Compito di Matematica Generale del 12/6/14 **MG1**

1) $f(x) = x \cdot e^{x-1} - 3e^x = \frac{x}{e} e^x - 3e^x = e^x\left(\frac{x}{e} - 3\right)$. C.E.: $\mathbb{R}$.

$\lim_{x \to -\infty} f(x) = 0^-$; $\lim_{x \to +\infty} f(x) = +\infty$. $f(x) \geq 0$ per $\frac{x}{e} - 3 \geq 0 \Rightarrow x \geq 3e$ (−) 3e (+)

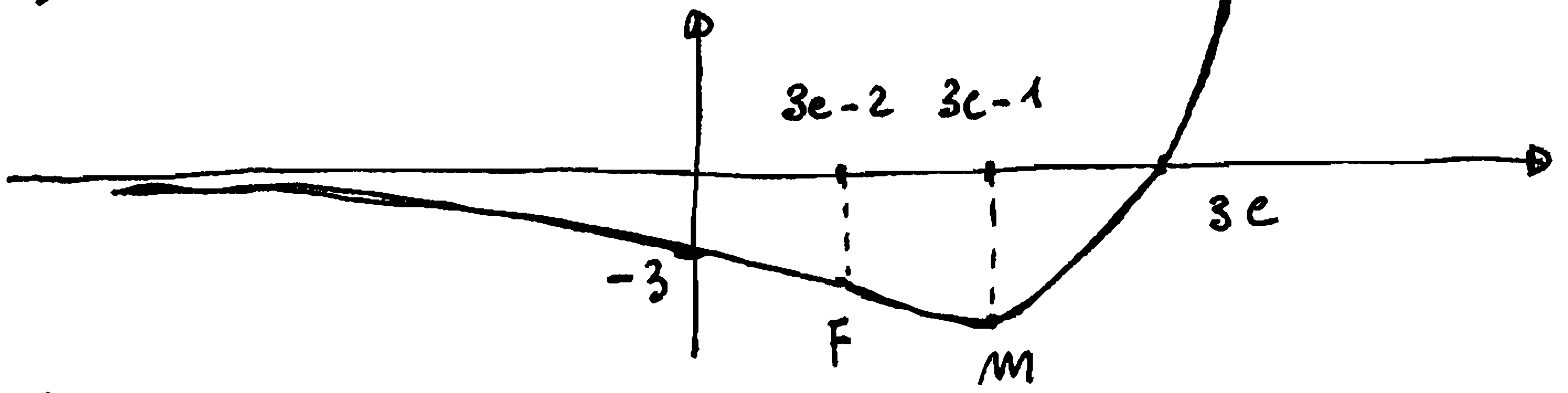
$f'(x) = e^x\left(\frac{x}{e} - 3\right) + e^x \cdot \frac{1}{e} = e^x\left(\frac{x}{e} - 3 + \frac{1}{e}\right) \geq 0$ per $\frac{x}{e} \geq 3 - \frac{1}{e} \Rightarrow x \geq 3e - 1$ 3e−1 m

$f''(x) = e^x\left(\frac{x}{e} - 3 + \frac{1}{e}\right) + e^x \cdot \frac{1}{e} = e^x\left(\frac{x}{e} - 3 + \frac{2}{e}\right) \geq 0$ per $\frac{x}{e} \geq 3 - \frac{2}{e} \Rightarrow x \geq 3e - 2$ 3e−2

Grafico:

$f(0) = -3$

$f(3e) = 0$.



2) $\lim_{x \to 0} \frac{(1+\operatorname{tg} x)^2 - 1}{\operatorname{sen} x} = \lim_{x \to 0} \frac{(1+\operatorname{tg} x)^2 - 1}{\operatorname{tg} x} \cdot \frac{\operatorname{tg} x}{\operatorname{sen} x} = \lim_{t \to 0} \frac{(1+t)^2 - 1}{t} \cdot \lim_{x \to 0} \frac{\operatorname{tg} x}{\operatorname{sen} x} = 2 \cdot 1 = 2$.

$\lim_{x \to +\infty} \left(\frac{1 - 3x + x^2}{2x+1}\right)^{\frac{1-x}{1+x}} = (\to +\infty)^{(\to -1)} = 0^+$.

3) Retta passante per l'origine: $y = mx$; se tangente a $y = \log x \Rightarrow m = \frac{1}{x}$.

Dovrà risultare: $\begin{cases} \text{Intersezione}: mx = \log x \\ \text{Tangenza}: m = \frac{1}{x} \end{cases} \Rightarrow \begin{cases} \frac{1}{x} \cdot x = \log x \\ m = \frac{1}{x} \end{cases} \Rightarrow \begin{cases} \log x = 1 \\ m = \frac{1}{x} \end{cases} \Rightarrow \begin{cases} x = e \\ m = \frac{1}{e} \end{cases}$.

Equazione retta tangente: $y - \log e = \frac{1}{e}(x - e) \Rightarrow y - 1 = \frac{1}{e}x - 1 \Rightarrow y = \frac{1}{e} \cdot x$.

4) $\int_0^1 x \cdot e^{2x} - \frac{1}{2x+1}\, dx \Rightarrow x \cdot \frac{1}{2} e^{2x} - \int 1 \cdot \frac{1}{2} e^{2x} dx - \frac{1}{2} \int \frac{1}{2x+1} d(2x+1) =$

$= \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx - \frac{1}{2} \log(2x+1) \Rightarrow \left(\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} - \frac{1}{2} \log(2x+1)\right)\Big|_0^1 = \frac{1}{2} e^2 - \frac{1}{4} e^2 - \frac{1}{2} \log 3 - \left(0 - \frac{1}{4} - 0\right) =$

$= \frac{1}{4} e^2 - \log\sqrt{3} + \frac{1}{4} = \frac{1}{4}(e^2 + 1) - \log\sqrt{3}$.

