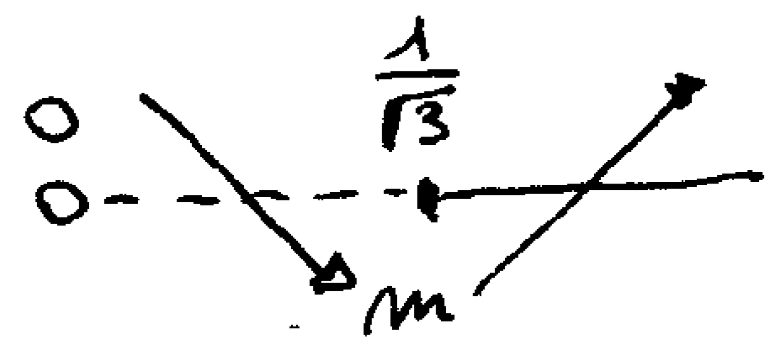
Compito di Matematica Generale del 19/1/2015 Compito A $\boxed{\text{MGA1}}$

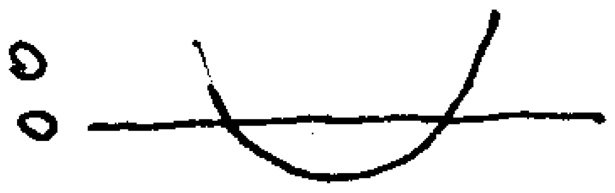
1) $f(x)=3x^2-2\log x$. C.E.: $x>0$. $\lim_{x\to 0^+} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=+\infty$.

$f(x)>0 \ \forall x>0$.

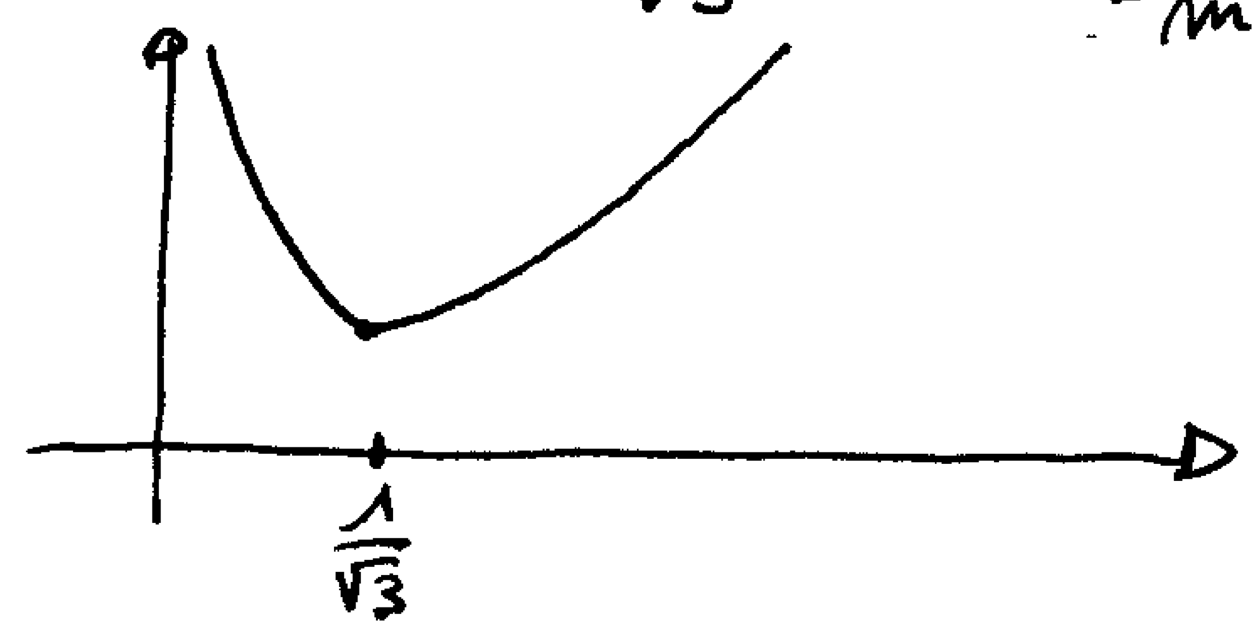
$f'(x)=6x-\frac{2}{x}=2\cdot\frac{3x^2-1}{x}\geq 0 \Rightarrow 3x^2\geq 1 \Rightarrow x^2\geq\frac{1}{3}\Rightarrow x\geq\frac{1}{\sqrt{3}}$:



$f''(x)=6+\frac{2}{x^2}>0 \ \forall x>0$:



$f\left(\frac{1}{\sqrt{3}}\right)=3\cdot\frac{1}{3}-2\log 3^{-\frac{1}{2}}=1+\log 3>0$. Grafico:



2) $\lim_{x\to 0}\frac{\operatorname{sen}3x-\operatorname{arctg}2x}{2x}=\lim_{x\to 0}\frac{\operatorname{sen}3x}{3x}\cdot\frac{3}{2}-\frac{\operatorname{arctg}2x}{2x}=\lim_{t\to 0}\frac{3}{2}\frac{\operatorname{sen}t}{t}-\frac{\operatorname{arctg}t}{t}=\frac{3}{2}\cdot 1-1=\frac{1}{2}$.

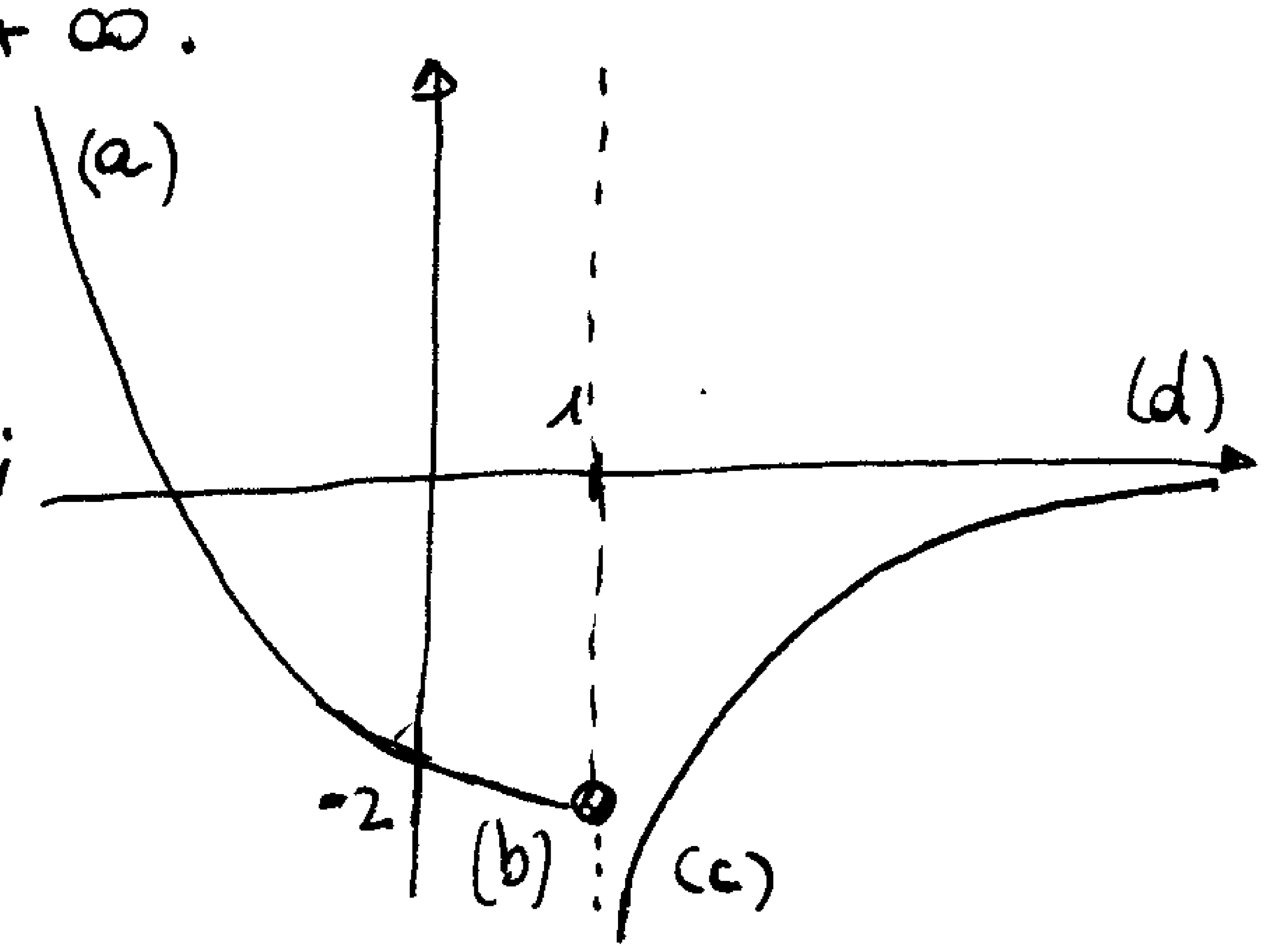
$\lim_{x\to+\infty}\left(\frac{1+x}{1+x+x^2}\right)^{1-x}=\left(\to 0^+\right)^{(\to-\infty)}=+\infty$.

