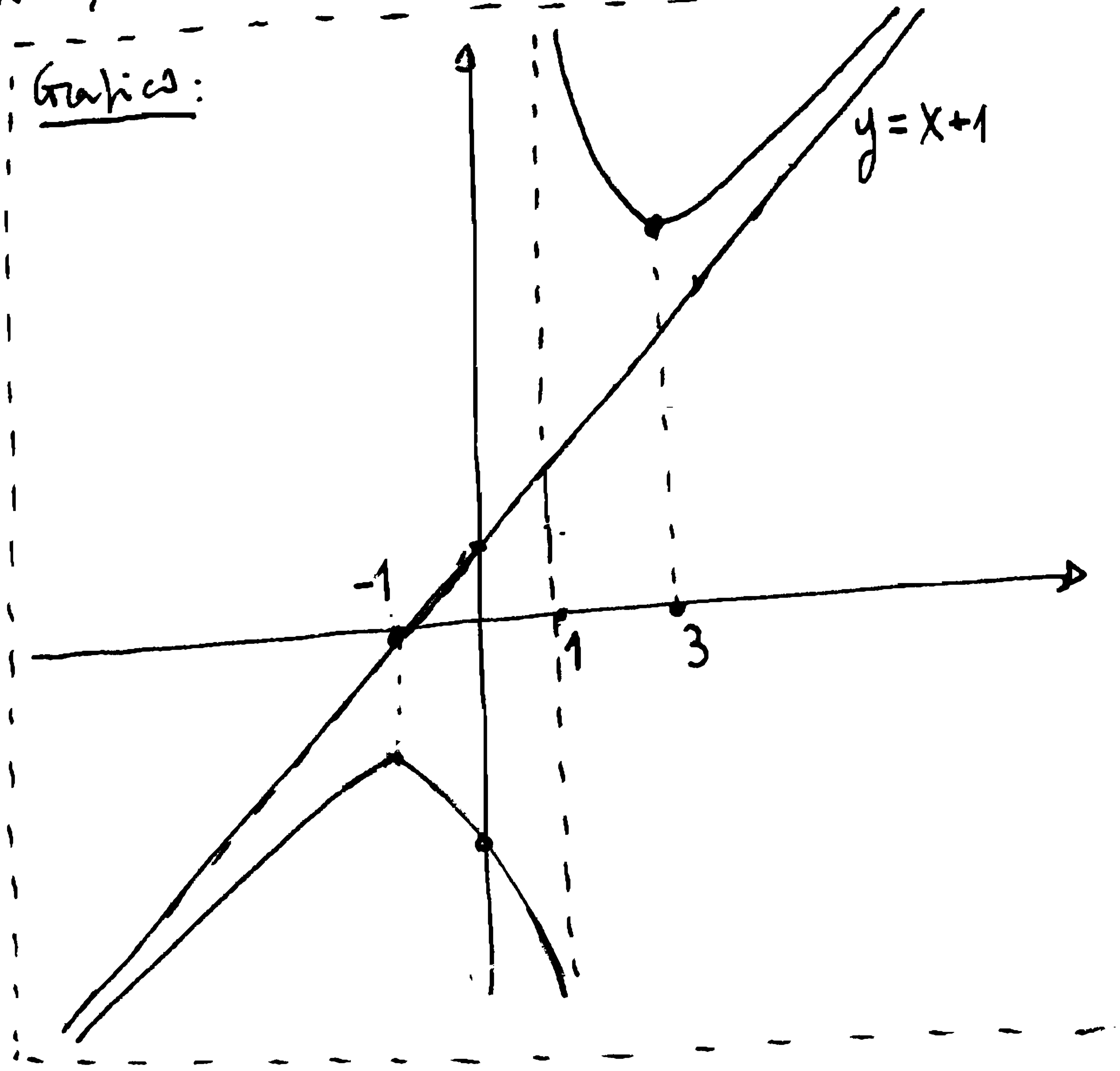
$f(-1)=-2$; $f(3)=6$.

$\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^2+3}{x^2-x}=1=m$?

$\lim\limits_{x\to\infty} f(x)-1\cdot x=\lim\limits_{x\to\infty}\dfrac{x+3}{x-1}=1=q \Rightarrow$

$\Rightarrow y=x+1$ Asintoto obliquo.

