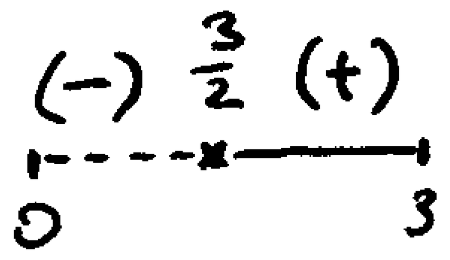
Compito di Matematica Generale del 5/9/2014 (MG1)

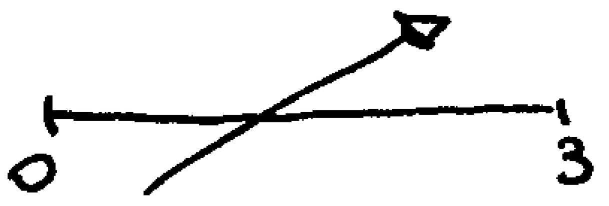
1) $f(x)=\log x-\log(3-x)$. C.E.: $\begin{cases} x>0 \\ 3-x>0 \end{cases} \Rightarrow \begin{cases} x>0 \\ x<3 \end{cases} \Rightarrow$ C.E.: $0<x<3$.

$\lim_{x\to 0^+} f(x) = (-\infty)-\log 3 = -\infty$; $\lim_{x\to 3^-} f(x) = \log 3-(-\infty) = +\infty$.

$f(x)\geq 0: f(x)=\log\frac{x}{3-x}\geq 0 \Rightarrow \frac{x}{3-x}\geq 1 \Rightarrow \frac{x}{3-x}-1\geq 0 \Rightarrow \frac{2x-3}{3-x}\geq 0 \Rightarrow x\geq \frac{3}{2}$:



$f'(x)=\frac{1}{x}+\frac{1}{3-x}=\frac{3}{x(3-x)}>0 \quad \forall x\in$ C.E.



$f''(x)=3\cdot\left(-\frac{3-2x}{x^2(3-x)^2}\right)\geq 0 \Rightarrow 3-2x\leq 0 \Rightarrow x\geq\frac{3}{2}$

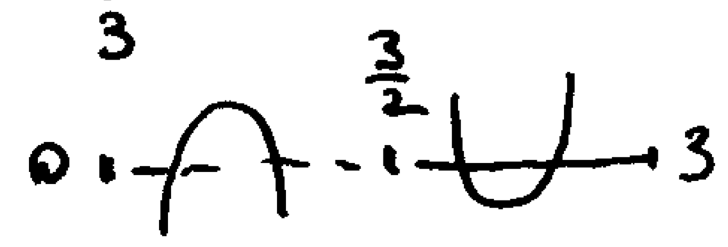
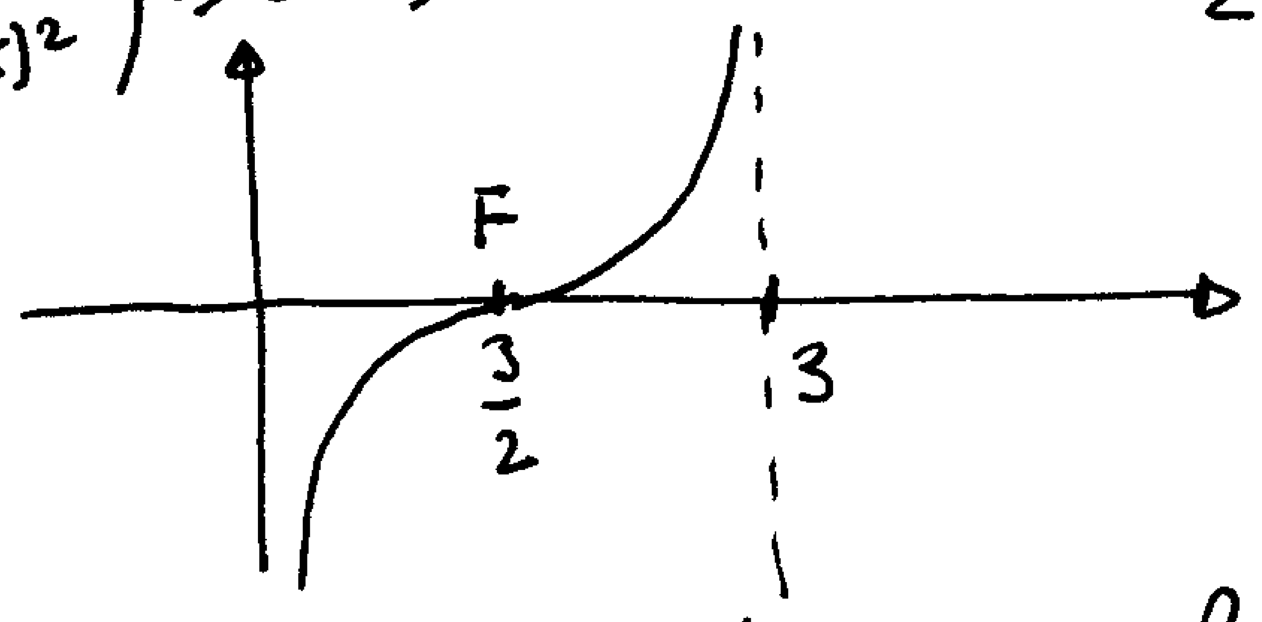