3) a) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon$: $\lim_{x\to-\infty} f(x)=+\infty$;

b) $\forall\varepsilon>0\ \exists\delta(\varepsilon): 1-\delta<x<1\Rightarrow |f(x)+2|<\varepsilon$: $\lim_{x\to 1^-} f(x)=-2$;

c) $\forall\varepsilon\ \exists\delta(\varepsilon): 1<x<1+\delta\Rightarrow f(x)<\varepsilon$: $\lim_{x\to 1^+} f(x)=-\infty$;

d) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow |f(x)|<\varepsilon$: $\lim_{x\to+\infty} f(x)=0$.



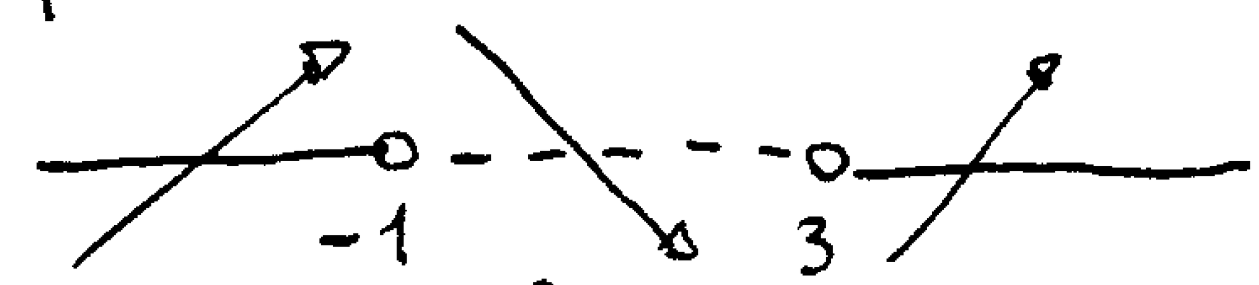
4) $f(x;y;z)=x^2y-e^{y-z}+\log(x-z)$

$\nabla f(x;y;z)=\left(2xy-0+\frac{1}{x-z};\ x^2-e^{y-z}+0;\ 0+e^{y-z}-\frac{1}{x-z}\right)$; $\nabla f(2;1;1)=(5;3;0)$.

$\|\nabla f(2;1;1)\|=\sqrt{25+9}=\sqrt{34}$; $\nabla f(2;1;1)\cdot v=\|\nabla f\|\cdot\|v\|\cdot\cos\frac{\pi}{3}=\sqrt{34}\cdot 2\cdot\frac{1}{2}=\sqrt{34}$.

5) $X_1\cdot X_2=(x^2;-3x;x)\cdot(x;x;-9)=x^3-3x^2-9x=f(x)$. $f'(x)=3x^2-6x-9\geq 0\Rightarrow$

$3\cdot(x^2-2x-3)=3\cdot(x-3)(x+1)\geq 0$ per $x\leq -1$ oppure $x\geq 3$:



$x=-1$ è il punto di Massimo; $x=3$ è il punto di minimo. $f(-1)=5$; $f(3)=-27$.

6) $f(x)=\frac{2^x+1}{2^x-4}$. C.E.: $2^x\neq 4\Rightarrow x\neq 2$. $\lim_{x\to-\infty} f(x)=-\frac{1}{4}$; $\lim_{x\to 2^-} f(x)=-\infty$; $\lim_{x\to 2^+} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=1$.

$\boxed{\text{MGA2}}$

$f(x)=\dfrac{2^x-4+5}{2^x-4}=1+5\cdot\dfrac{1}{2^x-4} \Rightarrow f'(x)=5\cdot\left(-\dfrac{1}{(2^x-4)^2}\right)\cdot 2^x\log 2<0 \quad \forall x\in C.E.$ $\quad 2$

$f: \,]-\infty;2[\,\cup\,]2;+\infty[\, \to \,]-\infty;-\frac{1}{4}[\,\cup\,]1;+\infty[$; funzione invertibile in tutto C.E..

$f^{-1}: \,]-\infty;-\frac{1}{4}[\,\cup\,]1;+\infty[\, \to \,]-\infty;2[\,\cup\,]2;+\infty[$.

$1+5\cdot\dfrac{1}{2^x-4}=y \Rightarrow 5\cdot\dfrac{1}{2^x-4}=y-1 \Rightarrow 2^x-4=\dfrac{5}{y-1} \Rightarrow 2^x=4+\dfrac{5}{y-1}=\dfrac{4y+1}{y-1} \Rightarrow$

$\Rightarrow x=\log_2\dfrac{4y+1}{y-1}$. Funzione inversa: $y=\log_2\dfrac{4x+1}{x-1}$.

7) $f(x)=\log(3x-2)$; C.E. $x>\frac{2}{3}$; $f'(x)=\dfrac{3}{3x-2}$. $2y+x=10 \Rightarrow y=-\frac{1}{2}x+5$. Per avere la perpendicolarità: $\dfrac{3}{3x_0-2}=-\dfrac{1}{m}=2 \Rightarrow 6x_0-4=3 \Rightarrow x_0=\frac{7}{6}$. $f(\frac{7}{6})=\log(\frac{7}{2}-2)=\log\frac{3}{2}$.

Equazione retta tangente: $y-\log\frac{3}{2}=2\left(x-\frac{7}{6}\right) \Rightarrow y=2x-\frac{7}{3}+\log\frac{3}{2}$.

8) $f(x;y)=x^2+y^3-xy+x-3y$.

$$\begin{cases} f'_x=2x-y+1=0 \\ f'_y=3y^2-x-3=0 \end{cases} \Rightarrow \begin{cases} 6y^2-6-y+1=0 \\ x=3y^2-3 \end{cases} \Rightarrow \begin{cases} 6y^2-y-5=0 \\ x=3y^2-3 \end{cases} \Rightarrow \begin{cases} y=\dfrac{1\pm 11}{12} \\ x=3y^2-3 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=1 \end{cases} \cup \begin{cases} x=-\frac{11}{12} \\ y=-\frac{5}{6} \end{cases}.$$

$H=\left\|\begin{matrix}2 & -1\\ -1 & 6y\end{matrix}\right\|$, $H(0;1)=\left\|\begin{matrix}2 & -1\\ -1 & 6\end{matrix}\right\|$: P. di Minimo. $H\left(-\frac{11}{12};-\frac{5}{6}\right)=\left\|\begin{matrix}2 & -1\\ -1 & -5\end{matrix}\right\|$: P. di Sella.

9) $\displaystyle\int_{-1}^{0}\frac{x+k}{x+2}\,dx=\int_{-1}^{0}\frac{x+2+(k-2)}{x+2}\,dx=\int_{-1}^{0}1+(k-2)\frac{1}{x+2}\,dx=\Big(x+(k-2)\cdot\log(x+2)\Big|_{-1}^{0}=$

$=\left(0+(k-2)\log 2\right)-\left(-1+(k-2)\cdot\log 1\right)=(k-2)\log 2+1=3 \Rightarrow k-2=\dfrac{2}{\log 2} \Rightarrow k=2+2\cdot\log_2 e$.

