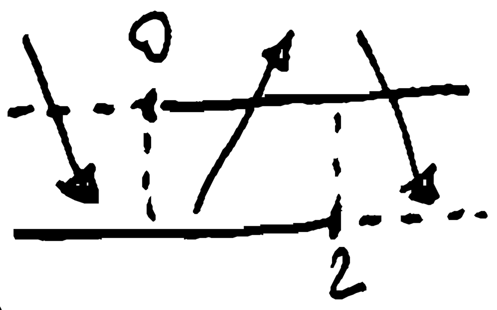
Compito di Matematica Generale del 11/10/2014. MG1

1) $f(x) = x^2 \cdot e^{1-x}$. C.E.: $\mathbb{R}$. $\lim_{x \to -\infty} f(x) = +\infty$; $\lim_{x \to +\infty} f(x) = 0^+$.

$f(x) \geq 0 \ \forall x \in \mathbb{R}$; $f(0) = 0$.

$f'(x) = 2x e^{1-x} + x^2(-e^{1-x}) = e^{1-x}(2x - x^2) \geq 0 \Rightarrow x(2-x) \geq 0 \left\langle \begin{array}{l} x \geq 0 \\ x \leq 2 \end{array} \right.$ 

$f''(x) = -e^{1-x}(2x - x^2) + e^{1-x}(2-2x) = e^{1-x}(x^2 - 4x + 2) \geq 0$: $x = 2 \pm \sqrt{4-2} = 2 \pm \sqrt{2}$

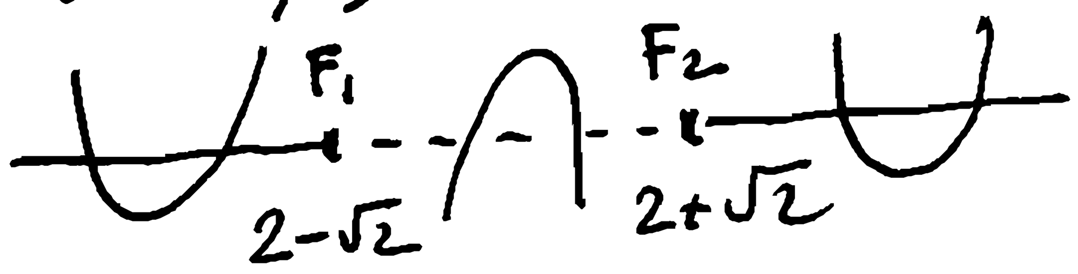
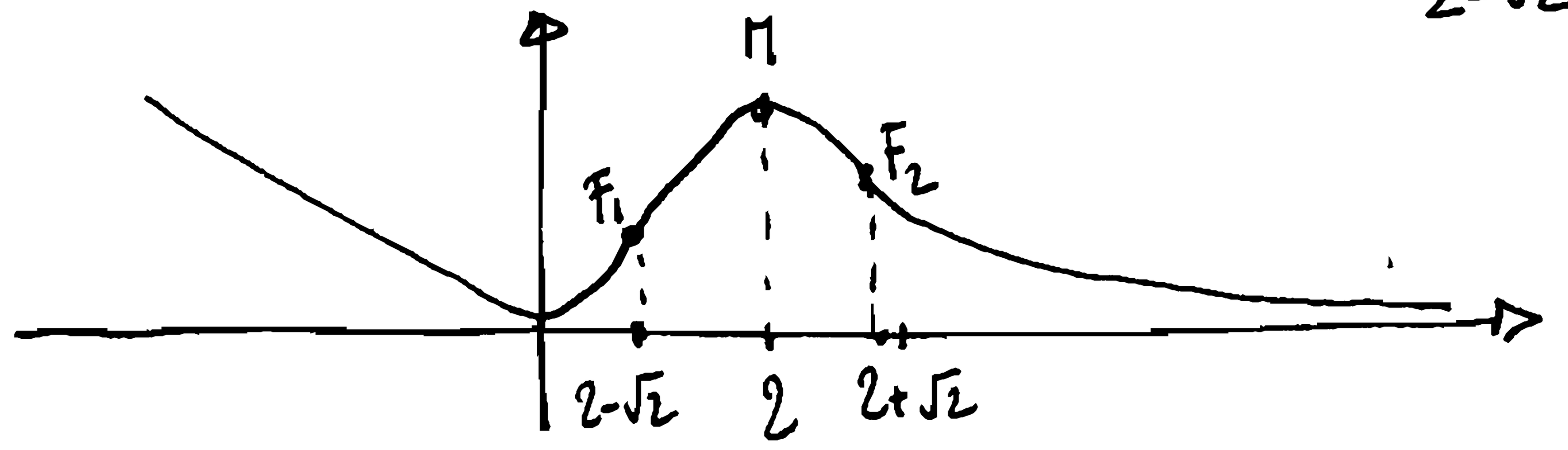
$f''(x) \geq 0$ per $x \leq 2 - \sqrt{2}$ oppure $x \geq 2 + \sqrt{2}$:  . $f(2) = \frac{4}{e}$