Grafico:



2) $\lim_{x\to+\infty} x\cdot \text{sen}\frac{1}{x} = (\infty\cdot 0) \Rightarrow$ pongo $\frac{1}{x}=t \Rightarrow \lim_{t\to 0}\frac{1}{t}\cdot\text{sen}\, t = \lim_{t\to 0}\frac{\text{sen}\, t}{t} = 1$.

$\lim_{x\to+\infty}\left(\frac{x}{2+x}\right)^x = \lim_{x\to+\infty}\left(\frac{2+x-2}{2+x}\right)^x = \lim_{x\to+\infty}\left(1+\frac{(-2)}{2+x}\right)^x = e^{-2} = \frac{1}{e^2}$.

3) $f(x)\sim g(x)$ per $x\to x_0 \Rightarrow \lim_{x\to x_0}\frac{f(x)}{g(x)} = 1$.

$f(x)=o(g^2(x)) \Rightarrow \lim_{x\to x_0}\frac{f(x)}{g^2(x)}=0 \Rightarrow \lim_{x\to x_0}\frac{f(x)}{g(x)}\cdot\frac{1}{g(x)} = \lim_{x\to x_0} 1\cdot\frac{1}{g(x)} = 0$ se $g(x)\to\infty$.

4) $f(x)=1-\log_3(1-\sqrt{x})$. C.E.: $\begin{cases} x\geq 0 \\ 1-\sqrt{x}>0 \end{cases} \Rightarrow \begin{cases} x\geq 0 \\ \sqrt{x}<1 \end{cases} \Rightarrow \begin{cases} x\geq 0 \\ x<1 \end{cases} \Rightarrow$ C.E.: $0\leq x<1$.

$f'(x) = -\frac{1}{1-\sqrt{x}}\cdot\left(-\frac{1}{2\sqrt{x}}\right)\cdot\log_3 e > 0 \quad \forall x\in$ C.E.. $\lim_{x\to 1^-} f(x)=+\infty$; $f(0)=1$.

Funzione invertibile in tutto il C.E..

$y=1-\log_3(1-\sqrt{x}) \Rightarrow \log_3(1-\sqrt{x}) = 1-y \Rightarrow 1-\sqrt{x} = 3^{1-y} \Rightarrow \sqrt{x} = 1-3^{1-y} \Rightarrow$

$\Rightarrow x = (1-3^{1-y})^2$. Inversa: $y=(1-3^{1-x})^2$.

5) Se $y=2x+k$ è la tangente a $f(x) \Rightarrow f'(x_0)=3e^{3x_0}=2 \Rightarrow e^{3x_0}=\frac{2}{3} \Rightarrow$

$\Rightarrow 3x_0=\log\frac{2}{3} \Rightarrow x_0=\frac{1}{3}\log\frac{2}{3}$. Equazione retta tangente: $y-f(x_0)=f'(x_0)\cdot(x-x_0) \Rightarrow$

$\Rightarrow y-e^{3\cdot\left(\frac{1}{3}\log\frac{2}{3}\right)}+1 = 2\left(x-\frac{1}{3}\log\frac{2}{3}\right) \Rightarrow y=2x-\frac{2}{3}\log\frac{2}{3}+e^{\log\frac{2}{3}}-1 = 2x-\frac{2}{3}\log\frac{2}{3}+\frac{2}{3}-1 \Rightarrow$

$\Rightarrow k = -\frac{1}{3}-\frac{2}{3}\log\frac{2}{3}$.