10) $X_1 /\!/ X_2$: $\frac{1}{2}=\frac{1}{k}=\frac{m}{4} \Rightarrow k=2$ e $m=2$.

$X_1 \perp X_2$: $(1;1;m)\cdot(2;k;4)=0 \Rightarrow 2+k+4m=0 \Rightarrow k=-4m-2$.

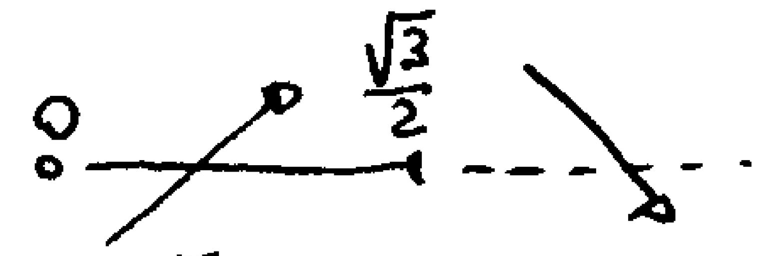
$$A\cdot Y=\left\|\begin{matrix}1 & 1 & m\\ 2 & k & 4\end{matrix}\right\|\cdot\left\|\begin{matrix}1\\ -1\\ 1\end{matrix}\right\|=\left\|\begin{matrix}1-1+m\\ 2-k+4\end{matrix}\right\|=\left\|\begin{matrix}m\\ 6-k\end{matrix}\right\|=\left\|\begin{matrix}0\\ 0\end{matrix}\right\| \Rightarrow \begin{cases} m=0\\ k=6\end{cases}.$$

Compito di Matematica Generale del 19/1/2015 Compito B $\boxed{\text{MGB1}}$

1) $f(x)=3\log x-2x^2$. C.E.: $x>0$. $\lim_{x\to0^+} f(x)=-\infty$; $\lim_{x\to+\infty} f(x)=-\infty$.

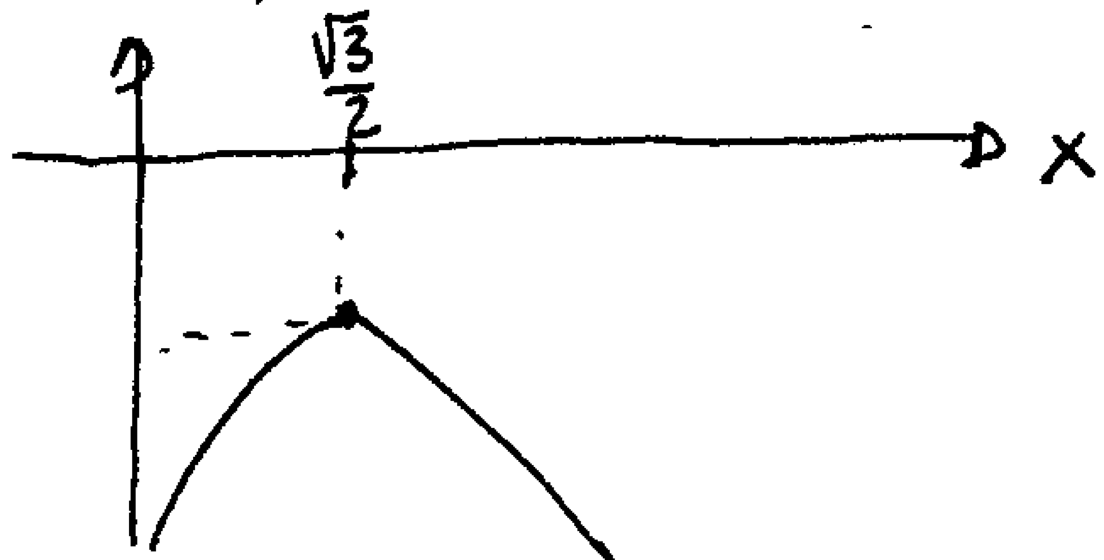
$f(x)<0\ \forall x>0$.

$f'(x)=\frac{3}{x}-4x=\frac{3-4x^2}{x}\geq0 \Rightarrow 4x^2\leq3\Rightarrow x^2\leq\frac{3}{4}\Rightarrow 0<x\leq\frac{\sqrt3}{2}$.



$f''(x)=-\frac{3}{x^2}-4<0\ \forall x>0$.

$f\left(\frac{\sqrt3}{2}\right)=3\cdot\log\frac{\sqrt3}{2}-2\cdot\frac{3}{4}=\frac{3}{2}(\log3-1)-3\log2<0$. Grafico:



2) $\lim_{x\to0}\frac{\operatorname{arcsen}x-\operatorname{sen}3x}{4x}=\lim_{x\to0}\frac{1}{4}\cdot\frac{\operatorname{arcsen}x}{x}-\frac{3}{4}\cdot\frac{\operatorname{sen}3x}{3x}=\lim_{t\to0}\frac{1}{4}\frac{\operatorname{arcsen}t}{t}-\frac{3}{4}\frac{\operatorname{sen}t}{t}=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$.

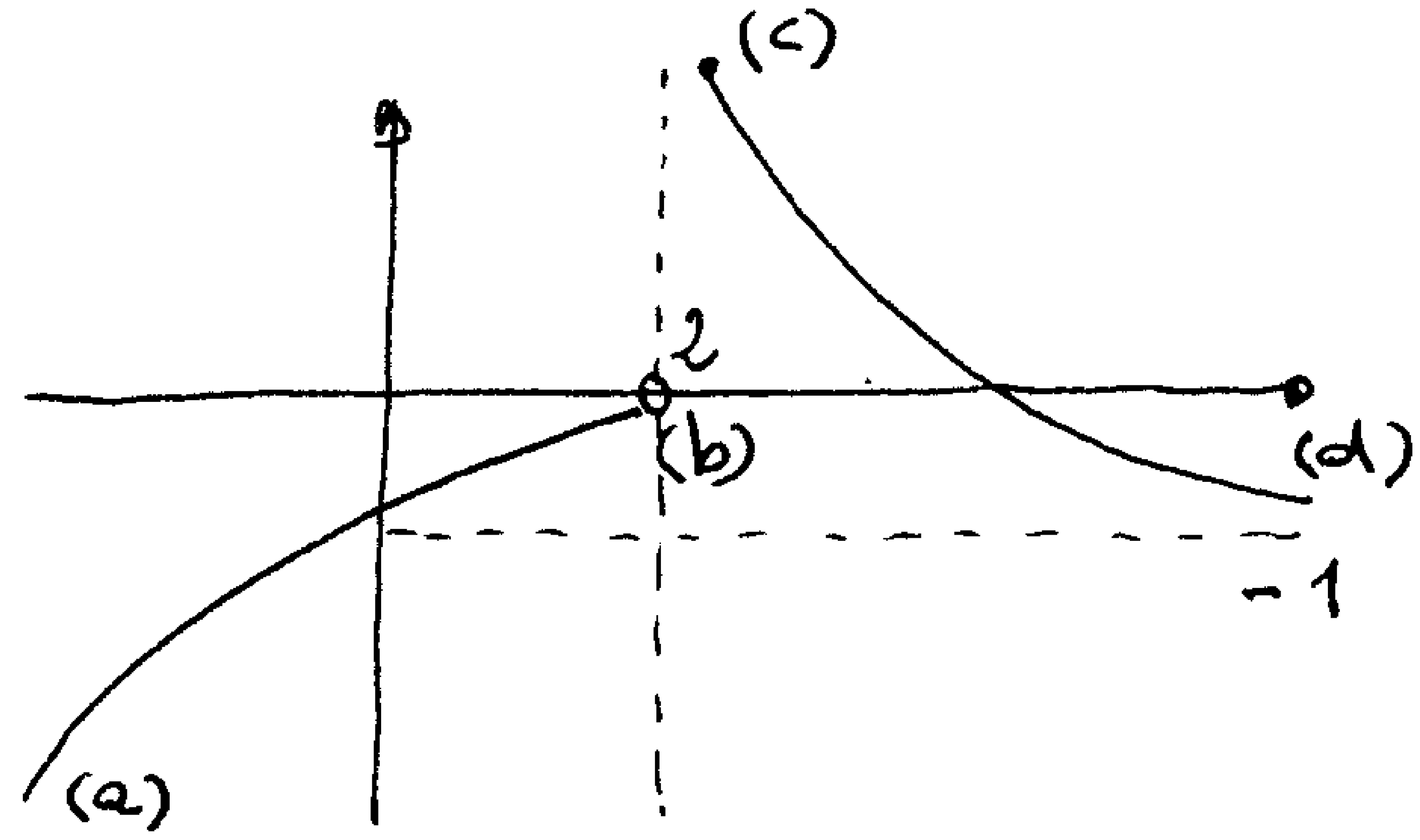
$\lim_{x\to+\infty}\left(\frac{1+x+x^2}{1+2x}\right)^{x-x^2}=\left(\to+\infty\right)^{(\to-\infty)}=0^+$.