5) $f(x;y) = x^3 - 3xy + y^3$

$\begin{cases} f'_x = 3x^2 - 3y = 0 \\ f'_y = -3x + 3y^2 = 0 \end{cases} \Rightarrow \begin{cases} y = x^2 \\ x^4 - x = x(x^3 - 1) = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \cup \begin{cases} x = 1 \\ y = 1. \end{cases}$ $H(x;y) = \begin{Vmatrix} 6x & -3 \\ -3 & 6y \end{Vmatrix}$.

$H(0;0) = \begin{Vmatrix} 0 & -3 \\ -3 & 0 \end{Vmatrix} \Rightarrow |H| = -9 < 0$: Sella; $H(1;1) = \begin{Vmatrix} 6 & -3 \\ -3 & 6 \end{Vmatrix} \Rightarrow \begin{cases} 6 > 0 \\ 36 - 9 = 27 > 0 \end{cases}$: Minimo.

6) $A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$

$A \cdot X = 2X \Rightarrow \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 0 \end{Vmatrix} = \begin{Vmatrix} 2 \\ 0 \end{Vmatrix} \Rightarrow \begin{cases} a_{11} \cdot 1 + a_{12} \cdot 0 = 2 \\ a_{21} \cdot 1 + a_{22} \cdot 0 = 0 \end{cases} \Rightarrow \begin{cases} a_{11} = 2 \\ a_{21} = 0 \end{cases}$;

$A \cdot Y = Y \Rightarrow \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} \Rightarrow \begin{cases} a_{11} \cdot 1 + a_{12} \cdot 1 = 1 \\ a_{21} \cdot 1 + a_{22} \cdot 1 = 1 \end{cases} \Rightarrow \begin{cases} a_{12} = 1 - a_{11} = 1 - 2 = -1 \\ a_{22} = 1 - a_{21} = 1 - 0 = 1 \end{cases} \Rightarrow$

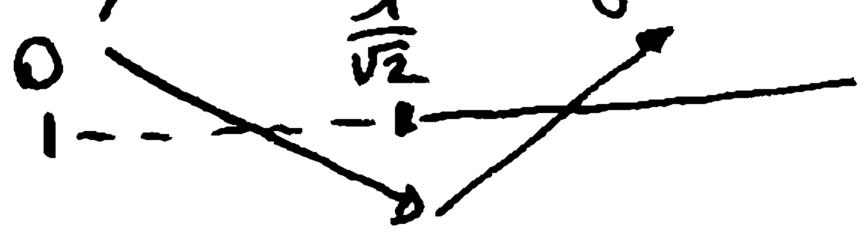
$A = \begin{Vmatrix} 2 & -1 \\ 0 & 1 \end{Vmatrix}$.

7) $f(x) = \log x \cdot \log 2x = \log x (\log x + \log 2) = \log^2 x + \log 2 \cdot \log x$.   C.E.: $x > 0$ $\boxed{MG2}$

$\lim_{x \to 0^+} f(x) = +\infty$; $\lim_{x \to +\infty} f(x) = +\infty$.

$f'(x) = 2 \log x \cdot \frac{1}{x} + \log 2 \cdot \frac{1}{x} = \frac{1}{x}(2 \log x + \log 2) \geq 0 \Rightarrow \log x \geq -\frac{1}{2} \log 2 \Rightarrow$

$\Rightarrow x \geq e^{-\frac{1}{2}\log 2} = (e^{\log 2})^{-\frac{1}{2}} = 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}$:



Si tratta, in $x = \frac{1}{\sqrt{2}}$, di un punto di minimo assoluto.

8) $f(x) = e^{x-1} - \log x$; $f(1) = e^0 - \log 1 = 1 - 0 = 1$;

$f'(x) = e^{x-1} - \frac{1}{x}$; $f'(1) = e^0 - \frac{1}{1} = 1 - 1 = 0$;

$f''(x) = e^{x-1} + \frac{1}{x^2}$; $f''(1) = e^0 + \frac{1}{1} = 1 + 1 = 2$;

$f'''(x) = e^{x-1} + (-2)\frac{1}{x^3}$; $f'''(1) = e^0 - 2 \cdot 1 = 1 - 2 = -1$.

$P_3(x;1) = 1 + 0 \cdot (x-1) + 2 \cdot \frac{1}{2!}(x-1)^2 + (-1) \cdot \frac{1}{3!}(x-1)^3 = 1 + (x-1)^2 - \frac{1}{6}(x-1)^3$.

9)

A B C	(A⇒B)	(A⇒C)	(A⇒B) e (A⇒C)	B e C	$P_1 \Leftrightarrow P_2$
1 1 1	1	1	1	1	1
1 1 0	1	0	0	0	1
1 0 1	0	1	0	0	1
1 0 0	0	0	0	0	1
0 1 1	1	1	1	1	1
0 1 0	1	1	1	0	0
0 0 1	1	1	1	0	0
0 0 0	1	1	1	0	0

Le due proposizioni non sono logicamente equivalenti.

10) Risulta $1 + x^2 = o(x)$ se $\lim_{x \to x_0} \frac{1+x^2}{x} = 0$.

Si può avere questo risultato se:

•) il numeratore tendesse a zero, ma questo non è possibile;

••) il denominatore tendesse ad infinito, ma risulta:

$\lim_{x \to -\infty} \frac{1+x^2}{x} = -\infty$; $\lim_{x \to +\infty} \frac{1+x^2}{x} = +\infty$.

Quindi non risulta mai che $1 + x^2 = o(x)$.