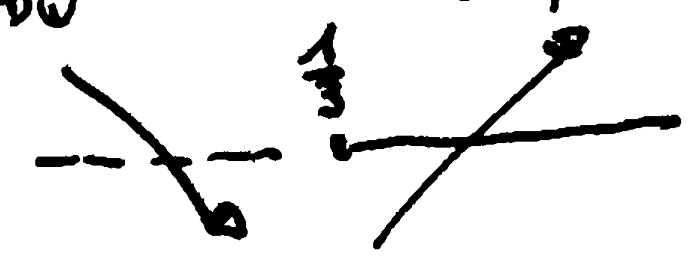
Compito di Matematica Generale del 22/3/2014 $\boxed{MG1}$

1) $f(x)=\dfrac{e^{1+3x}}{x}$. C.E.: $x\neq 0$. $f(x)>0$ per $x>0$.

$\lim\limits_{x\to-\infty} f(x)=\left(\dfrac{\to 0^+}{\to -\infty}\right)=0^-$; $\lim\limits_{x\to 0^-} f(x)=\left(\dfrac{\to e}{\to 0^-}\right)=-\infty$; $\lim\limits_{x\to 0^+} f(x)=\left(\dfrac{\to e}{\to 0^+}\right)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$

$x=0$ (e^{1+3x})

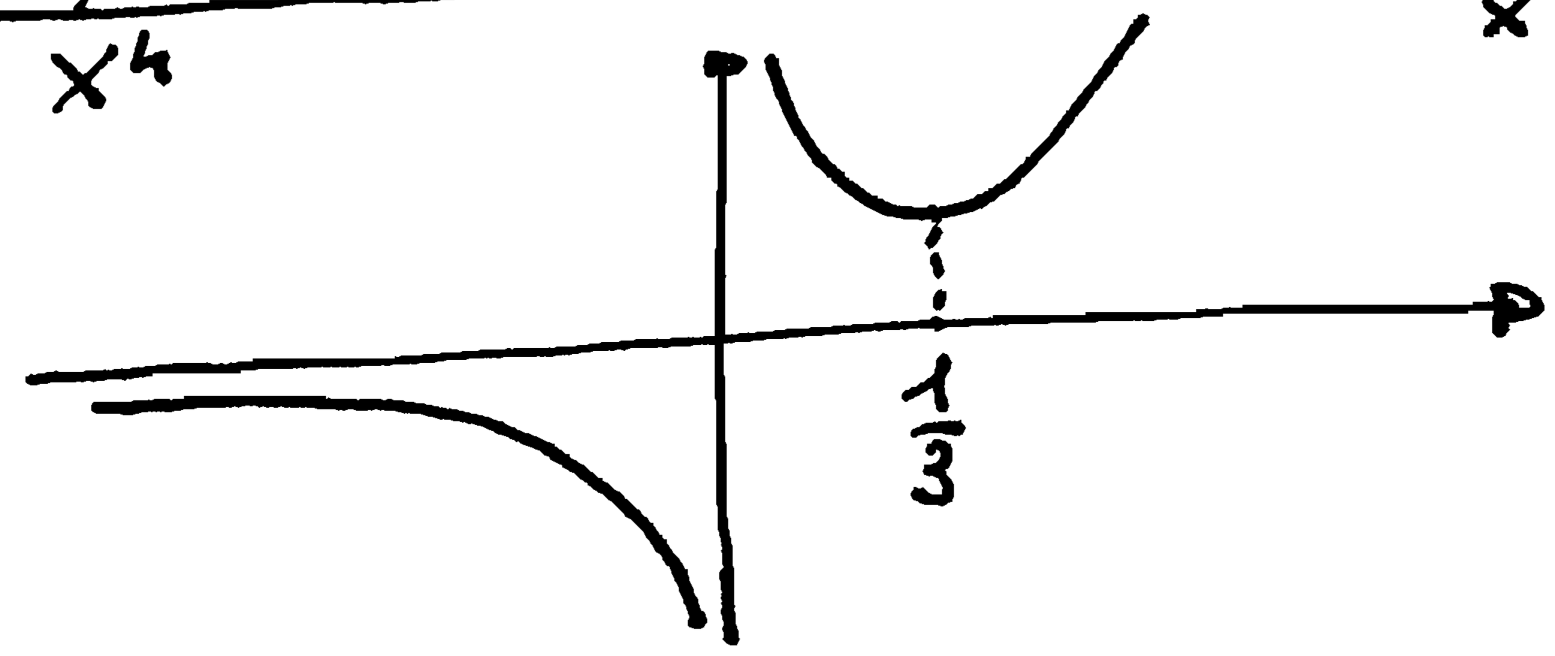
$f'(x)=\dfrac{3e^{1+3x}\cdot x-e^{1+3x}}{x^2}=\dfrac{e^{1+3x}}{x^2}\cdot(3x-1)\geq 0:\ x\geq\dfrac{1}{3}$