Grafico:



2) $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{(1+x)^3}-1}{\log(1+x)}=\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{3}{4}}-1}{x}\cdot\dfrac{x}{\log(1+x)}=\dfrac{3}{4}\cdot 1=\dfrac{3}{4}$ $\left(\text{da } \lim\limits_{x\to 0}\dfrac{(1+x)^\alpha-1}{x}=\alpha\right)$.

$\lim\limits_{x\to+\infty}\dfrac{2^{1-x}-\log x+2x}{x^2-2x+\operatorname{sen}x}=\lim\limits_{x\to+\infty}\dfrac{2x}{x^2}=0$. $\left(2^{1-x}\to 0;\ \log x=o(2x);\ \operatorname{sen}x-2x=o(x^2)\right)$.

3) $f(x)=2^{x-2}$. Coefficiente angolare retta perpendicolare: $-\dfrac{1}{m}=-\dfrac{1}{-3}=\dfrac{1}{3}=f'(x_0)\Rightarrow$

$\Rightarrow 2^{x-2}\cdot\log 2=\dfrac{1}{3}\Rightarrow 2^{x-2}=\dfrac{1}{3\log 2}\Rightarrow x-2=\log_2\left(\dfrac{1}{3\log 2}\right)\Rightarrow x=2+\log_2\left(\dfrac{1}{3\log 2}\right)=2-\log_2(3\log 2)$.

4) $\displaystyle\int_0^1 ke^x-2e^{3x}dx=\left(ke^x-\frac{2}{3}e^{3x}\right|_0^1=\left(k\cdot e-\frac{2}{3}e^3\right)-\left(k-\frac{2}{3}\right)=1\Rightarrow$

$\Rightarrow k\cdot e-k=1-\dfrac{2}{3}+\dfrac{2}{3}e^3\Rightarrow k(e-1)=\dfrac{1}{3}(1+2e^3)\Rightarrow k=\dfrac{1+2e^3}{3(e-1)}$.

5) $f(x)=x^3-2x^2+k^2x-1$. $f'(x)=3x^2-4x+k^2\geq 0$.

$x=\dfrac{2\pm\sqrt{4-3k^2}}{3}$. Se $\Delta=4-3k^2\leq 0\Rightarrow k^2\geq\dfrac{4}{3}\Rightarrow k\leq-\dfrac{2}{\sqrt{3}}$ oppure $k\geq\dfrac{2}{\sqrt{3}}$

risulta $f'(x)\geq 0\ \forall x\in\mathbb{R}$ e quindi la funzione è strettamente monotona. Se $k=\pm\dfrac{2}{\sqrt{3}}$ risulta $f'(x)=0$ per $x=\dfrac{2}{3}$ e questo sarà un punto di flesso a tangente orizzontale.

MGB2

6) $f(x) = e^{2+x} - e^{-x}$; $f'(x) = e^{2+x} + e^{-x} > 0 \ \forall x \in \mathbb{R}$.

$f(x)$ è invertibile su tutto $\mathbb{R}$. $\lim_{x \to -\infty} f(x) = -\infty$; $\lim_{x \to +\infty} f(x) = +\infty$.

$$y = e^2 \cdot e^x - \frac{1}{e^x} = \frac{e^2 \cdot e^{2x} - 1}{e^x} \Rightarrow e^2 \cdot e^{2x} - y \cdot e^x - 1 = 0 \Rightarrow (e^x = t) : e^2 \cdot t^2 - y \cdot t - 1 = 0 \Rightarrow$$

$$t = e^x = \frac{y \pm \sqrt{y^2 + 4e^2}}{2e^2} \Rightarrow x = \log\left(\frac{y + \sqrt{y^2+4e^2}}{2e^2}\right). \left(\frac{y - \sqrt{y^2+4e^2}}{2e^2} < 0 \ \forall y \text{ va scartata}\right)$$

Inversa : $y = \log\left(\frac{x + \sqrt{x^2+4e^2}}{2e^2}\right)$.

7) $\log x = o(x + \log x) \Rightarrow \lim_{x \to x_0} \frac{\log x}{x + \log x} = 0$.

Tale relazione è soddisfatta se : $x \to 1$ e se $x \to +\infty$.

8) $f(x;y) = y^2 - y + 2xy + x^2 y$.

$$\begin{cases} f'_x = 2y + 2xy = 2y(1+x) = 0 \\ f'_y = 2y - 1 + 2x + x^2 = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x^2 + 2x - 1 = 0 \Rightarrow x = -1 \pm \sqrt{2} \end{cases} \cup \begin{cases} x = -1 \\ 2y = 2 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = 1 \end{cases}.$$

$H = \begin{Vmatrix} 2y & 2+2x \\ 2+2x & 2 \end{Vmatrix}$; $H(-1;1) = \begin{Vmatrix} 2 & 0 \\ 0 & 2 \end{Vmatrix} : \begin{cases} 2 > 0 \\ 4 > 0 \end{cases}$: punto di minimo.