6) $A\cdot X = Y \Rightarrow \begin{Vmatrix} 1 & k & m \\ 2 & -1 & k \\ 1 & m & 2 \end{Vmatrix} \cdot \begin{Vmatrix} 2 \\ -1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 2-k+m \\ 4+1+k \\ 2-m+2 \end{Vmatrix} = \begin{Vmatrix} 2-k+m \\ 5+k \\ 4-m \end{Vmatrix} = \begin{Vmatrix} 3 \\ 7 \\ 1 \end{Vmatrix} \Rightarrow \begin{cases} m-k=1 \\ k=2 \\ m=3 \end{cases}$.

Soluzione: $k=2$ e $m=3$.

$\boxed{MG2}$

7) $\displaystyle\int_0^1 \sqrt{x} - e^{1-x}\,dx = \int_0^1 x^{\frac{1}{2}} - e^{1-x}\,dx = \left(\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + e^{1-x}\right|_0^1 =$

$\displaystyle = \left(\frac{2}{3}\sqrt{x^3} + e^{1-x}\right|_0^1 = \left(\frac{2}{3}+e^0\right) - \left(0+e^1\right) = \frac{2}{3}+1-e = \frac{5}{3}-e.$

8) $f(x;y) = x^3+3x^2+y^2-xy.$

$$\begin{cases} f'_x = 3x^2+6x-y=0 \\ f'_y = 2y-x=0 \end{cases} \Rightarrow \begin{cases} y = 3x^2+6x \\ 6x^2+12x-x = 6x^2+11x = x(6x+11)=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} e \begin{cases} x=-\frac{11}{6} \\ y=-\frac{11}{12} \end{cases}.$$

$P_1=(0;0)$; $P_2=\left(-\frac{11}{6};-\frac{11}{12}\right)$. $H = \begin{Vmatrix} 6x+6 & -1 \\ -1 & 2 \end{Vmatrix}$

$H(0;0) = \begin{Vmatrix} 6 & -1 \\ -1 & 2 \end{Vmatrix} \Rightarrow \begin{cases} 6>0;\ 2>0 \\ 12-1>0 \end{cases}$: Minimo; $H\left(-\frac{11}{6};-\frac{11}{12}\right) = \begin{Vmatrix} -5 & -1 \\ -1 & 2 \end{Vmatrix} \Rightarrow \begin{cases} -5<0 \\ 2>0 \end{cases}$: Sella.

9) $f(x) = x^{10}\cdot e^{-x^2}$. Dato che $e^x = 1+x+\frac{x^2}{2}+\frac{x^3}{6}+o(x^3)$ sostituendo:

$e^{-x^2} = 1-x^2+\frac{x^4}{2}-\frac{x^6}{6}+o(x^6) \Rightarrow x^{10}\cdot e^{-x^2} = x^{10}-x^{12}+\frac{x^{14}}{2}-\frac{x^{16}}{6}+o(x^{16}).$

10)

A B C	A∘B	(A∘B) e C	non ((A∘B) e C)	B e C	non (B e C)	P	Q
1 1 1	1	1	0	1	0	0	0
1 1 0	1	0	1	0	1	1	1
~~1 0 1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~1~~		
~~1 0 0~~	~~1~~	~~0~~	~~1~~	~~0~~	~~1~~		
0 1 1	1	1	0	1	0	0	0
0 1 0	1	0	1	0	1	1	1
0 0 1	0	0	1	0	1	1	1
0 0 0	0	0	1	0	1	1	1

Si tolgono le righe 3 e 4 dove $A \Rightarrow B$ risulta falsa.
Sotto questa ipotesi P e Q risultano proposizioni logicamente equivalenti.