Prova Intermedia di Matematica Generale del 10/11/2014 Compito A1

1) $\lim_{x\to 0}\dfrac{\operatorname{sen}(x^2-3x)}{2x-x^3}=\lim_{x\to 0}\dfrac{\operatorname{sen}(x^2-3x)}{x^2-3x}\cdot\lim_{x\to 0}\dfrac{x^2-3x}{2x-x^3}=1\cdot\lim_{x\to 0}\dfrac{\cancel{x}(x-3)}{\cancel{x}(2-x^2)}=1\cdot\left(-\dfrac{3}{2}\right)=-\dfrac{3}{2}.$

$\lim_{x\to +\infty}\left(\dfrac{x-\operatorname{sen}x+x^2}{x+2x^2}\right)^{\frac{1-x^2}{x}}=\left(\to\dfrac{1}{2}\right)^{(\to -\infty)}=+\infty.$

2) $f(x)=3x-2$; $g(x)=2^x$; $h(x)=??$

$f(g(h(x)))=f\left(2^{h(x)}\right)=3\cdot 2^{h(x)}-2=\operatorname{sen}x \Rightarrow 3\cdot 2^{h(x)}=2+\operatorname{sen}x \Rightarrow$

$\Rightarrow 2^{h(x)}=\dfrac{1}{3}(2+\operatorname{sen}x)\Rightarrow h(x)=\log_2\left(\dfrac{1}{3}(2+\operatorname{sen}x)\right).$

3) $\log\left(\dfrac{3^x+1}{2-3^x}\right)>0$. C.E.: $\dfrac{3^x+1}{2-3^x}>0 \Rightarrow 2-3^x>0 \Rightarrow 3^x<2 \Rightarrow x<\log_3 2.$

$\log\dfrac{3^x+1}{2-3^x}>0 \Rightarrow \dfrac{3^x+1}{2-3^x}>1 \Rightarrow \dfrac{3^x+1}{2-3^x}-1>0 \Rightarrow \dfrac{3^x+1-2+3^x}{2-3^x}>0 \Rightarrow \dfrac{2\cdot 3^x-1}{2-3^x}>0 \Rightarrow$

$\Rightarrow 2\cdot 3^x-1>0 \Rightarrow 2\cdot 3^x>1 \Rightarrow 3^x>\dfrac{1}{2} \Rightarrow x>\log_3\dfrac{1}{2}=-\log_3 2.$

$\log_3 2$

$-\log_3 2.$

Soluzione: $-\log_3 2<x<\log_3 2.$

4)

A B C	(A⇒B)	non C	(B⇒non C)	(A⇒B) o (B⇒non C)	A⇔C	P e Q	A⇔B
1 1 1	1	0	0	1	1	1	1
1 1 0	1	1	1	1	0	0	1
~~1 0 1~~	~~0~~	~~0~~	~~1~~	~~1~~	1	1	0
~~1 0 0~~	~~0~~	~~1~~	~~1~~	~~1~~	0	0	0
~~0 1 1~~	~~1~~	~~0~~	~~0~~	~~1~~	0	0	0
~~0 1 0~~	~~1~~	~~1~~	~~1~~	~~1~~	1	1	0
0 0 1	1	0	1	1	0	0	1
0 0 0	1	1	1	1	1	1	1
						*	

La proposizione P e Q è nella colonna *.