3) a) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow f(x)<\varepsilon$: $\lim_{x\to-\infty}f(x)=-\infty$;

b) $\forall\varepsilon>0\ \exists\delta(\varepsilon): 2-\delta<x<2\Rightarrow |f(x)|<\varepsilon$: $\lim_{x\to2^-}f(x)=0$;

c) $\forall\varepsilon\ \exists\delta(\varepsilon): 2<x<2+\delta\Rightarrow f(x)>\varepsilon$: $\lim_{x\to2^+}f(x)=+\infty$;

d) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow |f(x)+1|<\varepsilon$: $\lim_{x\to+\infty}f(x)=-1$.



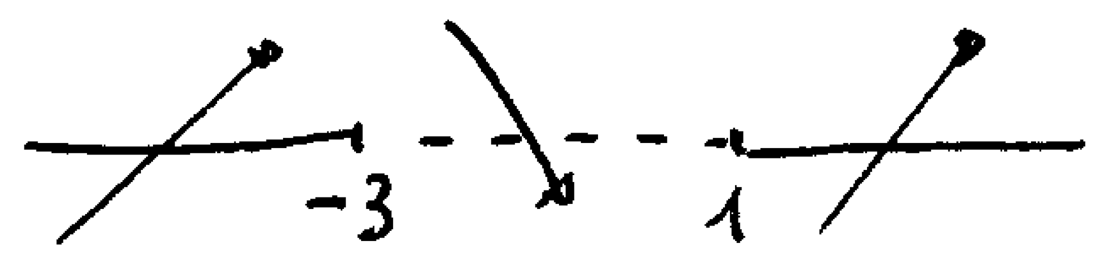
4) $f(x;y;z)=e^{x-y}+\log(z-x)+y^2z$

$\nabla f(x;y;z)=\left(e^{x-y}-\frac{1}{z-x}+0;-e^{x-y}+0+2yz;0+\frac{1}{z-x}+y^2\right)$; $\nabla f(1;1;2)=(0;3;2)$.

$\|\nabla f(1;1;2)\|=\sqrt{9+4}=\sqrt{13}$. $\nabla f(1;1;2)\cdot v=\|\nabla f\|\cdot\|v\|\cdot\cos\frac{\pi}{4}=\sqrt{13}\cdot1\cdot\frac{\sqrt2}{2}=\frac{\sqrt{26}}{2}$.

5) $x_1\cdot x_2=(x;3;-9)\cdot(x^2;x^2;x)=x^3+3x^2-9x=f(x)$. $f'(x)=3x^2+6x-9\geq0\Rightarrow$

$3\cdot(x^2+2x-3)=3\cdot(x+3)(x-1)\geq0$ per $x\leq-3$ oppure $x\geq1$:



$x=-3$ è il punto di massimo; $x=1$ è il punto di minimo. $f(-3)=27$; $f(1)=-5$.

6) $f(x)=\frac{3^x+2}{3^x-1}$. C.E.: $3^x\neq1\Rightarrow x\neq0$. $\lim_{x\to-\infty}f(x)=-2$; $\lim_{x\to0^-}f(x)=-\infty$; $\lim_{x\to0^+}f(x)=+\infty$; $\lim_{x\to+\infty}f(x)=1$.

MGB2

$f(x)=\frac{3^x-1+3}{3^x-1}=1+3\cdot\frac{1}{3^x-1}\Rightarrow f'(x)=3\cdot\left(-\frac{1}{(3^x-1)^2}\right)\cdot 3^x\log 3<0\ \forall x\in C.E.$

$f:\,]-\infty;0[\,\cup\,]0;+\infty[\,\to\,]-\infty;-2[\,\cup\,]1;+\infty[$; funzione invertibile in tutto C.E..

$f^{-1}:\,]-\infty;-2[\,\cup\,]1;+\infty[\,\to\,]-\infty;0[\,\cup\,]0;+\infty[$.

$1+3\cdot\frac{1}{3^x-1}=y\Rightarrow 3\cdot\frac{1}{3^x-1}=y-1\Rightarrow 3^x-1=\frac{3}{y-1}\Rightarrow 3^x=1+\frac{3}{y-1}=\frac{y+2}{y-1}\Rightarrow$

$\Rightarrow x=\log_3\frac{y+2}{y-1}$. Funzione inversa: $y=\log_3\frac{x+2}{x-1}$.

7) $f(x)=\log(2x+3)$. C.E.: $x>-\frac{3}{2}$; $f'(x)=\frac{2}{2x+3}$. $3y+x=6\Rightarrow y=-\frac{1}{3}x+2$. Per avere la perpendicolarità: $\frac{2}{2x_0+3}=-\frac{1}{m}=3\Rightarrow 6x_0+9=2\Rightarrow x_0=-\frac{7}{6}$. $f\left(-\frac{7}{6}\right)=\log\left(-\frac{7}{3}+3\right)=\log\frac{2}{3}$.

Equazione retta tangente: $y-\log\frac{2}{3}=3\cdot\left(x+\frac{7}{6}\right)\Rightarrow y=3x+\frac{7}{2}+\log\frac{2}{3}$.

8) $f(x;y)=y^3-x^2+xy+x-3y$.

$\begin{cases}f'_x=-2x+y+1=0\\ f'_y=3y^2+x-3=0\end{cases}\Rightarrow\begin{cases}6y^2-6+y+1=0\\ x=3-3y^2\end{cases}\Rightarrow\begin{cases}6y^2+y-5=0\\ x=3-3y^2\end{cases}\Rightarrow\begin{cases}y=\frac{-1\pm 11}{12}\\ x=3-3y^2\end{cases}\Rightarrow\begin{cases}x=0\\ y=-1\end{cases}\cup\begin{cases}x=\frac{11}{12}\\ y=\frac{5}{6}\end{cases}$.

$H=\begin{Vmatrix}-2 & 1\\ 1 & 6y\end{Vmatrix}$. $H(0;-1)=\begin{Vmatrix}-2 & 1\\ 1 & -6\end{Vmatrix}$: P. di Massimo; $H\left(\frac{11}{12};\frac{5}{6}\right)=\begin{Vmatrix}-2 & 1\\ 1 & 5\end{Vmatrix}$: P. di Sella.

9) $\int_0^1\frac{x-k}{x+1}dx=\int_0^1\frac{x+1-(k+1)}{x+1}dx=\int_0^1 1-(k+1)\cdot\frac{1}{x+1}dx=\Big(x-(k+1)\cdot\log(x+1)\Big|_0^1=$

$=\big(1-(k+1)\cdot\log 2\big)-\big(0-(k+1)\cdot\log 1\big)=1-(k+1)\log 2=2\Rightarrow k+1=\frac{-1}{\log 2}\Rightarrow k=-\log_2 e-1$.

10) $X_1//X_2:\ \frac{2}{1}=\frac{-1}{k}=\frac{m}{2}\Rightarrow k=-\frac{1}{2}$ e $m=4$.

