Prova Intermedia di Matematica Generale del 10/11/2014 Compito A2

1) $\lim_{x\to 0}\frac{\operatorname{sen}^3 x^2}{7x^6+x^8}=\lim_{x\to 0}\frac{\operatorname{sen}^3 x^2}{(x^2)^3}\cdot\frac{x^6}{7x^6+x^8}=\lim_{x\to 0}\left(\frac{\operatorname{sen} x^2}{x^2}\right)^3\cdot\lim_{x\to 0}\frac{\cancel{x^6}\,1}{\cancel{x^6}(7+x)}=1^3\cdot\frac{1}{7}=\frac{1}{7}.$

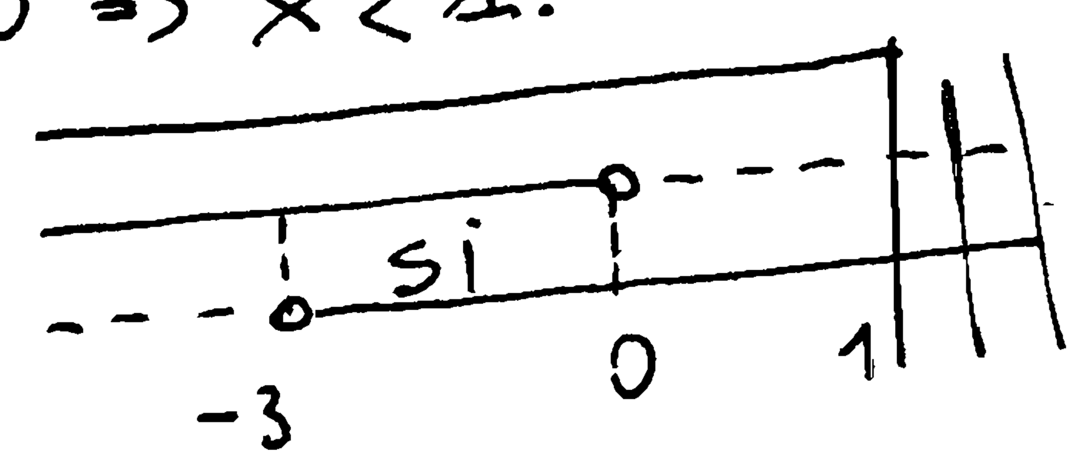
$\lim_{x\to+\infty}\left(\frac{x+2}{x}\right)^{\frac{x^2+1}{3x}}=\lim_{x\to+\infty}\left[\left(1+\frac{2}{x}\right)^x\right]^{\frac{x^2+1}{3x^2}}=\left(\to e^2\right)^{\left(\to\frac{1}{3}\right)}=e^{\frac{2}{3}}.$

2) $f(x)=\frac{2x-1}{3x+4}$; $f(g(x))=\frac{2g(x)-1}{3g(x)+4}=2^x \Rightarrow 2g(x)-1=3\cdot 2^x\cdot g(x)+4\cdot 2^x \Rightarrow$

$\Rightarrow g(x)\cdot(2-3\cdot 2^x)=4\cdot 2^x+1 \Rightarrow g(x)=\frac{4\cdot 2^x+1}{2-3\cdot 2^x}=\frac{2^{x+2}+1}{2-3\cdot 2^x}.$

3) $0<\log_2(1-x)<2.$ C.E.: $1-x>0 \Rightarrow x<1.$

$\begin{cases}\log_2(1-x)>0\\ \log_2(1-x)<2\end{cases} \Rightarrow \begin{cases}1-x>1\\ 1-x<4\end{cases} \Rightarrow \begin{cases}x<0\\ x>-3\end{cases}$



Soluzione: $-3<x<0.$

4)

A	B	$A \subset B$	e(B)	e(B) $\subset$ A	P	M	P e M
1	1	1	0	1	1	1	1
1	0	0	1	1	0	1	0
0	1	1	0	1	1	1	1
0	0	1	1	0	1	0	0

5) $f(x)=(x-2)\cdot\log_3(3x-k)$; $g(x)=x^2-5x+6=(x-2)(x-3).$

$f(x)=o(g(x))$ per $x\to 1$ se $\lim_{x\to 1}\frac{f(x)}{g(x)}=0.$

$\lim_{x\to 1}\frac{(x-2)\log_3(3x-k)}{(x-2)(x-3)}=\lim_{x\to 1}\frac{x-2}{(x-2)(x-3)}\cdot\lim_{x\to 1}\log_3(3x-k)=$