5) $\lim_{x\to 0}\dfrac{(1+kx)^{-3}-1}{\log(1+2x)}=\lim_{x\to 0}\dfrac{(1+kx)^{3}-1}{kx}\cdot\dfrac{kx}{2x}\cdot\dfrac{2x}{\log(1+2x)}=$

$=\lim_{t\to 0}\dfrac{(1+t)^{-3}-1}{t}\cdot\dfrac{k}{2}\cdot\lim_{t\to 0}\dfrac{1}{\frac{\log(1+t)}{t}}=-3\cdot\dfrac{k}{2}\cdot 1=5 \Rightarrow$

$\Rightarrow k=-\dfrac{10}{3}.$

Prova Intermedia di Matematica Generale del 10/11/2014 Compito B1

1) $\lim_{x\to 0} \frac{\log(1+x-x^2)}{2x^2+x} = \lim_{x\to 0} \frac{\log(1+x-x^2)}{x-x^2} \cdot \lim_{x\to 0} \frac{x-x^2}{2x^2+x} = \lim_{t\to 0} \frac{\log(1+t)}{t} \cdot \lim_{x\to 0} \frac{\cancel{x}(1-x)}{\cancel{x}(2x+1)} = 1\cdot 1 = 1.$

$\lim_{x\to +\infty} \left(\frac{3x - \operatorname{sen} x + 2x^2}{x^2-x}\right)^{\frac{1+x}{x^2}} = \left(\to \frac{2}{1}\right)^{(\to 0^+)} = 2^0 = 1.$

2) $f(x) = 3^x - 2$; $g(x) = 2x+3$; $h(x) = ??.$

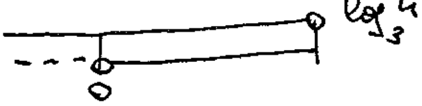
$f(g(h(x))) = f(2h(x)+3) = 3^{2h(x)+3} - 2 = \cos x \Rightarrow 3^{2h(x)+3} = 2 + \cos x \Rightarrow$

$\Rightarrow 2h(x) + 3 = \log_3(2+\cos x) \Rightarrow 2h(x) = \log_3(2+\cos x) - 3 \Rightarrow h(x) = \frac{1}{2}\left(\log_3(2+\cos x) - 3\right).$

3) $\log\left(\frac{3^x+2}{4-3^x}\right) > 0.$ C.E.: $\frac{3^x+2}{4-3^x} > 0 \Rightarrow 4 - 3^x > 0 \Rightarrow 3^x < 4 \Rightarrow x < \log_3 4.$

$\log\left(\frac{3^x+2}{4-3^x}\right) > 0 \Rightarrow \frac{3^x+2}{4-3^x} > 1 \Rightarrow \frac{3^x+2}{4-3^x} - 1 > 0 \Rightarrow \frac{3^x+2-4+3^x}{4-3^x} > 0 \Rightarrow \frac{2\cdot 3^x - 2}{4-3^x} > 0 \Rightarrow$

$\Rightarrow 2\cdot 3^x - 2 > 0 \Rightarrow 3^x > 1 \Rightarrow x > 0.$



soluzione: $0 < x < \log_3 4.$

4)

A B C	$(A \Leftrightarrow C)$	$(B \Leftrightarrow A)$	$(A \Leftrightarrow C)$ e $(B \Leftrightarrow A)$	non C	(non C $\Rightarrow$ B)	P∘Q	$(A \Leftrightarrow B)$
1 1 1	1	1	1	0	1	1	1
1 1 0	0	1	0	1	1	1	1
1 0 1	1	0	0	0	1	1	0
1 0 0	0	0	0	1	0	0	0
0 1 1	0	0	0	0	1	1	0
0 1 0	1	0	0	1	1	1	0
0 0 1	0	1	0	0	1	1	1
0 0 0	1	1	1	1	0	1 *	1

La proposizione P∘Q è nella colonna *.