$H(-1-\sqrt{2};0) = \begin{Vmatrix} 0 & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{Vmatrix} : -8 < 0$: Sella; $H(-1+\sqrt{2};0) = \begin{Vmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & 2 \end{Vmatrix} : -8 < 0$: Sella.

9) $f(x;y) = xy - 3y^2$. $\nabla f = (y ; x - 6y)$

$\nabla f \parallel (-1;3) \Rightarrow \frac{y}{-1} = \frac{x-6y}{3} \Rightarrow 3y = -x + 6y \Rightarrow x = 3y$ oppure $y = \frac{1}{3}x$. $\nabla f = \left(\frac{1}{3}x ; -x\right), x \in \mathbb{R}$

$\nabla f \perp (-1;3) \Rightarrow (y ; x-6y) \cdot (-1;3) = -y + 3x - 18y = 0 \Rightarrow 3x = 19y \Rightarrow y = \frac{3}{19}x$. $\nabla f = \left(\frac{3}{19}x ; \frac{1}{19}x\right), x \in \mathbb{R}$.

10) $f(x) = \log(2 - \log_3(10-x))$.

$$C.E. : \begin{cases} 10 - x > 0 \\ 2 - \log_3(10-x) > 0 \end{cases} \Rightarrow \begin{cases} x < 10 \\ \log_3(10-x) < 2 \end{cases} \Rightarrow \begin{cases} x < 10 \\ 10 - x < 3^2 = 9 \end{cases} \Rightarrow \begin{cases} x < 10 \\ x > 1 \end{cases}.$$

$C.E. = 1 < x < 10 \Rightarrow C.E. =]1;10[$.

Compito di Matematica Generale del 17/2/14 Compito C $\boxed{\text{MGC1}}$

1) $f(x)=\dfrac{x^2-5}{x-3}$. C.E.: $x\neq 3$. $\lim\limits_{x\to-\infty} f(x)=-\infty$; $\lim\limits_{x\to3^-} f(x)=-\infty$; $\lim\limits_{x\to3^+} f(x)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0:\begin{cases}x^2\geq 5\\ x>3\end{cases} \Rightarrow \begin{cases}x\leq-\sqrt5 \cup x\geq\sqrt5\\ x>3\end{cases}$ $\quad -\sqrt5 \quad \sqrt5 \quad 3$ $\quad (-)\ (+)\ (-)\ (+)$. $f(\pm\sqrt5)=0$; $f(0)=\frac{5}{3}$.

$$f'(x)=\frac{2x(x-3)-1\cdot(x^2-5)}{(x-3)^2}=\frac{x^2-6x+5}{(x-3)^2}=\frac{(x-1)(x-5)}{(x-3)^2}\geq 0$$

$\Rightarrow (x-1)(x-5)\geq 0 \Rightarrow x\leq 1 \cup x\geq 5$

$1 \quad 5$

$$f''(x)=\frac{(2x-6)(x-3)^2-2(x-3)\cdot(x^2-6x+5)}{(x-3)^4}=$$

$$=\frac{2(x-3)}{(x-3)^4}\cdot(\cancel{x^2}\cancel{-6x}+9\cancel{-x^2}\cancel{+6x}-5)=\frac{8}{(x-3)^3}>0$$