$X_1\perp X_2:\ (2;-1;m)\cdot(1;k;2)=0\Rightarrow 2-k+2m=0\Rightarrow k=2m+2$.

$A\circ Y=\begin{Vmatrix}2 & -1 & m\\ 1 & k & 2\end{Vmatrix}\cdot\begin{Vmatrix}1\\ -1\\ 1\end{Vmatrix}=\begin{Vmatrix}2+1+m\\ 1-k+2\end{Vmatrix}=\begin{Vmatrix}m+3\\ 3-k\end{Vmatrix}=\begin{Vmatrix}0\\ 0\end{Vmatrix}\Rightarrow\begin{cases}m=-3\\ k=3\end{cases}$.

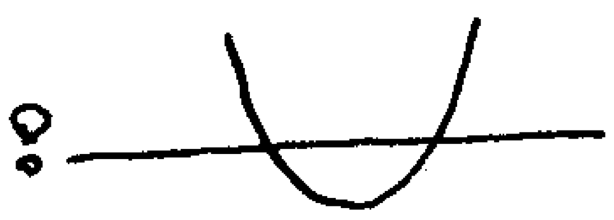
Compito di Matematica Generale del 19/1/2015 Compito C $\boxed{MGC1}$

1) $f(x)=x^2-4\log x$. C.E.: $x>0$. $\lim_{x\to 0^+} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=+\infty$.

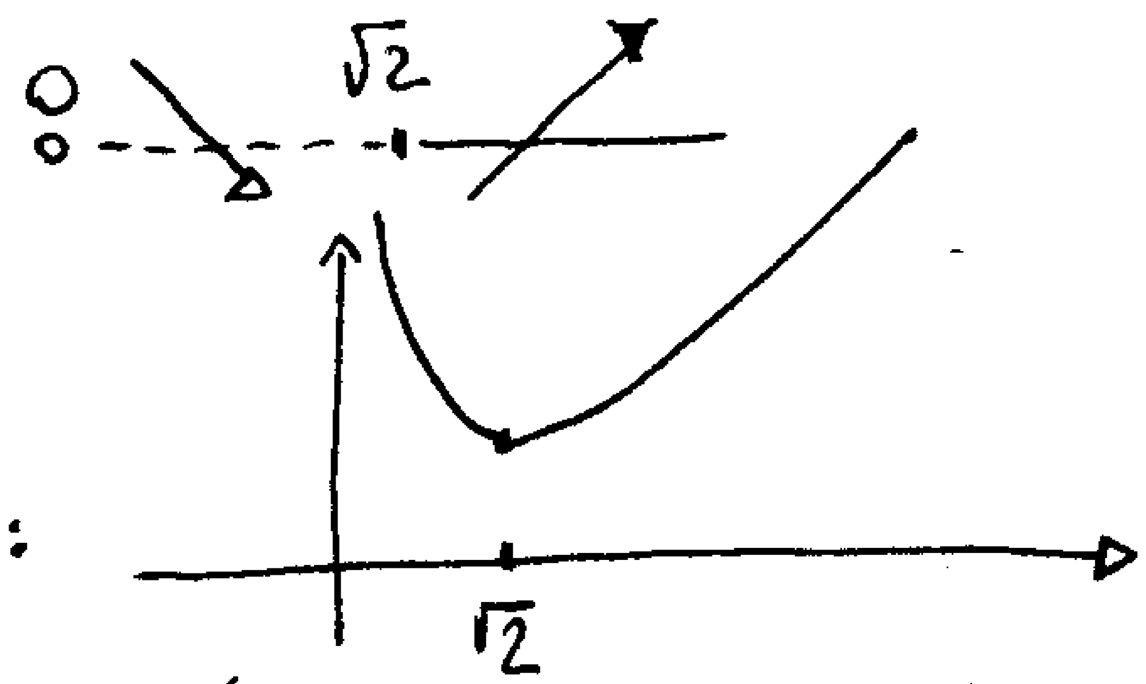
$f(x)>0\ \forall x>0$.

$f'(x)=2x-\frac{4}{x}=2\cdot\frac{x^2-2}{x}\geq 0 \Rightarrow x^2\geq 2 \Rightarrow x\geq\sqrt{2}$

$f''(x)=2+\frac{4}{x^2}>0\ \forall x>0$



$f(\sqrt{2})=2-4\log\sqrt{2}=2-2\log 2=2(1-\log 2)>0$. Grafico:



2) $\lim_{x\to 0}\frac{\operatorname{arctg}4x-\operatorname{sen}3x}{3x}=\lim_{x\to 0}\frac{4}{3}\cdot\frac{\operatorname{arctg}4x}{4x}-\frac{\operatorname{sen}3x}{3x}=\lim_{t\to 0}\frac{4}{3}\frac{\operatorname{arctg}t}{t}-\frac{\operatorname{sen}t}{t}=\frac{4}{3}-1=\frac{1}{3}$.

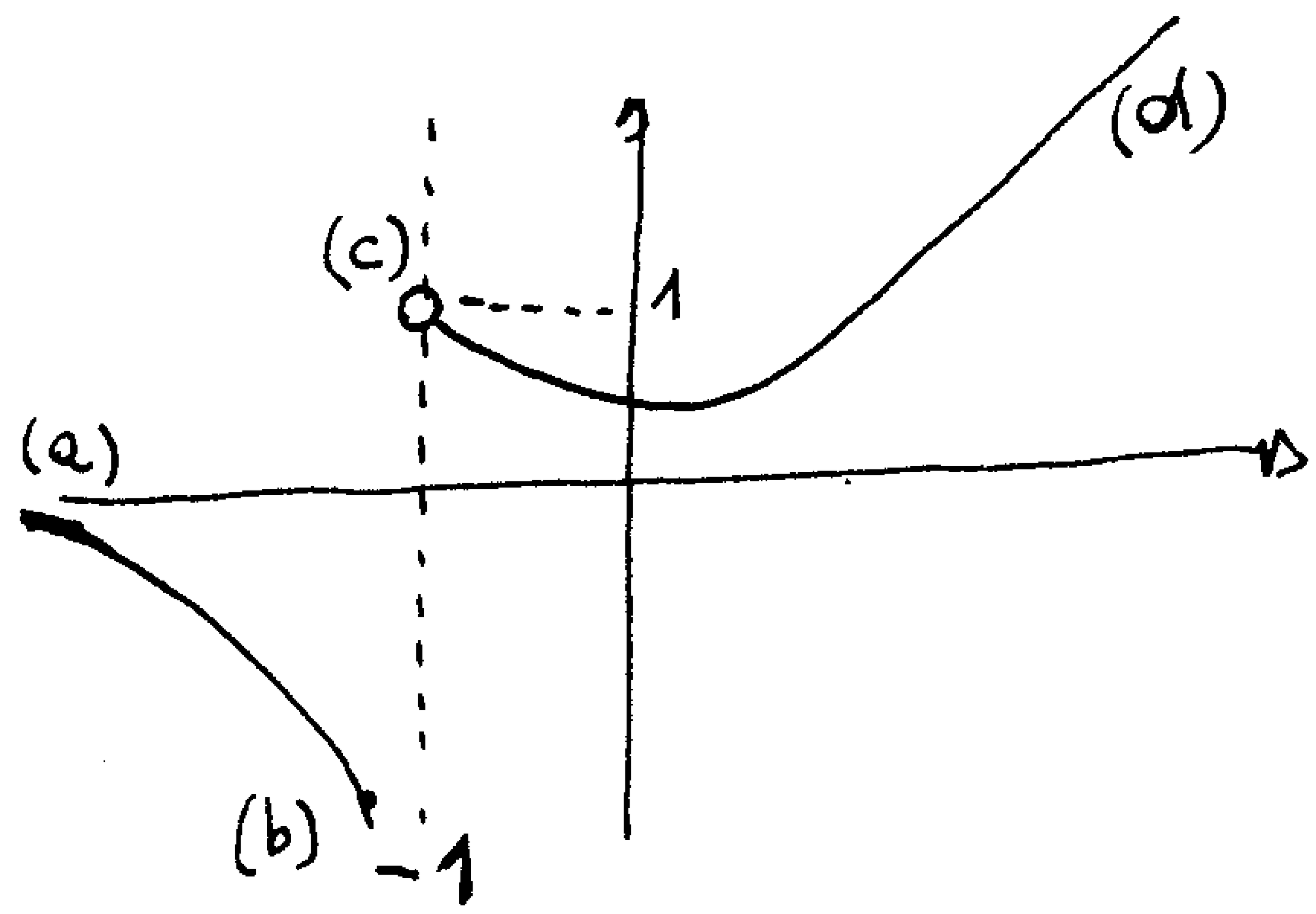
$\lim_{x\to+\infty}\left(\frac{1+2x}{1-x+x^2}\right)^{x-1}=\left(\to 0^+\right)^{(\to+\infty)}=0^+$.