$=\lim_{x\to 1}\frac{1}{x-3}\cdot\lim_{x\to 1}\log_3(3x-k)=-\frac{1}{2}\cdot\lim_{x\to 1}\log_3(3x-k)=-\frac{1}{2}\cdot\log_3(3-k)=0$

se $3-k=1 \Rightarrow k=2.$

Prova Intermedia di Matematica Generale del 10/11/2014 Compito B2

1) $\lim_{x\to 0} \frac{1-\cos 3x^2}{5x^4} = \lim_{x\to 0} \frac{1-\cos 3x^2}{9x^4} \cdot \frac{9x^4}{5x^4} = \lim_{t\to 0} \frac{1-\cos t}{t} \cdot \frac{9}{5} = \frac{1}{2} \cdot \frac{9}{5} = \frac{9}{10}.$

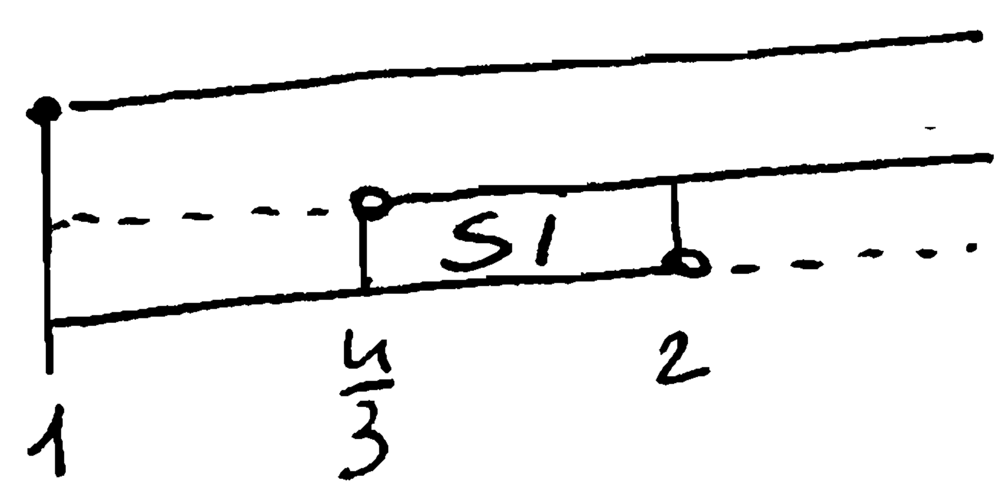
$\lim_{x\to +\infty} \left(\frac{x-2}{x}\right)^{\frac{x^2-1}{2x}} = \lim_{x\to +\infty} \left[\left(1+\frac{(-2)}{x}\right)^x\right]^{\frac{x^2-1}{2x^2}} = (\to e^{-2})^{(\to \frac{1}{2})} = e^{-2\cdot\frac{1}{2}} = e^{-1} = \frac{1}{e}.$

2) $f(x) = \frac{x-1}{3x-2}$; $f(g(x)) = \frac{g(x)-1}{3g(x)-2} = \operatorname{sen} x \Rightarrow g(x) - 1 = 3 \cdot \operatorname{sen} x \cdot g(x) - 2 \operatorname{sen} x \Rightarrow$

$\Rightarrow g(x)(3 \operatorname{sen} x - 1) = 2 \operatorname{sen} x - 1 \Rightarrow g(x) = \frac{2 \operatorname{sen} x - 1}{3 \operatorname{sen} x - 1}.$

3) $-1 < \log_3 (x-1) < 0$. C.E.: $x - 1 > 0 \Rightarrow x > 1.$

$\begin{cases} \log_3 (x-1) > -1 \\ \log_3 (x-1) < 0 \end{cases} \Rightarrow \begin{cases} x-1 > \frac{1}{3} \\ x-1 < 1 \end{cases} \Rightarrow \begin{cases} x > \frac{4}{3} \\ x < 2 \end{cases}$



Soluzione: $\frac{4}{3} < x < 2.$

4)

A	B	$\mathcal{C}(A)$	$B \subset \mathcal{C}(A)$	$A \subset B$	P	M	$P \vee M$
1	1	0	0	1	0	1	1
1	0	0	1	0	1	0	1
0	1	1	1	1	1	1	1
0	0	1	1	1	1	1	1

5) $f(x) = (x-1) \cdot (2^x - 3k)$; $g(x) = x^2 - 1 = (x-1)(x+1).$

$f(x) = o(g(x))$ per $x \to 2$ se $\lim_{x\to 2} \frac{f(x)}{g(x)} = 0.$

$\lim_{x\to 2} \frac{(x-1)(2^x-3k)}{(x-1)(x+1)} = \lim_{x\to 2} \frac{x-1}{(x-1)(x+1)} \cdot \lim_{x\to 2} (2^x - 3k) =$