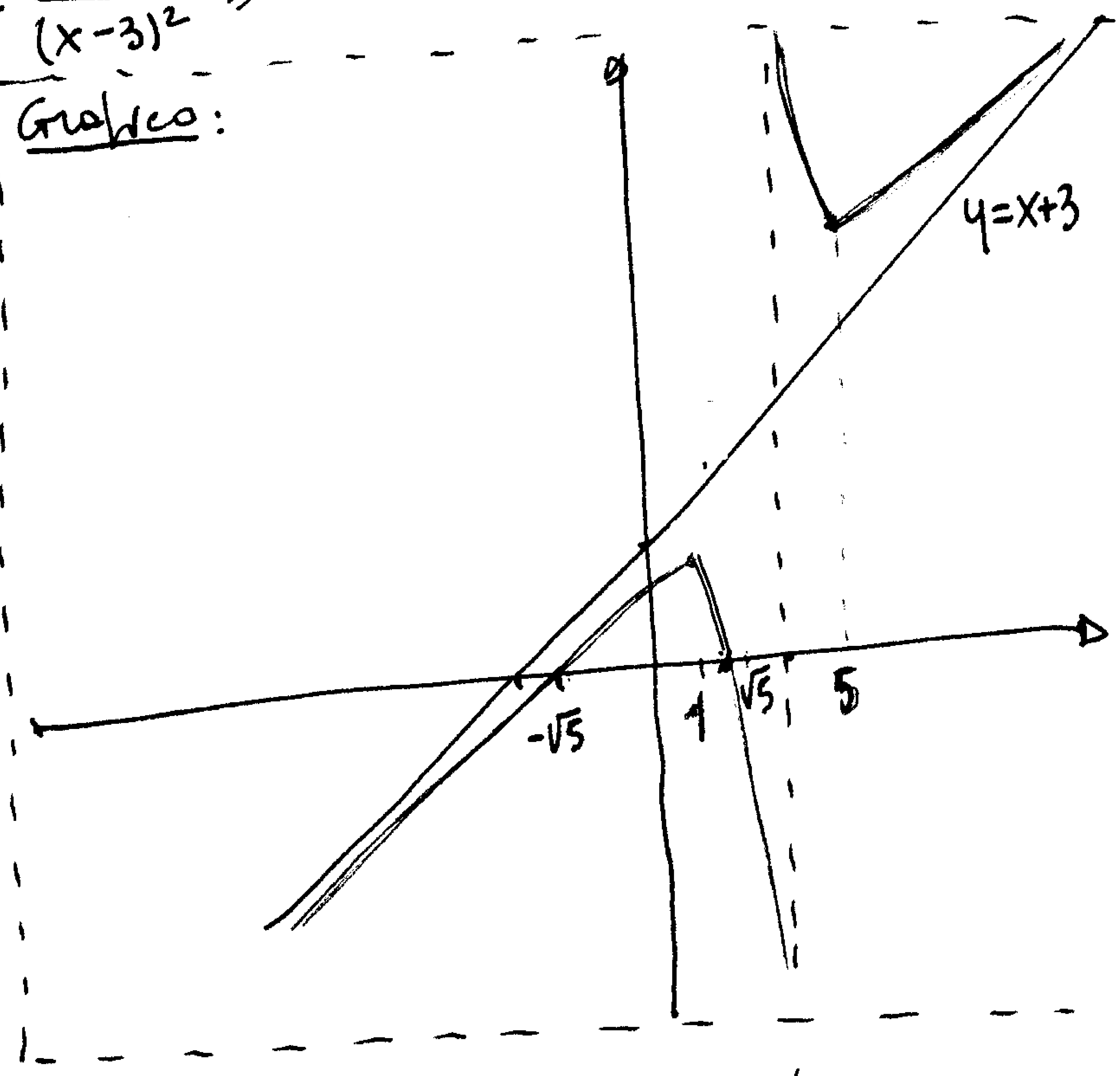
per $x-3>0 \Rightarrow x>3$ $\quad 3$

$f(1)=2$; $f(5)=10$.

$\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{x^2-5}{x^2-3x}=1=m$?

$\lim\limits_{x\to\infty} f(x)-1\cdot x=\lim\limits_{u\to\infty}\frac{3x-5}{x-3}=3=q \Rightarrow$

$\Rightarrow y=x+3$ Asintoto obliquo



2) $\lim\limits_{x\to0}\dfrac{\sqrt[3]{(1+x)^2}-1}{\operatorname{tg}x}=\lim\limits_{x\to0}\dfrac{(1+x)^{\frac{2}{3}}-1}{x}\cdot\dfrac{x}{\operatorname{tg}x}=\dfrac{2}{3}\cdot1=\dfrac{2}{3}$. $\left(\text{da} \lim\limits_{x\to0}\dfrac{(1+x)^{\alpha}-1}{x}=\alpha\right)$.

$\lim\limits_{x\to-\infty}\dfrac{\cos x-\log(1-x)+3x}{2^x-x}=\lim\limits_{x\to-\infty}\dfrac{3x}{-x}=-3$. $\left(\cos x-\log(1-x)=o(3x);\ 2^x\to0\right)$

3) $f(x)=3^{2-x}$. Coefficiente angolare retta perpendicolare: $-\frac{1}{m}=-\frac{1}{2}=f'(x_0)\Rightarrow$

$\Rightarrow -3^{2-x}\cdot\log3=-\frac{1}{2}\Rightarrow 3^{2-x}=\frac{1}{2\log3}\Rightarrow 2-x=\log_3\left(\frac{1}{2\log3}\right)\Rightarrow x=2-\log_3\left(\frac{1}{2\log3}\right)=2+\log_3(2\log3)$.