5) $\lim_{x\to 0} \frac{\sqrt[3]{1+kx}-1}{\operatorname{sen} 3x} = \lim_{x\to 0} \frac{(1+kx)^{\frac{1}{3}}-1}{kx} \cdot \frac{kx}{3x} \cdot \frac{3x}{\operatorname{sen} 3x} =$

$= \lim_{t\to 0} \frac{(1+t)^{\frac{1}{3}}-1}{t} \cdot \frac{k}{3} \cdot \lim_{t\to 0} \frac{1}{\frac{\operatorname{sen} t}{t}} = \frac{1}{3}\cdot\frac{k}{3}\cdot 1 = 2 \Rightarrow$

$\Rightarrow k = 18.$

Prova Intermedia di Matematica Generale del 10/11/2014 Compito C1

1) $\lim_{x\to 0} \dfrac{x^3+x^2+x}{tg(x^2+2x)} = \lim_{x\to 0} \dfrac{x^2+2x}{tg(x^2+2x)} \lim_{x\to 0} \dfrac{x^3+x^2+x}{x^2+2x} = \lim_{t\to 0} \dfrac{1}{\frac{tg\,t}{t}} \cdot \lim_{x\to 0} \dfrac{\cancel{x}(x^2+x+1)}{\cancel{x}(x+2)} = 1 \cdot \dfrac{1}{2} = \dfrac{1}{2}.$

$\lim_{x\to +\infty} \left(\dfrac{x+\operatorname{sen} x - 2x^2}{1-x} \right)^{\frac{x^2}{1-x}} = (\to +\infty)^{(\to -\infty)} = 0^+.$

2) $f(x) = 2x-3$; $g(x) = \log_2 x$; $h(x) = ??.$

$f(g(h(x))) = f(\log_2 h(x)) = 2\log_2 h(x) - 3 = \operatorname{sen} x \Rightarrow 2\log_2 h(x) = 3 + \operatorname{sen} x \Rightarrow$

$\Rightarrow \log_2 h(x) = \frac{1}{2}(3+\operatorname{sen} x) \Rightarrow h(x) = 2^{\frac{1}{2}(3+\operatorname{sen} x)}.$

3) $\log\left(\dfrac{3-2^x}{2^x+1}\right) > 0.$ C.E.: $\dfrac{3-2^x}{2^x+1} > 0 \Rightarrow 3-2^x > 0 \Rightarrow 2^x < 3 \Rightarrow x < \log_2 3.$

$\log\left(\dfrac{3-2^x}{2^x+1}\right) > 0 \Rightarrow \dfrac{3-2^x}{2^x+1} > 1 \Rightarrow \dfrac{3-2^x}{2^x+1} - 1 > 0 \Rightarrow \dfrac{3-2^x-2^x-1}{2^x+1} > 0 \Rightarrow \dfrac{2-2\cdot 2^x}{2^x+1} > 0 \Rightarrow$

$\Rightarrow 2 - 2\cdot 2^x > 0 \Rightarrow 2^x < 1 \Rightarrow x < 0.$

$0 \qquad \log_2 3$

Soluzione: $-\infty < x < 0.$

4)

A B C	non A	(B ⇒ non A)	(A ⇔ C)	(B ⇔ C)	(A ⇔ C) o (B ⇔ C)	P e Q	(A ⇔ B)
1 1 1	0	0	1	1	1	0	1
1 1 0	0	0	0	0	0	0	1
~~1 0 1~~	0	1	1	0	1	1	0
~~1 0 0~~	0	1	0	1	1	1	0
~~0 1 1~~	1	1	0	1	1	1	0
~~0 1 0~~	1	1	1	0	1	1	0
0 0 1	1	1	0	0	0	0	1
0 0 0	1	1	1	1	1	1	1
						*	

La Proposizione P e Q è nella colonna *.

5) $\lim_{x\to 0} \dfrac{(1+kx)^{-2}-1}{e^{2x}-1} = \lim_{x\to 0} \dfrac{(1+kx)^{-2}-1}{kx} \cdot \dfrac{kx}{2x} \cdot \dfrac{2x}{e^{2x}-1} =$

$= \lim_{t\to 0} \dfrac{(1+t)^{-2}-1}{t} \cdot \dfrac{k}{2} \cdot \lim_{t\to 0} \dfrac{1}{\frac{e^t-1}{t}} = -2 \cdot \dfrac{k}{2} \cdot 1 = 3 \Rightarrow$