3) a) $\forall\varepsilon\,\exists\,\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow|f(x)|<\varepsilon$: $\lim_{x\to-\infty}f(x)=0$;

b) $\forall\varepsilon\,\exists\,\delta(\varepsilon): -1-\delta<x<-1\Rightarrow f(x)<\varepsilon$: $\lim_{x\to-1^-}f(x)=-\infty$;

c) $\forall\varepsilon>0\,\exists\,\delta(\varepsilon): -1<x<-1+\delta\Rightarrow|f(x)-1|<\varepsilon$: $\lim_{x\to-1^+}f(x)=1$;

d) $\forall\varepsilon\,\exists\,\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow f(x)>\varepsilon$: $\lim_{x\to+\infty}f(x)=+\infty$.



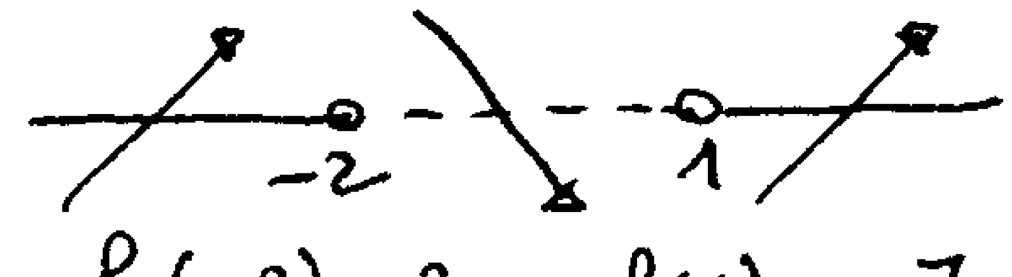
4) $f(x;y;z)=\log(y-z)+e^{z-x}+2xy$.

$\nabla f(x;y;z)=\left(0-e^{z-x}+2y;\ \frac{1}{y-z}+0+2x;\ -\frac{1}{y-z}+e^{z-x}+0\right)$; $\nabla f(1;2;1)=(3;3;0)$.

$\|\nabla f(1;2;1)\|=\sqrt{9+9}=3\sqrt{2}$. $\nabla f(1;2;1)\cdot v=\|\nabla f\|\cdot\|v\|\cdot\cos\frac{\pi}{6}=3\cdot\sqrt{2}\cdot 2\cdot\frac{\sqrt{3}}{2}=3\sqrt{6}$.

5) $X_1\cdot X_2=(x^2;x;-3)\cdot(2x;3x;4x)=2x^3+3x^2-12x=f(x)$. $f'(x)=6x^2+6x-12\geq 0\Rightarrow$

$6(x^2+x-2)=6\cdot(x-1)(x+2)\geq 0$ per $x\leq -2$ oppure $x\geq 1$:



$x=-2$ è il punto di massimo; $x=1$ è il punto di minimo. $f(-2)=20$; $f(1)=-7$.

6) $f(x)=\frac{2^x+2}{2^x-1}$. C.E.: $2^x\neq 1\Rightarrow x\neq 0$. $\lim_{x\to-\infty}f(x)=-2$; $\lim_{x\to 0^-}f(x)=-\infty$; $\lim_{x\to 0^+}f(x)=+\infty$; $\lim_{x\to+\infty}f(x)=1$.

MGC2

$f(x)=\frac{2^x-1+3}{2^x-1}=1+3\cdot\frac{1}{2^x-1} \Rightarrow f'(x)=3\cdot\left(-\frac{1}{(2^x-1)^2}\right)\cdot 2^x\log 2<0\ \forall x\in C.E.$

$f:]-\infty;0[\cup]0;+\infty[\to]-\infty;-2[\cup]1;+\infty[$; funzione invertibile in tutto C. E..

$f^{-1}:]-\infty;-2[\cup]1;+\infty[\to]-\infty;0[\cup]0;+\infty[$.

$1+3\cdot\frac{1}{2^x-1}=y \Rightarrow 3\cdot\frac{1}{2^x-1}=y-1 \Rightarrow 2^x-1=\frac{3}{y-1} \Rightarrow 2^x=1+\frac{3}{y-1}=\frac{y+2}{y-1} \Rightarrow$

$\Rightarrow x=\log_2\frac{y+2}{y-1}$. Funzione inversa: $y=\log_2\frac{x+2}{x-1}$.

7) $f(x)=\log(2x-3)$. C.E.: $x>\frac{3}{2}$; $f'(x)=\frac{2}{2x-3}$. $y+2x=1 \Rightarrow y=-2x+1$. Per avere la perpendicolarità: $\frac{2}{2x_0-3}=-\frac{1}{m}=\frac{1}{2} \Rightarrow 2x_0-3=4 \Rightarrow x_0=\frac{7}{2}$. $f\left(\frac{7}{2}\right)=\log(7-3)=\log 4$.

Equazione retta tangente: $y-\log 4=\frac{1}{2}\left(x-\frac{7}{2}\right) \Rightarrow y=\frac{1}{2}x-\frac{7}{4}+\log 4$.

8) $f(x;y)=x^2+y^3+xy-x-3y$.

$\begin{cases} f'_x=2x+y-1=0 \\ f'_y=3y^2+x-3=0 \end{cases} \Rightarrow \begin{cases} -6y^2+6+y-1=0 \\ x=3-3y^2 \end{cases} \Rightarrow \begin{cases} 6y^2-y-5=0 \\ x=-3y^2+3 \end{cases} \Rightarrow \begin{cases} y=\frac{1\pm 11}{12} \\ x=3-3y^2 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=1 \end{cases} \cup \begin{cases} x=\frac{11}{12} \\ y=-\frac{5}{6} \end{cases}$.

$H=\begin{Vmatrix} 2 & 1 \\ 1 & 6y \end{Vmatrix}$. $H(0;1)=\begin{Vmatrix} 2 & 1 \\ 1 & 6 \end{Vmatrix}$: P. di minimo; $H\left(\frac{11}{12};-\frac{5}{6}\right)=\begin{Vmatrix} 2 & 1 \\ 1 & -5 \end{Vmatrix}$: P. di Sella.

9) $\int_{-1}^{0}\frac{x-k}{x+2}dx=\int_{-1}^{0}\frac{x+2-(k+2)}{x+2}dx=\int_{-1}^{0}1-(k+2)\cdot\frac{1}{x+2}dx=\Big(x-(k+2)\cdot\log(x+2)\Big|_{-1}^{0}=$

$=\big(0-(k+2)\cdot\log 2\big)-\big(-1-(k+2)\log 1\big)=1-(k+2)\log 2=\frac{1}{2} \Rightarrow k+2=\frac{1}{2\log 2} \Rightarrow k=\frac{1}{2}\log_2 e-2$.

10) $X_1 /\!/ X_2$: $\frac{k}{1}=\frac{3}{1}=\frac{2}{m} \Rightarrow k=3$ e $m=\frac{2}{3}$.

$X_1\perp X_2$: $(k;3;2)\cdot(1;1;m)=0 \Rightarrow k+3+2m=0 \Rightarrow k=-3-2m$.