4) $\int_0^1 e^{3x}-ke^x\,dx=\left[\frac{1}{3}e^{3x}-ke^x\right]_0^1=\left(\frac{1}{3}e^3-ke\right)-\left(\frac{1}{3}-k\right)=1\Rightarrow$

$\Rightarrow k-ke=1+\frac{1}{3}-\frac{1}{3}e^3\Rightarrow k(1-e)=\frac{1}{3}(4-e^3)\Rightarrow k=\frac{1}{3}\cdot\frac{4-e^3}{1-e}=\frac{1}{3}\,\frac{e^3-4}{e-1}$.

5) $f(x)=x^3-2kx^2+4x-5$. $f'(x)=3x^2-4kx+4\geq0$.

$x=\dfrac{2k\pm\sqrt{4k^2-12}}{3}$. Se $\Delta=4k^2-12\leq0\Rightarrow k^2\leq3\Rightarrow-\sqrt3\leq k\leq\sqrt3$ risulta $f'(x)\geq0\ \forall x\in\mathbb{R}$ e quindi la funzione è strettamente monotona.

Se $k=\pm\sqrt3$ risulta $f'(x)=0$ per $x=\pm\frac{2}{\sqrt3}$ e questi saranno punti di flesso a tangente orizzontale.

MGC2

6) $f(x)=e^x-e^{2-x}$. $f'(x)=e^x+e^{2-x}>0 \quad \forall x\in\mathbb{R}$.

$f(x)$ è invertibile su tutto $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=-\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$y=e^x-\frac{e^2}{e^x}=\frac{e^{2x}-e^2}{e^x} \Rightarrow e^{2x}-ye^x-e^2=0 \; (e^x=t) \Rightarrow t^2-yt-e^2=0 \Rightarrow$

$\Rightarrow t=e^x=\frac{y\pm\sqrt{y^2+4e^2}}{2} \Rightarrow x=\log\left(\frac{y+\sqrt{y^2+4e^2}}{2}\right)$. $\left(\frac{y-\sqrt{y^2+4e^2}}{2}<0 \; \forall y \text{ va scartata}\right)$

Inversa: $y=\log\left(\frac{x+\sqrt{x^2+4e^2}}{2}\right)$.

7) $e^x-1=o(x+1) \Rightarrow \lim\limits_{x\to x_0}\frac{e^x-1}{x+1}=0$.

Tale relazione è soddisfatta se $x\to 0$ e se $x\to-\infty$.

8) $f(x;y)=xy^2-2xy+x^2-x$.

$\begin{cases} f'_x=y^2-2y+2x-1=0 \\ f'_y=2xy-2x=2x(y-1)=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y^2-2y-1=0 \Rightarrow y=1\pm\sqrt{2} \end{cases} \cup \begin{cases} y=1 \\ 2x=2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases}$.

$H=\begin{Vmatrix} 2 & 2y-2 \\ 2y-2 & 2x \end{Vmatrix}$. $H(1;1)=\begin{Vmatrix} 2 & 0 \\ 0 & 2 \end{Vmatrix}$: $\begin{cases} 2>0 \\ 4>0 \end{cases}$: punto di minimo.

$H(0;1-\sqrt{2})=\begin{Vmatrix} 2 & -2\sqrt{2} \\ -2\sqrt{2} & 0 \end{Vmatrix}$: $-8<0$: Sella. $H(0;1+\sqrt{2})=\begin{Vmatrix} 2 & 2\sqrt{2} \\ 2\sqrt{2} & 0 \end{Vmatrix}$: $-8<0$: Sella.

9) $f(x;y)=x^2-3xy$. $\nabla f=(2x-3y;-3x)$

$\nabla f \parallel (-1;2)$: $\frac{2x-3y}{-1}=\frac{-3x}{2} \Rightarrow 4x-6y=3x \Rightarrow x=6y \;\delta\; y=\frac{1}{6}x$. $\nabla f=\left(\frac{3}{2}x;-3x\right)$.