$f''(x)=\dfrac{\left(3\cdot e^{1+3x}\cdot(3x-1)+3\cdot e^{1+3x}\right)\cdot x^2-2x\cdot e^{1+3x}\cdot(3x-1)}{x^4}=\dfrac{x\cdot e^{1+3x}\,(9x^2-6x+2)}{x^4}>0$ per $x>0$.

$(9x^2-6x+2)>0\ \forall x$!

Grafico:



2) $\lim\limits_{x\to 0}\dfrac{\operatorname{sen}(\operatorname{sen}x)}{e^x-1}=\lim\limits_{x\to 0}\dfrac{\operatorname{sen}(\operatorname{sen}x)}{\operatorname{sen}x}\cdot\dfrac{\operatorname{sen}x}{x}\cdot\dfrac{x}{e^x-1}=1\cdot1\cdot1=1.$

$\lim\limits_{x\to+\infty}\dfrac{x-\log(1+x^2)}{3^{1-x}+2x}=\lim\limits_{x\to+\infty}\dfrac{x}{2x}=\dfrac{1}{2}$ $\left(\log(1+x^2)=o(x);\ 3^{1-x}\to 0\right)$

3) $f(x)=\log(1+x)$; $g(x)=e^{2x-1}$; $f(g(x))=\log\left(1+e^{2x-1}\right)$.

$D(f(g(x)))=\dfrac{1}{1+e^{2x-1}}\cdot e^{2x-1}\cdot 2>0\ \forall x$: funzione strettamente crescente.

C.E. $(f(g(x)))=\mathbb{R}$ $(1+e^{2x-1}>0\ \forall x)$. $\lim\limits_{x\to-\infty} f(g(x))=0$; $\lim\limits_{x\to+\infty} f(g(x))=+\infty$.

$f(g(x)):\ \mathbb{R}\to\,]0;+\infty[$.

$\log(1+e^{2x-1})=y\Rightarrow 1+e^{2x-1}=e^y\Rightarrow e^{2x-1}=e^y-1\Rightarrow 2x-1=\log(e^y-1)\Rightarrow$

$\Rightarrow 2x=1+\log(e^y-1)\Rightarrow x=\dfrac{1}{2}\left(1+\log(e^y-1)\right)$. Inversa: $y=\dfrac{1}{2}\left(1+\log(e^x-1)\right)$.

$[f(g(x))]^{-1}:\ ]0;+\infty[\,\to\mathbb{R}$.

4) $f(x)=2x^3-9x^2+12x-4\in\mathcal{C}(\mathbb{R})$; $f'(x)=6x^2-18x+12\in\mathcal{C}(\mathbb{R})\Rightarrow$

$f'(x_0)=6x_0^2-18x_0+12=\dfrac{f(3)-f(0)}{3-0}=\dfrac{5-(-4)}{3}=3\Rightarrow 6x_0^2-18x_0+9=0\Rightarrow$

$\Rightarrow 2x_0^2-6x_0+3=0\Rightarrow x_0=\dfrac{3\pm\sqrt{9-6}}{2}=\dfrac{3\pm\sqrt{3}}{2}$. Ci sono due punti che soddisfano al Teorema di Lagrange: $x=\dfrac{3-\sqrt{3}}{2}$ e $x=\dfrac{3+\sqrt{3}}{2}$.