Grafico:



2) $\lim_{x \to 0} \frac{\operatorname{sen} 2x - \operatorname{sen} 3x}{x} = \lim_{x \to 0} \frac{\operatorname{sen} 2x}{2x} \cdot 2 - \frac{\operatorname{sen} 3x}{3x} \cdot 3 = 1 \cdot 2 - 1 \cdot 3 = 2 - 3 = -1$.

$\lim_{x \to +\infty} \frac{e^x - 3^x + \log x}{e^x - 2^x + \operatorname{sen} x} = \lim_{x \to +\infty} \frac{-3^x}{e^x} = \lim_{x \to +\infty} -\left(\frac{3}{e}\right)^x = -\infty$. $\left(\begin{array}{l} e^x = o(3^x);\ \log x = o(3^x) \\ 2^x = o(e^x);\ \operatorname{sen} x = o(e^x) \end{array} \right)$.

3) $\lim_{x \to 0} \frac{\log(1+Kx)}{3x} = \lim_{x \to 0} \frac{\log(1+Kx)}{K \cdot x} \cdot \frac{K}{3} = \frac{K}{3} \cdot 1 = 5 \Rightarrow K = 15$.

4) $f(x) = \log x$; $f'(x) = \frac{1}{x}$ per la tangente: $\frac{1}{x} = 3 \Rightarrow x_0 = \frac{1}{3}$.

Equazione retta tangente in $x_0 = \frac{1}{3}$: $y - \log\frac{1}{3} = 3\left(x - \frac{1}{3}\right) \Rightarrow y = 3x - 1 + \log\frac{1}{3} \Rightarrow$

$\Rightarrow y = 3x - 1 - \log 3$. Se $y = 3x + K \Rightarrow K = -1 - \log 3$.

5) $\int_0^1 x - \sqrt{x} - e^{-x}\, dx = \left(\frac{x^2}{2} - \frac{1}{\frac{1}{2}+1} \cdot x^{\frac{1}{2}+1} + e^{-x} \right|_0^1 = \left(\frac{x^2}{2} - \frac{2}{3} x^{\frac{3}{2}} + e^{-x} \right|_0^1 =$

$= \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{e} \right) - (0 - 0 + 1) = \left(\frac{1}{2} - \frac{2}{3} - 1 \right) + \frac{1}{e} = \frac{1}{e} - \frac{7}{6}$.

6) $f(x;y) = x^2 - xy^2 + xy$.

$\begin{cases} f'_x = 2x - y^2 + y = 0 \\ f'_y = -2xy + x = x(1-2y) = 0 \end{cases} \Rightarrow \begin{cases} y - y^2 = y(1-y) = 0 \\ x = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$ e $\begin{cases} x = 0 \\ y = 1 \end{cases}$ oppure $\begin{cases} y = \frac{1}{2} \\ 2x - \frac{1}{4} + \frac{1}{2} = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y = \frac{1}{2} \\ x = -\frac{1}{8} \end{cases}$. Ci sono 3 punti stazionari: $P_1 = (0;0)$; $P_2 = (0;1)$ e $P_3 = \left(-\frac{1}{8}; \frac{1}{2}\right)$.

$H(x;y) = \left\| \begin{array}{cc} 2 & 1-2y \\ 1-2y & -2x \end{array} \right\|$. $H(0;0) = \left\| \begin{array}{cc} 2 & 1 \\ 1 & 0 \end{array} \right\|$: $|H| = -1 < 0$: Sella;

$H(0;1) = \left\| \begin{array}{cc} 2 & -1 \\ -1 & 0 \end{array} \right\|$: $|H| = -1 < 0$: Sella; $H\left(-\frac{1}{8}; \frac{1}{2}\right) = \left\| \begin{array}{cc} 2 & 0 \\ 0 & \frac{1}{4} \end{array} \right\| \Rightarrow \begin{cases} 2 > 0;\ \frac{1}{4} > 0 \\ 2 \cdot \frac{1}{4} - 0 > 0 \end{cases}$: Minimo.