$\nabla f \perp (-1;2)$: $(2x-3y;-3x)\cdot(-1;2)=-2x+3y-6x=3y-8x=0 \Rightarrow y=\frac{8}{3}x$. $\nabla f=(-6x;-3x)$.

10) $f(x)=\log(1-\log_2(3-x))$.

C.E.: $\begin{cases} 3-x>0 \\ 1-\log_2(3-x)>0 \end{cases} \Rightarrow \begin{cases} x<3 \\ \log_2(3-x)<1 \end{cases} \Rightarrow \begin{cases} x<3 \\ 3-x<2^1=2 \end{cases} \Rightarrow \begin{cases} x<3 \\ x>1 \end{cases}$.

C.E.: $1<x<3 \Rightarrow$ C.E. $=\,]1;3[$.

Compito di Matematica Generale del 17/2/14 Compito D $\boxed{MGD1}$

1) $f(x) = \dfrac{x^2-3}{x-2}$. C.E.: $x \neq 2$. $\lim\limits_{x\to-\infty} f(x) = -\infty$; $\lim\limits_{x\to 2^-} f(x) = -\infty$; $\lim\limits_{x\to 2^+} f(x) = +\infty$; $\lim\limits_{x\to+\infty} f(x) = +\infty$.

$f(x) \geq 0: \begin{cases} x^2 \geq 3 \\ x > 2 \end{cases} \Rightarrow \begin{cases} x \leq -\sqrt{3} \cup x \geq \sqrt{3} \\ x > 2 \end{cases}$ $(-)$ $(+)$ $(-)$ $(+)$ $-\sqrt{3}$ $\sqrt{3}$ 2. $f(\pm\sqrt{3}) = 0$; $f(0) = \dfrac{3}{2}$.

$$f'(x) = \frac{2x(x-2) - 1\cdot(x^2-3)}{(x-2)^2} = \frac{x^2-4x+3}{(x-2)^2} = \frac{(x-1)(x-3)}{(x-2)^2} \geq 0$$

$\Rightarrow (x-1)(x-3) \geq 0 \Rightarrow x \leq 1 \cup x \geq 3$.

1 3

$$f''(x) = \frac{(2x-4)(x-2)^2 - 2(x-2)(x^2-4x+3)}{(x-2)^4} =$$

$$= \frac{2(x-2)}{(x-2)^4}\cdot(x^2 \cancel{-4x} + 4 \cancel{-x^2} \cancel{+4x} - 3) = \frac{2}{(x-2)^3} > 0$$

per $x-2>0 \Rightarrow x > 2$ 2

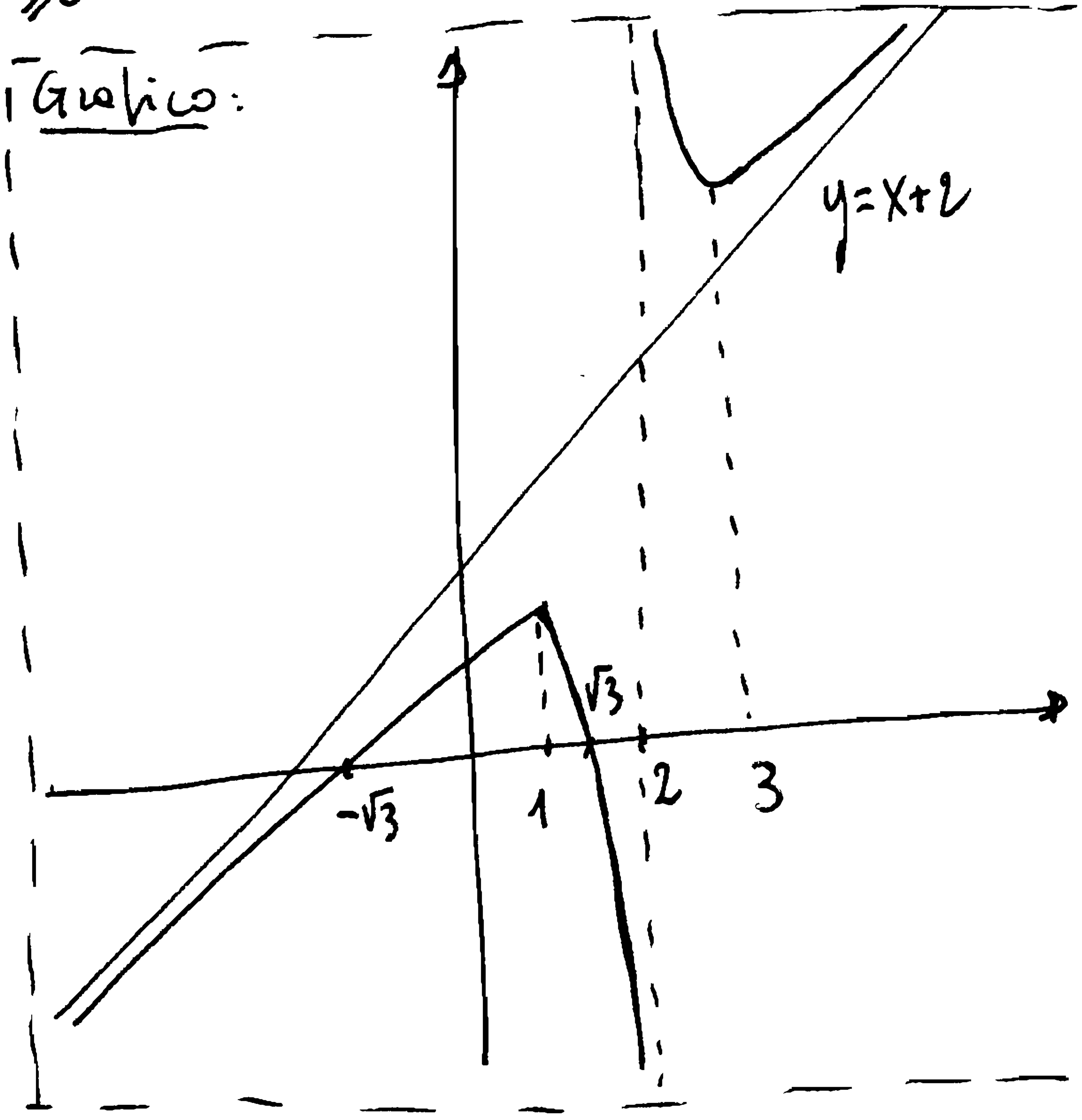
$f(1) = 2$; $f(3) = 6$.

