Compito di Matematica Generale del 23/9/2014 $\boxed{MG1}$

1) $f(x)=2e^{x}+e^{-x}=2e^{x}+\frac{1}{e^{x}}=\frac{2e^{2x}+1}{e^{x}}$. C.E.: $\mathbb{R}$

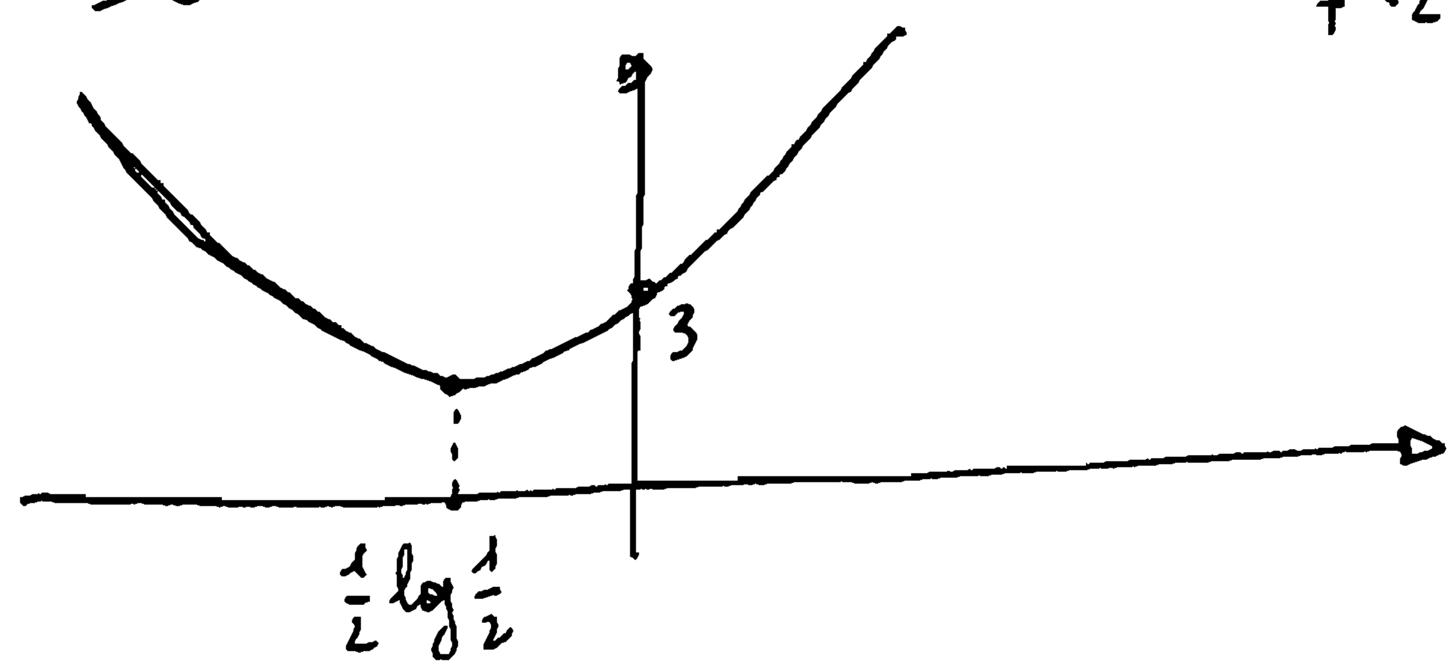
$\lim_{x\to-\infty} f(x)=(0+\infty)=+\infty$; $\lim_{x\to+\infty} f(x)=(+\infty+0)=+\infty$. $f(x)>0\ \forall x$; $f(0)=3$.

$f'(x)=2e^{x}-e^{-x}=\frac{2e^{2x}-1}{e^{x}}\geq 0 : 2e^{2x}\geq 1\Rightarrow e^{2x}\geq\frac{1}{2}\Rightarrow 2x\geq\log\frac{1}{2}\Rightarrow x\geq\frac{1}{2}\log\frac{1}{2}$ $\qquad \frac{1}{2}\log\frac{1}{2}$

$f''(x)=2e^{x}+e^{-x}>0\quad\forall x\in\mathbb{R}$

$f\left(\frac{1}{2}\log\frac{1}{2}\right)=2\left(e^{\log\frac{1}{2}}\right)^{\frac{1}{2}}+e^{\left(\log\frac{1}{2}\right)\left(-\frac{1}{2}\right)}=2\left(\frac{1}{2}\right)^{\frac{1}{2}}+\left(\frac{1}{2}\right)^{-\frac{1}{2}}=$
$=\frac{2}{\sqrt{2}}+\sqrt{2}=2\sqrt{2}$.