$= \lim_{x\to 2} \frac{1}{x+1} \cdot \lim_{x\to 2} (2^x - 3k) = \frac{1}{3} \cdot (4 - 3k) = 0$ se $4 - 3k = 0 \Rightarrow k = \frac{4}{3}.$

Prova Intermedia di Matematica Generale del 10/11/2014 Compito C2

1) $\lim\limits_{x\to 0} \frac{\operatorname{sen}^2 x^3}{\operatorname{tg}^3 x^2} = \lim\limits_{x\to 0} \frac{\operatorname{sen}^2 x^3}{(x^3)^2} \cdot \frac{(x^2)^3}{\operatorname{tg}^3 x^2} = \lim\limits_{x\to 0} \left(\frac{\operatorname{sen} x^3}{x^3}\right)^2 \cdot \frac{1}{\left(\frac{\operatorname{tg} x^2}{x^2}\right)^3} = 1 \cdot 1 = 1.$

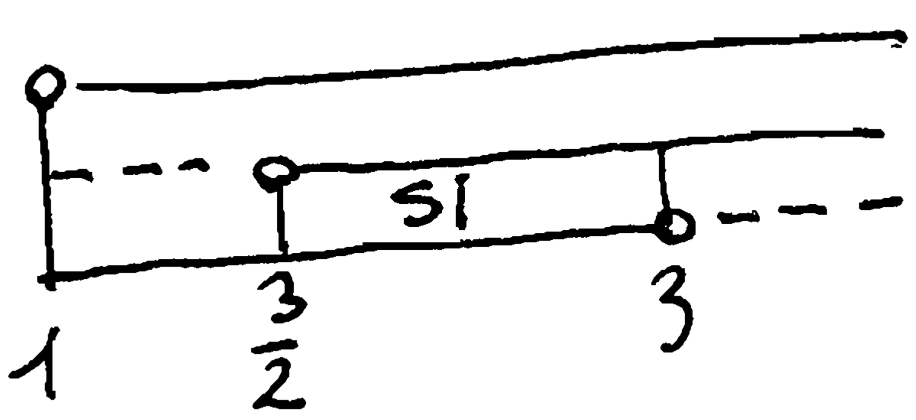
$\lim\limits_{x\to +\infty} \left(\frac{1+2x}{2x+3}\right)^{x-1} = \lim\limits_{x\to +\infty} \left(\frac{3+2x-2}{2x+3}\right)^{x-1} = \lim\limits_{x\to +\infty} \left[\left(1+\frac{(-2)}{2x+3}\right)^{2x+3}\right]^{\frac{x-1}{2x+3}} = \left(e^{-2}\right)^{\frac{1}{2}} = e^{-1} = \frac{1}{e}.$

2) $f(x) = 3^{1-2x}$; $f(g(x)) = 3^{1-2g(x)} = \frac{x+1}{x-1} \Rightarrow 1-2g(x) = \log_3 \frac{x+1}{x-1} \Rightarrow$

$\Rightarrow 2g(x) = 1 - \log_3 \frac{x+1}{x-1} \Rightarrow g(x) = \frac{1}{2}\left(1 - \log_3 \frac{x+1}{x-1}\right).$

3) $-1 < \log_2 (x-1) < 1.$ C.E.: $x-1>0 \Rightarrow x>1.$

$\begin{cases} \log_2 (x-1) > -1 \\ \log_2 (x-1) < 1 \end{cases} \Rightarrow \begin{cases} x-1 > \frac{1}{2} \\ x-1 < 2 \end{cases} \Rightarrow \begin{cases} x > \frac{3}{2} \\ x < 3 \end{cases}$



Soluzione: $\frac{3}{2} < x < 3.$

4)

A B	$A\subset B$	$\mathcal{C}(A)$	$B \subset \mathcal{C}(A)$	P	M	P e M
1 1	1	0	0	1	0	0
1 0	0	0	1	0	1	0
0 1	1	1	1	1	1	1
0 0	1	1	1	1	1	1

5) $f(x) = (x-3)\cdot \log_2 (2x+k)$; $g(x) = x^2-4x+3 = (x-1)(x-3).$

$f(x) = o(g(x))$ per $x \to 2$ se $\lim\limits_{x\to 2} \frac{f(x)}{g(x)} = 0.$

$\lim\limits_{x\to 2} \frac{(x-3)\cdot \log_2 (2x+k)}{(x-1)(x-3)} = \lim\limits_{x\to 2} \frac{x-3}{(x-1)(x-3)} \cdot \lim\limits_{x\to 2} \log_2 (2x+k) =$