$\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x^2-3}{x^2-2x} = 1 = m$?

$\lim\limits_{x\to\infty} f(x) - 1\cdot x = \lim\limits_{x\to\infty} \dfrac{2x-3}{x-2} = 2 = q \Rightarrow$

$y = x + 2$ Asintoto obliquo.

Grafico:



2) $\lim\limits_{x\to 0} \dfrac{\sqrt[5]{(1+x)^4} - 1}{e^x - 1} = \lim\limits_{x\to 0} \dfrac{(1+x)^{\frac{4}{5}} - 1}{x} \cdot \dfrac{x}{e^x-1} = \dfrac{4}{5}\cdot 1 = \dfrac{4}{5}$ $\left(\text{da } \lim\limits_{x\to 0} \dfrac{(1+x)^\alpha - 1}{x} = \alpha\right)$.

$\lim\limits_{x\to+\infty} \dfrac{\log x - \cos x + x^2}{x - x^2 + 3^{1-x}} = \lim\limits_{x\to+\infty} \dfrac{x^2}{-x^2} = -1$ $\left(\log x - \cos x = o(x^2),\ 3^{1-x} \to 0;\ x = o(x^2)\right)$.

3) $f(x) = 2^{x-1}$. Coefficiente angolare retta perpendicolare: $-\dfrac{1}{m} = \dfrac{1}{3} = f'(x_0) \Rightarrow$

$\Rightarrow 2^{x-1}\cdot \log 2 = \dfrac{1}{3} \Rightarrow 2^{x-1} = \dfrac{1}{3\log 2} \Rightarrow x - 1 = \log_2\left(\dfrac{1}{3\log 2}\right) \Rightarrow x = 1 + \log_2\left(\dfrac{1}{3\log 2}\right) = 1 - \log_2(3\log 2)$.

4) $\displaystyle\int_0^1 k e^{2x} - e^{3x}\, dx = \left(\frac{k}{2}e^{2x} - \frac{1}{3}e^{3x}\right|_0^1 = \left(\frac{k}{2}e^2 - \frac{1}{3}e^3\right) - \left(\frac{k}{2} - \frac{1}{3}\right) = 1 \Rightarrow$

$\Rightarrow \dfrac{k}{2}e^2 - \dfrac{k}{2} = 1 - \dfrac{1}{3} + \dfrac{1}{3}e^3 \Rightarrow \dfrac{k}{2}(e^2-1) = \dfrac{1}{3}(2+e^3) \Rightarrow k = \dfrac{2}{3}\dfrac{e^3+2}{e^2-1}$.