Grafico:



2) $\lim_{x\to 0}\frac{\log(1+2x)}{\log(1+3x)}=\lim_{x\to 0}\frac{\log(1+2x)}{2x}\cdot\frac{2x}{3x}\cdot\frac{3x}{\log(1+3x)}=1\cdot\frac{2}{3}\cdot 1=\frac{2}{3}$. $\left(\text{da }\lim_{x\to 0}\frac{\log(1+x)}{x}=1\right)$.

$\lim_{x\to-\infty}\frac{3^{x}-2^{-x}}{2^{x}+3^{-x}}=\lim_{x\to-\infty}\frac{-2^{-x}}{3^{-x}}=0^{-}\quad\left(3^{x}\to 0; 2^{x}\to 0; 2^{-x}=o(3^{-x})\right)$.

3) $\lim_{x\to 0}\frac{\operatorname{sen}^{2}x-1+\cos x}{\operatorname{tg}^{2}x}=\lim_{x\to 0}\frac{\operatorname{sen}^{2}x}{\operatorname{tg}^{2}x}-\frac{1-\cos x}{x^{2}}\cdot\frac{x^{2}}{\operatorname{tg}^{2}x}=1-\frac{1}{2}\cdot 1=\frac{1}{2}$.

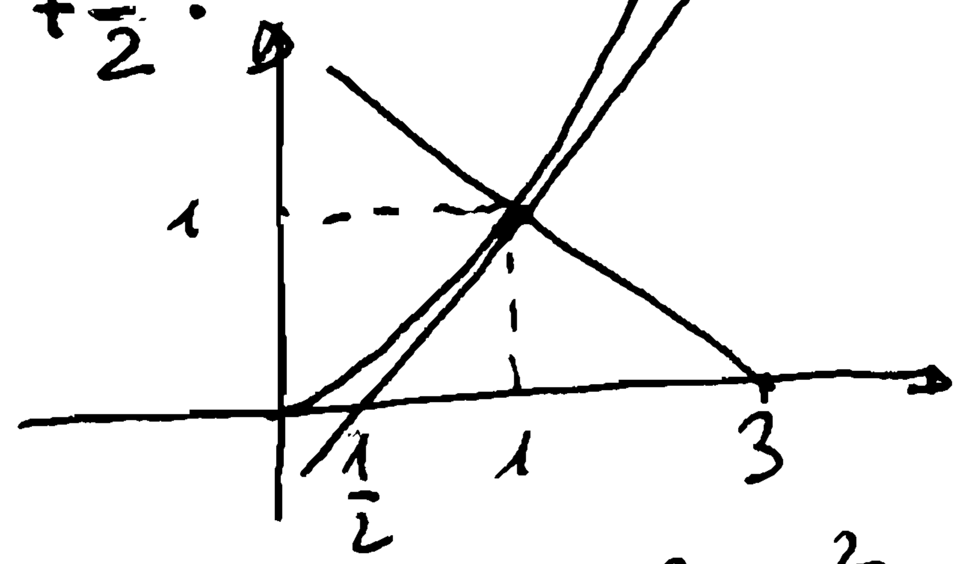
I due infinitesimi sono dello stesso ordine ma non asintoticamente equivalenti.

4) $f(x)=x^{2}$; $f(1)=1$; $f'(x)=2x$; $f'(1)=2$. Equazione retta tangente: $y-1=2(x-1)\Rightarrow$
$\Rightarrow y=2x-1$. Punto di tangenza: $P(1;1)$. Equazione retta perpendicolare alla tangente passante per P: $y-1=-\frac{1}{2}(x-1)\Rightarrow y=-\frac{1}{2}x+\frac{3}{2}$.

$y=2x-1$ taglia l'asse x in $x=\frac{1}{2}$.

$y=-\frac{1}{2}x+\frac{3}{2}$ taglia l'asse x in $x=3$.

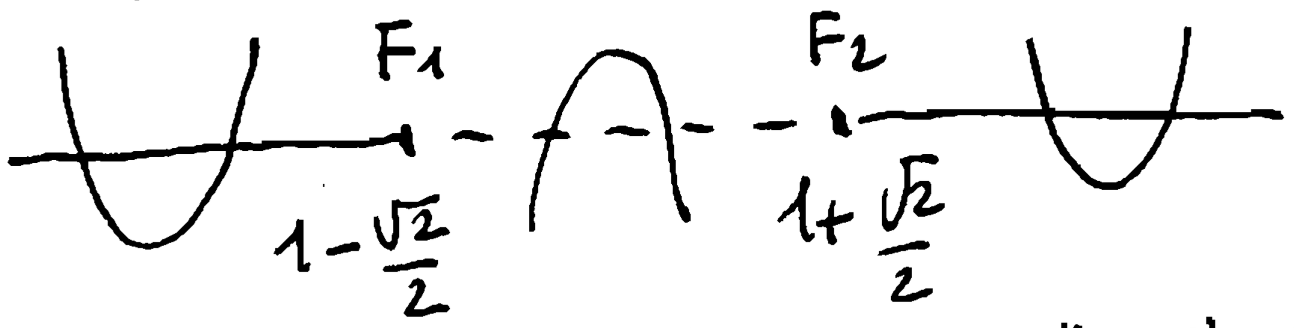
Area triangolo: $A=\frac{1}{2}\cdot 1\cdot\left(3-\frac{1}{2}\right)=\frac{5}{4}$.