5) $e^x=1+x+\dfrac{x^2}{2}+\dfrac{x^3}{6}+o(x^3)$; $e^{x^2}=1+x^2+\dfrac{x^4}{2}+o(x^4)$.

$\cos x=1-\dfrac{x^2}{2}+\dfrac{x^4}{24}+o(x^4)$.

$e^{x^2}-\cos x=1+x^2+\dfrac{x^4}{2}+o(x^4)-\left(1-\dfrac{x^2}{2}+\dfrac{x^4}{24}+o(x^4)\right)=\left(1+\dfrac{1}{2}\right)x^2+\left(\dfrac{1}{2}-\dfrac{1}{24}\right)x^4+o(x^4)$

$e^x-\cos x=\dfrac{3}{2}x^2+\dfrac{11}{24}x^4+o(x^4)$

MG2

6) $f(x;y) = x^2 e^y - 3\operatorname{sen}x + 5\log(y+1)$

$\nabla f(x;y) = \left(2xe^y - 3\cos x + 0;\ x^2 e^y - 0 + \frac{5}{y+1}\right)$

$\nabla f(0;0) = (0\cdot 1 - 3\cdot 1\ ;\ 0\cdot 1 + 5) = (-3;5).$

7) $\int_0^1 5x - \frac{1}{x+1} + e^{3x}\,dx = \left[\frac{5}{2}x^2 - \log(x+1) + \frac{1}{3}e^{3x}\right|_0^1 =$

$= \frac{5}{2} - \log 2 + \frac{e^3}{3} - \left(0 - 0 + \frac{1}{3}\right) = \frac{13}{6} - \log 2 + \frac{e^3}{3}.$

8) $A\cdot B\cdot X = \begin{Vmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 2 & 1 \\ 2 & 1 \\ -1 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} K \\ K \end{Vmatrix} = \begin{Vmatrix} 1\cdot 2 - 1\cdot 2 + 2(-1) & 1\cdot 1 - 1\cdot 1 + 2\cdot 2 \\ 1\cdot 2 + 0\cdot 2 + 1(-1) & 1\cdot 1 + 0\cdot 1 + 1\cdot 2 \end{Vmatrix} \cdot \begin{Vmatrix} K \\ K \end{Vmatrix} =$

$= \begin{Vmatrix} -2 & 4 \\ 1 & 3 \end{Vmatrix} \cdot \begin{Vmatrix} K \\ K \end{Vmatrix} = \begin{Vmatrix} -2K + 4K \\ 1\cdot K + 3K \end{Vmatrix} = \begin{Vmatrix} 2K \\ 4K \end{Vmatrix}.\ (2K;4K) \parallel (3;6) \Rightarrow \frac{2k}{3} = \frac{4k}{6} = \frac{2}{3}k$ vere $\forall K$.

Per ogni valore di K il vettore $A\cdot B\cdot X$ è parallelo a $(3;6)$.

9)

A	B	A e B	non B	A ⇒ non B
1	1	1	0	0
1	0	0	1	1
0	1	0	0	1
0	0	0	1	1

$\Rightarrow (A \text{ e } B) \Leftrightarrow \text{non}\,(A \Rightarrow \text{non}\,B).$

$(A \text{ e } B)$ è la negazione di $(A \Rightarrow \text{non}\,B)$

10) $f(x) = x^3 - 3Kx^2 + 5x - 2.$

$f'(x) = 3x^2 - 6Kx + 5$

$f''(x) = 6x - 6K \geq 0 \Rightarrow 6x \geq 6K \Rightarrow x \geq K.$

f convessa per $x \geq -1 \Rightarrow K = -1.$