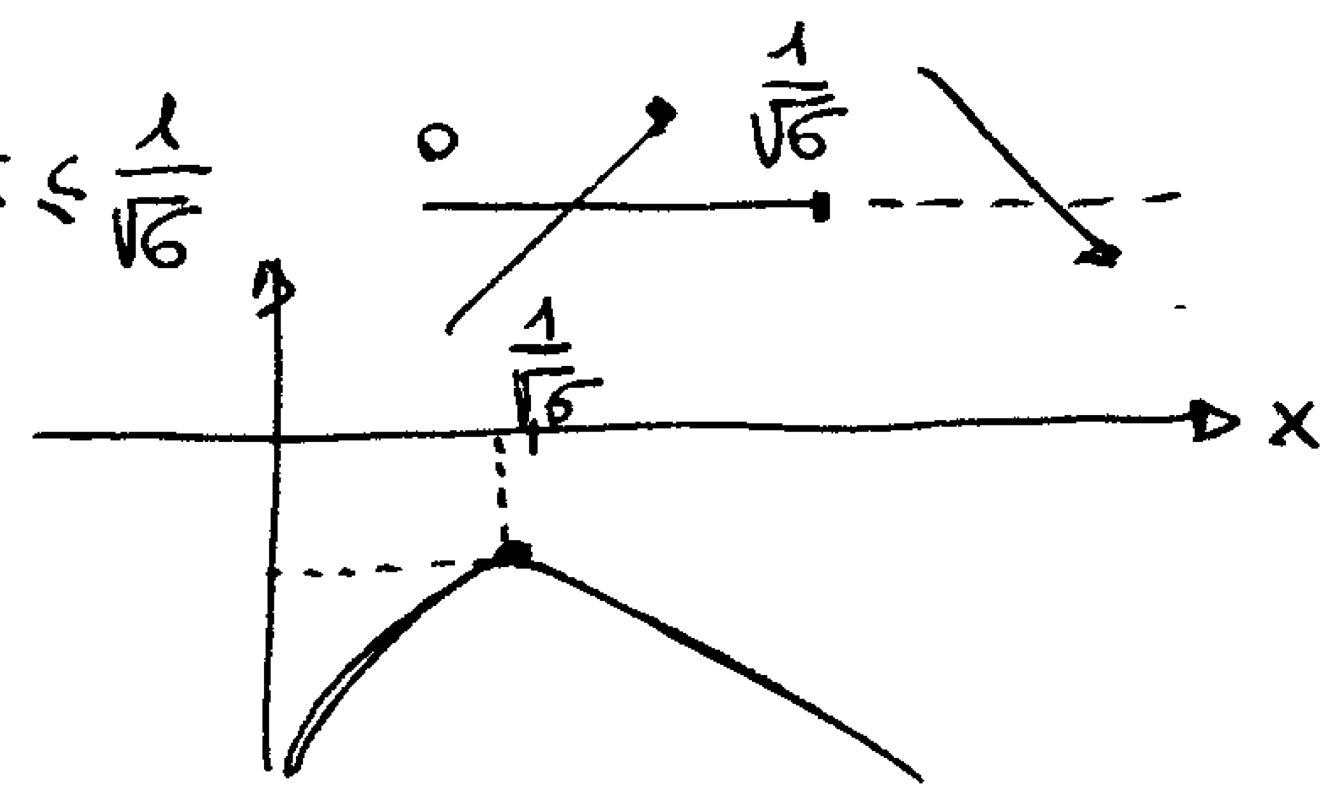
$A\cdot Y=\begin{Vmatrix} k & 3 & 2 \\ 1 & 1 & m \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ -1 \\ 1 \end{Vmatrix}=\begin{Vmatrix} k-3+2 \\ 1-1+m \end{Vmatrix}=\begin{Vmatrix} k-1 \\ m \end{Vmatrix}=\begin{Vmatrix} 0 \\ 0 \end{Vmatrix} \Rightarrow \begin{cases} k=1 \\ m=0 \end{cases}$.

Compito di Matematica Generale del 19/1/2015 Compito D $\boxed{\text{MGD1}}$

1) $f(x)=\log x-3x^2$. C.E.: $x>0$. $\lim\limits_{x\to 0^+} f(x)=-\infty$; $\lim\limits_{x\to+\infty} f(x)=-\infty$.

$f(x)<0\ \forall x>0$.

$f'(x)=\frac{1}{x}-6x=\frac{1-6x^2}{x}\geq 0 \Rightarrow 6x^2\leq 1 \Rightarrow 0<x\leq\frac{1}{\sqrt{6}}$



$f''(x)=-\frac{1}{x^2}-6<0\ \forall x>0$

$f\left(\frac{1}{\sqrt{6}}\right)=\log\frac{1}{\sqrt{6}}-\frac{1}{2}=-\frac{1}{2}-\frac{1}{2}\log 6<0$.

2) $\lim\limits_{x\to 0}\frac{\operatorname{sen}2x-\operatorname{arcsen}3x}{4x}=\lim\limits_{x\to 0}\frac{2}{4}\cdot\frac{\operatorname{sen}2x}{2x}-\frac{3}{4}\frac{\operatorname{arcsen}3x}{3x}=\lim\limits_{t\to 0}\frac{1}{2}\cdot\frac{\operatorname{sen}t}{t}-\frac{3}{4}\frac{\operatorname{arcsen}t}{t}=\frac{1}{2}-\frac{3}{4}=-\frac{1}{4}$.

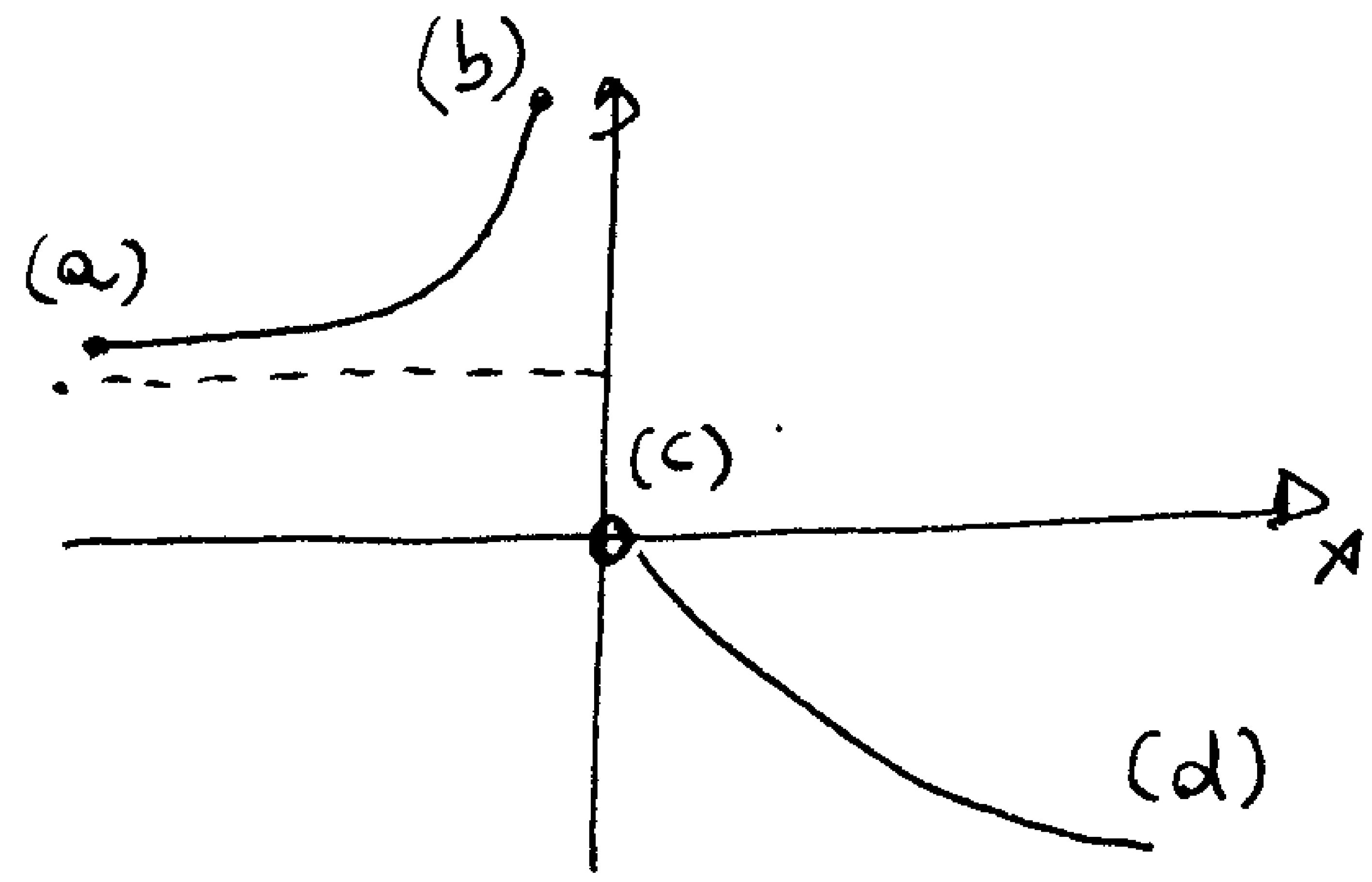
$\lim\limits_{x\to+\infty}\left(\frac{1-x+x^2}{1+x}\right)^{x^2-1}=\left(\to+\infty\right)^{(\to+\infty)}=+\infty$.