5) $f(x)=e^{2x-x^{2}}$; $f'(x)=(2-2x)e^{2x-x^{2}}$; $f''(x)=(-2)e^{2x-x^{2}}+(2-2x)(2-2x)e^{2x-x^{2}}=$

$f''(x)=e^{2x-x^{2}}\cdot(4x^{2}-8x+2)\geq 0\Rightarrow 2x^{2}-4x+1\geq 0 : x=\frac{2\pm\sqrt{4-2}}{2}=1\pm\frac{\sqrt{2}}{2}$

$f''(x)\geq 0$:



6) $A\circ X=\begin{Vmatrix}1&1&2\\0&K&1\\K&2&0\end{Vmatrix}\cdot\begin{Vmatrix}1\\1\\0\end{Vmatrix}=\begin{Vmatrix}1+1+0\\0+K+0\\K+2+0\end{Vmatrix}=\begin{Vmatrix}2\\K\\2+K\end{Vmatrix}$

$\|A\cdot X\|=\sqrt{4+k^{2}+(2+k)^{2}}=\sqrt{2k^{2}+4k+8}=3\Rightarrow 2k^{2}+4k+8=9\Rightarrow 2k^{2}+4k-1=0\Rightarrow$

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$\Rightarrow k = \frac{-2 \pm \sqrt{4+2}}{2} = \frac{-2 \pm \sqrt{6}}{2} = -1 \pm \frac{\sqrt{6}}{2}.$

$(2;k;2+k) \perp (-1;1;1) = -2+k+2+k = 0 \Rightarrow 2k = 0 \Rightarrow k = 0.$

7) $\int_0^1 e^x - kx\,dx = \left(e^x - k\cdot\frac{x^2}{2}\right)\Big|_0^1 = e - \frac{k}{2} - (1-0) = e - \frac{k}{2} - 1 = 1 \Rightarrow$

$\Rightarrow \frac{k}{2} = e - 2 \Rightarrow k = 2(e-2).$

8) $f(x;y;z) = \log(x-y) - e^{z^2-x} + 3xyz.$

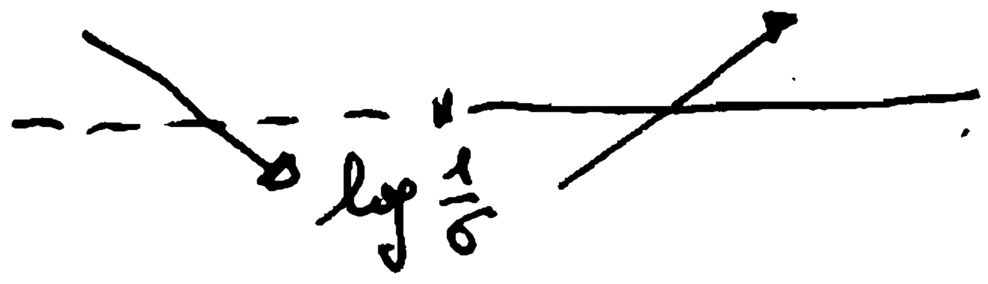
$\nabla f(x;y;z) = \left(\frac{1}{x-y} + e^{z^2-x} + 3yz; -\frac{1}{x-y} - 0 + 3xz; 0 - 2z e^{z^2-x} + 3xy\right).$

$\nabla f(1;0;1) = \left(\frac{1}{1-0} + e^0 + 3\cdot 0; -\frac{1}{1-0} - 0 + 3\cdot 1; 0 - 2\cdot 1\cdot e^0 + 3\cdot 0\right) = (2;2;-2).$

9) $f(x) = e^x(e^x-1)^5$

$f'(x) = e^x\cdot(e^x-1)^5 + e^x\cdot 5(e^x-1)^4\cdot e^x = e^x(e^x-1)^4\cdot(e^x-1+5e^x) \geq 0$

$e^x > 0\ \forall x;\ (e^x-1)^4 \geq 0\ \forall x;\ 6e^x - 1 \geq 0 \Rightarrow e^x \geq \frac{1}{6} \Rightarrow x \geq \log\frac{1}{6}$



Il punto $x = \log\frac{1}{6}$ è un punto di minimo assoluto.

10) $f(x) = k\cdot e^{3x-m}$. $f\left(\frac{1}{3}\right) = k\cdot e^{1-m} = k\cdot e\cdot e^{-m} = 1 \Rightarrow e^{-m} = \frac{1}{ke}.$

$f'(x) = 3k e^{3x-m} \Rightarrow f'(0) = 3k\cdot e^{-m} \Rightarrow f'(0) = 3k\cdot\frac{1}{ke} = \frac{3}{e}.$