$= \lim\limits_{x\to 2} \frac{1}{x-1} \cdot \lim\limits_{x\to 2} \log_2 (2x+k) = 1 \cdot \log_2 (4+k) = 0$ se $4+k = 1 \Rightarrow$

$\Rightarrow k = -3.$

Prova Intermedia di Matematica Generale del 10/11/2014 Compito D2

1) $\lim_{x\to 0} \frac{\log(1+x^4)}{3x^4-x^5} = \lim_{x\to 0} \frac{\log(1+x^4)}{x^4}\cdot\frac{x^4}{3x^4-x^5} = \lim_{t\to 0}\frac{\log(1+t)}{t}\cdot\lim_{x\to 0}\frac{\cancel{x^4}\,1}{\cancel{x^4}(3-x)} = 1\cdot\frac{1}{3} = \frac{1}{3}.$

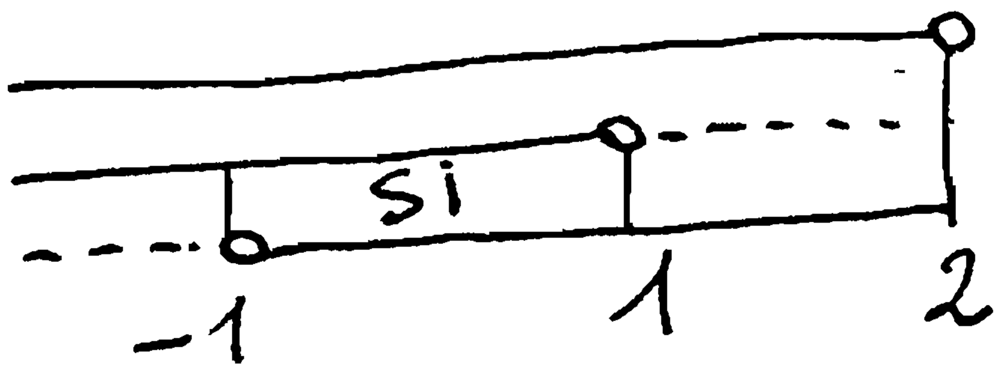
$\lim_{x\to+\infty}\left(\frac{3+2x}{2x+1}\right)^{1-x} = \lim_{x\to+\infty}\left(\frac{2x+1+2}{2x+1}\right)^{1-x} = \lim_{x\to+\infty}\left[\left(1+\frac{2}{2x+1}\right)^{2x+1}\right]^{\frac{1-x}{2x+1}} = \left(e^2\right)^{-\frac{1}{2}} = e^{-1} = \frac{1}{e}.$

2) $f(x) = 1-2\log x$; $f(g(x)) = 1-2\log g(x) = \log_3(1+x) \Rightarrow$

$\Rightarrow 2\log g(x) = 1-\log_3(1+x) \Rightarrow \log g(x) = \frac{1}{2}\left(1-\log_3(1+x)\right) \Rightarrow g(x) = e^{\frac{1}{2}\left(1-\log_3(1+x)\right)}.$

3) $0<\log_3(2-x)<1$. C.E.: $2-x>0 \Rightarrow x<2$.

$\begin{cases}\log_3(2-x)>0\\ \log_3(2-x)<1\end{cases} \Rightarrow \begin{cases}2-x>1\\ 2-x<3\end{cases} \Rightarrow \begin{cases}x<1\\ x>-1\end{cases}$



Soluzione: $-1<x<1$.

4)

A	B	C(B)	C(B) ⊂ A	A ⊂ B	P	M	P o M
1	1	0	1	1	1	1	1
1	0	1	1	0	1	0	1
0	1	0	1	1	1	1	1
0	0	1	0	1	0	1	1

5) $f(x) = (x+1)\cdot(3^x-2k)$; $g(x) = x^2-x-2 = (x+1)(x-2)$.

$f(x) = o(g(x))$ per $x\to 0$ se $\lim_{x\to 0}\frac{f(x)}{g(x)} = 0$.

$\lim_{x\to 0}\frac{(x+1)\cdot(3^x-2k)}{(x+1)\cdot(x-2)} = \lim_{x\to 0}\frac{x+1}{(x+1)(x-2)}\cdot\lim_{x\to 0}(3^x-2k) =$

$= \lim_{x\to 0}\frac{1}{x-2}\cdot\lim_{x\to 0}(3^x-2k) = -\frac{1}{2}\cdot(1-2k) = 0$ se $1-2k=0 \Rightarrow$

$\Rightarrow k = \frac{1}{2}.$