5) $f(x) = x^3 + 3x^2 + k^2 x + 2$. $f'(x) = 3x^2 + 6x + k^2 \geq 0$.

$x = \dfrac{-3 \pm \sqrt{9-3k^2}}{3}$. Se $\Delta = 9 - 3k^2 \leq 0 \Rightarrow k^2 \geq 3 \Rightarrow k \leq -\sqrt{3} \cup k \geq \sqrt{3}$ risulta $f'(x) \geq 0\ \forall x \in \mathbb{R}$ e quindi la funzione è strettamente monotòna. Se $k = \pm\sqrt{3}$ risulta $f'(x) = 0$ per $x = -1$ e questo sarà un punto di flesso a tangente orizzontale.

MGD2

6) $f(x)=e^{-x}-e^{3+x}$. $f'(x)=-e^{-x}-e^{3+x}=-(e^{-x}+e^{3+x})<0\ \forall x\in\mathbb{R}$.

$f(x)$ è invertibile su tutto $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=-\infty$.

$y=\frac{1}{e^x}-e^3\cdot e^x=\frac{1-e^3\cdot e^{2x}}{e^x} \Rightarrow e^3\cdot e^{2x}+ye^x-1=0\ (e^x=t) \Rightarrow e^3\cdot t^2+yt-1=0 \Rightarrow$

$\Rightarrow t=e^x=\frac{-y\pm\sqrt{y^2+4e^3}}{2e^3} \Rightarrow x=\log\left(\frac{\sqrt{y^2+4e^3}-y}{2e^3}\right).\ \left(\frac{-y-\sqrt{y^2+4e^3}}{2e^3}<0\ \forall y \text{ va scartata}\right)$

Inversa: $y=\log\left(\frac{\sqrt{x^2+4e^3}-x}{2e^3}\right)$.

7) $\log x-1=o(e^x+1) \Rightarrow \lim_{x\to x_0}\frac{\log x-1}{e^x+1}=0$.

Tale relazione è soddisfatta se $x\to e$ e se $x\to+\infty$.

8) $f(x;y)=x^2y-y^2-y+2xy$.

$\begin{cases} f'_x=2xy+2y=2y(x+1)=0 \\ f'_y=x^2-2y-1+2x=0 \end{cases} \Rightarrow \begin{cases} y=0 \\ x^2+2x-1=0 \Rightarrow x=-1\pm\sqrt{2} \end{cases} \cup \begin{cases} x=-1 \\ 2y=-2 \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=-1 \end{cases}$

$H=\begin{Vmatrix} 2y & 2x+2 \\ 2x+2 & -2 \end{Vmatrix}$. $H(-1;-1)=\begin{Vmatrix} -2 & 0 \\ 0 & -2 \end{Vmatrix}$: $\begin{cases} -2<0 \\ 4>0 \end{cases}$: punto di Massimo.

$H(-1-\sqrt{2};0)=\begin{Vmatrix} 0 & -2\sqrt{2} \\ -2\sqrt{2} & -2 \end{Vmatrix}$: $-8<0$: Sella. $H(-1+\sqrt{2};0)=\begin{Vmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & -2 \end{Vmatrix}$: $-8<0$: Sella.

9) $f(x;y)=2xy-y^2$. $\nabla f=(2y;2x-2y)$.

$\nabla f\,//\,(1;2)$: $\frac{2y}{1}=\frac{2x-2y}{2} \Rightarrow 4y=2x-2y \Rightarrow 2x=6y \Rightarrow y=\frac{1}{3}x$. $\nabla f=\left(\frac{2}{3}x;\frac{4}{3}x\right)$.

$\nabla f\perp(1;2)$: $(2y;2x-2y)\cdot(1;2)=2y+4x-4y=0 \Rightarrow 4x=2y \Rightarrow y=2x$. $\nabla f=(4x;-2x)$.

10) $f(x)=\log(1-\log_3(3-x))$.

C.E.: $\begin{cases} 3-x>0 \\ 1-\log_3(3-x)>0 \end{cases} \Rightarrow \begin{cases} x<3 \\ \log_3(3-x)<1 \end{cases} \Rightarrow \begin{cases} x<3 \\ 3-x<3^1=3 \end{cases} \Rightarrow \begin{cases} x<3 \\ x>0 \end{cases}$.

C.E.: $0<x<3 \Rightarrow$ C.E. $=]0;3[$.