3) a) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow|f(x)-1|<\varepsilon$: $\lim\limits_{x\to-\infty} f(x)=1$;

b) $\forall\varepsilon\ \exists\delta(\varepsilon): -\delta<x<0\Rightarrow f(x)>\varepsilon$: $\lim\limits_{x\to 0^-} f(x)=+\infty$;

c) $\forall\varepsilon>0\ \exists\delta(\varepsilon): 0<x<\delta\Rightarrow|f(x)|<\varepsilon$: $\lim\limits_{x\to 0^+} f(x)=0$;

d) $\forall\varepsilon\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow f(x)<\varepsilon$: $\lim\limits_{x\to+\infty} f(x)=-\infty$.



4) $f(x;y;z)=z^2y-e^{x-y}+\log(x-z)$.

$\nabla f(x;y;z)=\left(0-e^{x-y}+\frac{1}{x-z};z^2+e^{x-y}+0;2zy-0-\frac{1}{x-z}\right)$; $\nabla f(2;2;1)=(0;2;3)$.

$\|\nabla f(2;2;1)\|=\sqrt{4+9}=\sqrt{13}$. $\nabla f(2;2;1)\cdot v=\|\nabla f\|\cdot\|v\|\cdot\cos\frac{3}{4}\pi=\sqrt{13}\cdot 1\cdot\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\sqrt{26}}{2}$.

5) $X_1\cdot X_2=(2x;-3;6)\cdot(x^2;x^2;-2x)=2x^3-3x^2-12x=f(x)$. $f'(x)=6x^2-6x-12\geq 0\Rightarrow$

$\Rightarrow 6(x^2-x-2)=6(x+1)(x-2)\geq 0$ per $x\leq -1$ oppure $x\geq 2$

$x=-1$ è il punto di massimo; $x=2$ è il punto di minimo. $f(-1)=7$; $f(2)=-20$.

6) $f(x)=\frac{3^x+1}{3^x-9}$. C.E.: $3^x\neq 9\Rightarrow x\neq 2$. $\lim\limits_{x\to-\infty} f(x)=-\frac{1}{9}$; $\lim\limits_{x\to 2^-} f(x)=-\infty$; $\lim\limits_{x\to 2^+} f(x)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=1$.

MGD2

$f(x)=\frac{3^x-9+10}{3^x-9}=1+10\cdot\frac{1}{3^x-9} \Rightarrow f'(x)=10\cdot\left(-\frac{1}{(3^x-9)^2}\right)\cdot 3^x\cdot\log 3<0\ \forall x\in C.E.$ $\searrow\ 2\ \searrow$

$f: \,]-\infty;2[\,\cup\,]2;+\infty[\,\to\,]-\infty;-\frac{1}{9}[\,\cup\,]1;+\infty[$; funzione invertibile in tutto C.E..

$f^{-1}: \,]-\infty;-\frac{1}{9}[\,\cup\,]1;+\infty[\,\to\,]-\infty;2[\,\cup\,]2;+\infty[$.

$1+10\cdot\frac{1}{3^x-9}=y = 10\cdot\frac{1}{3^x-9}=y-1 \Rightarrow 3^x-9=\frac{10}{y-1} \Rightarrow 3^x=9+\frac{10}{y-1}=\frac{9y+1}{y-1} \Rightarrow$

$\Rightarrow x=\log_3\frac{9y+1}{y-1}$. Funzione inversa: $y=\log_3\frac{9x+1}{x-1}$.

7) $f(x)=\log(3x+2)$. C.E.: $x>-\frac{2}{3}$; $f'(x)=\frac{3}{3x+2}$. $y+3x=2 \Rightarrow y=-3x+\frac{2}{3}$. Per avere la perpendicolarità: $\frac{3}{3x_0+2}=-\frac{1}{m}=\frac{1}{3} \Rightarrow 3x_0+2=9 \Rightarrow x_0=\frac{7}{3}$. $f\left(\frac{7}{3}\right)=\log(7+2)=\log 9$.

Equazione retta tangente: $y-\log 9=\frac{1}{3}\left(x-\frac{7}{3}\right) \Rightarrow y=\frac{1}{3}x-\frac{7}{9}+\log 9$.

8) $f(x;y)=x^3-y^2-xy-3x-y$.

$$\begin{cases} f'_x=3x^2-y-3=0 \\ f'_y=-2y-x-1=0 \end{cases} \Rightarrow \begin{cases} y=3x^2-3 \\ -6x^2+6-x-1=0 \end{cases} \Rightarrow \begin{cases} 6x^2+x-5=0 \\ y=3x^2-3 \end{cases} \Rightarrow \begin{cases} x=\frac{-1\pm 11}{12} \\ y=3x^2-3 \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=0 \end{cases} \cup \begin{cases} x=\frac{5}{6} \\ y=-\frac{11}{12} \end{cases}.$$

$H=\left\|\begin{matrix}6x & -1\\ -1 & -2\end{matrix}\right\|$. $H(-1;0)=\left\|\begin{matrix}-6 & -1\\ -1 & -2\end{matrix}\right\|$: P. di Massimo; $H\left(\frac{5}{6};-\frac{11}{12}\right)=\left\|\begin{matrix}5 & -1\\ -1 & -2\end{matrix}\right\|$: P. di Sella.

9) $\int_0^1 \frac{x+k}{x+1}dx=\int_0^1 \frac{x+1+(k-1)}{x+1}dx=\int_0^1 1+(k-1)\cdot\frac{1}{x+1}dx=\left(x+(k-1)\log(x+1)\right)\Big|_0^1=$

$=\left(1+(k-1)\log 2\right)-\left(0+(k-1)\log 1\right)=1+(k-1)\log 2=2 \Rightarrow k-1=\frac{1}{\log 2} \Rightarrow k=1+\log_2 e$.

10) $x_1 \parallel x_2$: $\frac{k}{-1}=\frac{1}{3}=\frac{-2}{m} \Rightarrow k=-\frac{1}{3}$ e $m=-6$.

$x_1 \perp x_2$: $(k;1;-2)\cdot(-1;3;m)=0 \Rightarrow -k+3-2m=0 \Rightarrow k=3-2m$.

$A\cdot Y=\left\|\begin{matrix}k & 1 & -2\\ -1 & 3 & m\end{matrix}\right\|\cdot\left\|\begin{matrix}1\\ -1\\ 1\end{matrix}\right\|=\left\|\begin{matrix}k-1-2\\ -1-3+m\end{matrix}\right\|=\left\|\begin{matrix}k-3\\ m-4\end{matrix}\right\|=\left\|\begin{matrix}0\\ 0\end{matrix}\right\| \Rightarrow \begin{cases} k=3 \\ m=4 \end{cases}.$