MG2

7) $A\cdot B\cdot A\cdot X = \begin{Vmatrix}1&2\\2&-1\end{Vmatrix}\cdot\begin{Vmatrix}1&K\\K&1\end{Vmatrix}\cdot\begin{Vmatrix}1&2\\2&-1\end{Vmatrix}\cdot\begin{Vmatrix}1\\1\end{Vmatrix} = \begin{Vmatrix}1&2\\2&-1\end{Vmatrix}\cdot\begin{Vmatrix}1&K\\K&1\end{Vmatrix}\cdot\begin{Vmatrix}3\\1\end{Vmatrix} =$

$= \begin{Vmatrix}1&2\\2&-1\end{Vmatrix}\cdot\begin{Vmatrix}3+K\\3K+1\end{Vmatrix} = \begin{Vmatrix}3+K+6K+2\\6+2K-3K-1\end{Vmatrix} = \begin{Vmatrix}7K+5\\5-K\end{Vmatrix}.$

$(7K+5;5-K)\perp(1;2) \Rightarrow 7K+5+10-2K=0 \Rightarrow 5K+15=0 \Rightarrow K=\frac{-15}{5} \Rightarrow K=-3.$

8) $f(x)=x^2$; $g(x)=\operatorname{sen}x$; $h(x)=2x-1.$

$f(g(h(x))) = f(g(2x-1)) = f(\operatorname{sen}(2x-1)) = (\operatorname{sen}(2x-1))^2 = \operatorname{sen}^2(2x-1).$

$\mathcal{D}(\operatorname{sen}^2(2x-1)) = 2\cdot\operatorname{sen}(2x-1)\cdot\cos(2x-1)\cdot 2.$

$h(g(f(x))) = h(g(x^2)) = h(\operatorname{sen}x^2) = 2\cdot\operatorname{sen}x^2-1.$

$\mathcal{D}(2\cdot\operatorname{sen}x^2-1) = 2\cdot\cos x^2\cdot 2x - 0 = 2\cos x^2\cdot 2x.$

9)

A	B	A ⇒ B	non (A ⇒ B)	non A	(non A ⇒ B)	[non (A ⇒ B)] ⇒ (non A ⇒ B)]
1	1	1	0	0	1	1
1	0	0	1	0	1	1
0	1	1	0	1	1	1
0	0	1	0	1	0	1

La proposizione data risulta una tautologia.

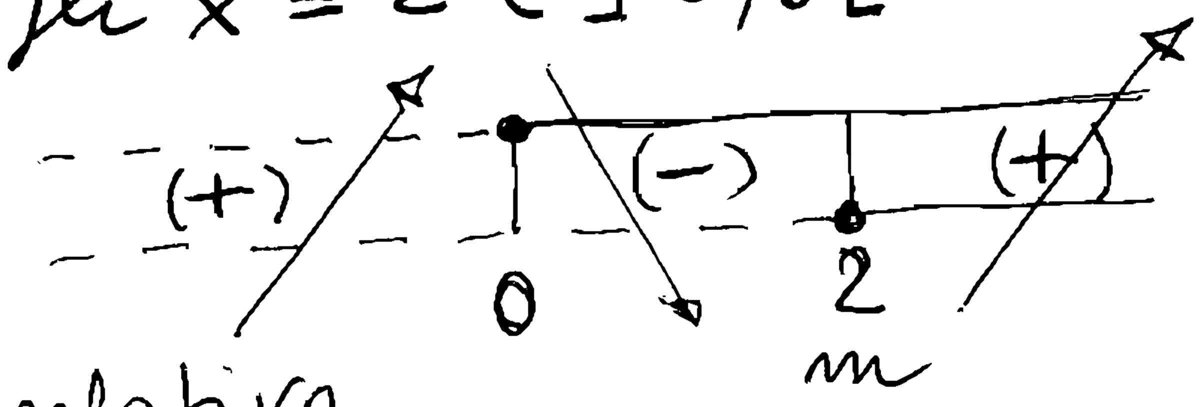
10) $f(x)=x^3-3x^2$: funzione continua e derivabile $\forall x\in\mathbb{R}$.

$f(0)=0=f(k)=k^3-3k^2=k^2(k-3)=0$ per $k=0$ e per $k=3$.

Quindi $k=3$ e l'intervallo è $[0;3]$.

$f'(x)=3x^2-6x=3x(x-2)=0$ per $x=2\in\,]0;3[$.

$f'(x)\geq 0$: $3x(x-2)\geq 0 \begin{cases} x\geq 0 \\ x\geq 2\end{cases}$:



Il punto $x=2$ è un punto di minimo relativo.