$\Rightarrow k = -3.$

Prova Intermedia di Matematica Generale del 10/11/2014 Compito D1

1) $\lim_{x\to 0} \frac{1-\cos x^2}{x^4+x^5} = \lim_{x\to 0} \frac{1-\cos x^2}{x^4} \cdot \lim_{x\to 0} \frac{x^4}{x^4+x^5} = \lim_{t\to 0} \frac{1-\cos t}{t} \cdot \lim_{x\to 0} \frac{\cancel{x^4}\, 1}{\cancel{x^4}(1+x)} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$

$\lim_{x\to +\infty} \left(\frac{\cos x + \operatorname{sen} x + x^2}{1+2x^2}\right)^{\frac{2x^2}{x-1}} = \left(\to \frac{1}{2}\right)^{(\to +\infty)} = 0^+.$

2) $f(x) = \log_2 x;\ g(x) = 3x+1;\ h(x) = ??$

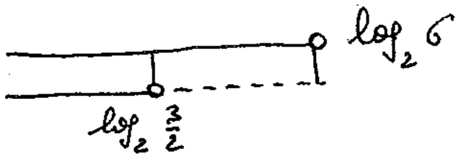
$f(g(h(x))) = f(3h(x)+1) = \log_2(3h(x)+1) = \cos x \Rightarrow 3h(x)+1 = 2^{\cos x} \Rightarrow$

$\Rightarrow 3h(x) = 2^{\cos x} - 1 \Rightarrow h(x) = \frac{1}{3}\left(2^{\cos x} - 1\right).$

3) $\log\left(\frac{6-2^x}{2^x+3}\right) > 0.$ C.E.: $\frac{6-2^x}{2^x+3} > 0 \Rightarrow 6-2^x > 0 \Rightarrow 2^x < 6 \Rightarrow x < \log_2 6.$

$\log\left(\frac{6-2^x}{2^x+3}\right) > 0 \Rightarrow \frac{6-2^x}{2^x+3} > 1 \Rightarrow \frac{6-2^x}{2^x+3} - 1 > 0 \Rightarrow \frac{6-2^x-2^x-3}{2^x+3} > 0 \Rightarrow \frac{3-2\cdot 2^x}{2^x+3} > 0 \Rightarrow$

$\Rightarrow 3 - 2\cdot 2^x > 0 \Rightarrow 2^x < \frac{3}{2} \Rightarrow x < \log_2 \frac{3}{2}.$



Soluzione: $-\infty < x < \log_2 \frac{3}{2}.$

4)

A B C	non C	(non C ⇒ A)	(B ⇔ C)	(B ⇒ A)	(B ⇔ C) e (B ⇒ A)	P o Q	A ⇔ B
1 1 1	0	1	1	1	1	1	1
1 1 0	1	1	0	1	0	1	1
1 0 1	0	1	0	1	0	1	0
1 0 0	1	1	1	1	1	1	0
0 1 1	0	1	1	0	0	1	0
0 1 0	1	0	0	0	0	0	0
0 0 1	0	1	0	1	0	1	1
0 0 0	1	0	1	1	1	1	1

La Proposizione P o Q è nella colonna ✱.

5) $\lim_{x\to 0} \frac{\sqrt{(1+Kx)^3} - 1}{\operatorname{tg} 4x} = \lim_{x\to 0} \frac{(1+Kx)^{\frac{3}{2}} - 1}{Kx} \cdot \frac{Kx}{4x} \cdot \frac{4x}{\operatorname{tg} 4x} =$

$= \lim_{t\to 0} \frac{(1+t)^{\frac{3}{2}} - 1}{t} \cdot \frac{K}{4} \cdot \lim_{t\to 0} \frac{1}{\frac{\operatorname{tg} t}{t}} = \frac{3}{2} \cdot \frac{K}{4} \cdot 1 = 1 \Rightarrow$

$\Rightarrow K = \frac{8}{3}.$