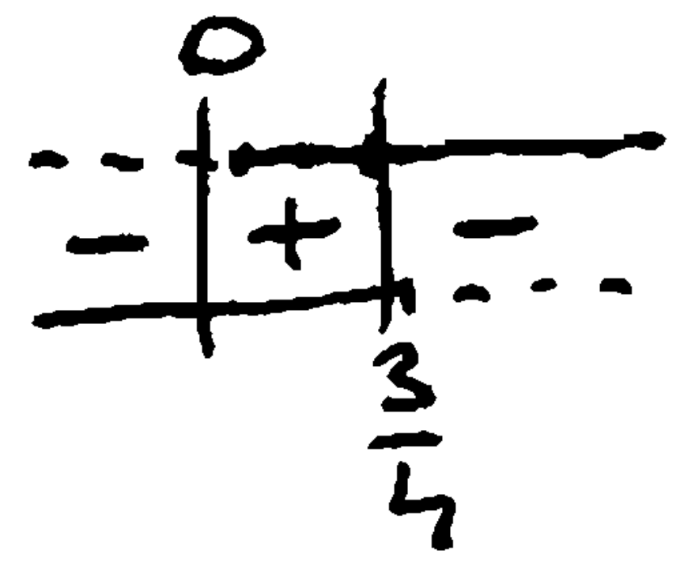
# Compito di Matematica Generale del 17/2/2015 Compito A $\boxed{\text{MGA1}}$

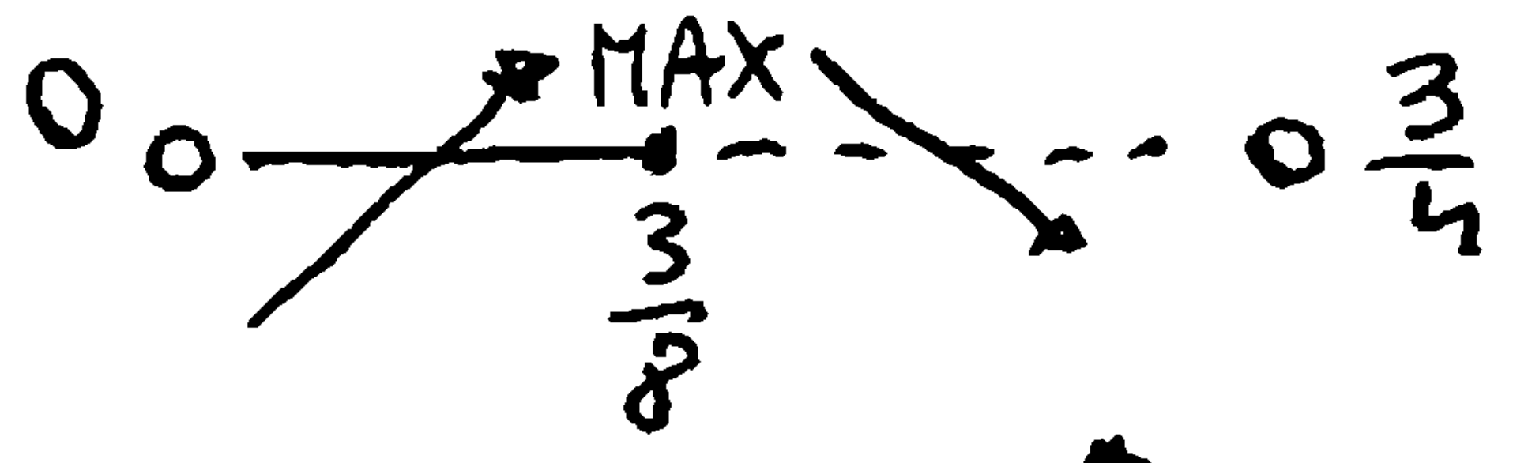
1) $f(x) = \log(3x-4x^2)$. C.E.: $3x-4x^2 = x(3-4x)>0 \begin{cases} x>0 \\ x<\frac{3}{4} \end{cases}$ C.E.$=\left]0;\frac{3}{4}\right[$.



$\lim_{x\to 0^+} f(x) = -\infty$; $\lim_{x\to \frac{3}{4}^-} f(x) = -\infty$.

$f(x) \geq 0 \Rightarrow 3x-4x^2>1 \Rightarrow 4x^2-3x+1<0 : \Delta = 9-16<0 \Rightarrow f(x)<0 \quad \forall x \in$ C.E.

$f'(x) = \frac{3-8x}{3x-4x^2} \geq 0 \Rightarrow 3-8x \geq 0 \Rightarrow x \leq \frac{3}{8}$



$$f''(x) = \frac{-8(3x-4x^2)-(3-8x)(3-8x)}{(3x-4x^2)^2} =$$

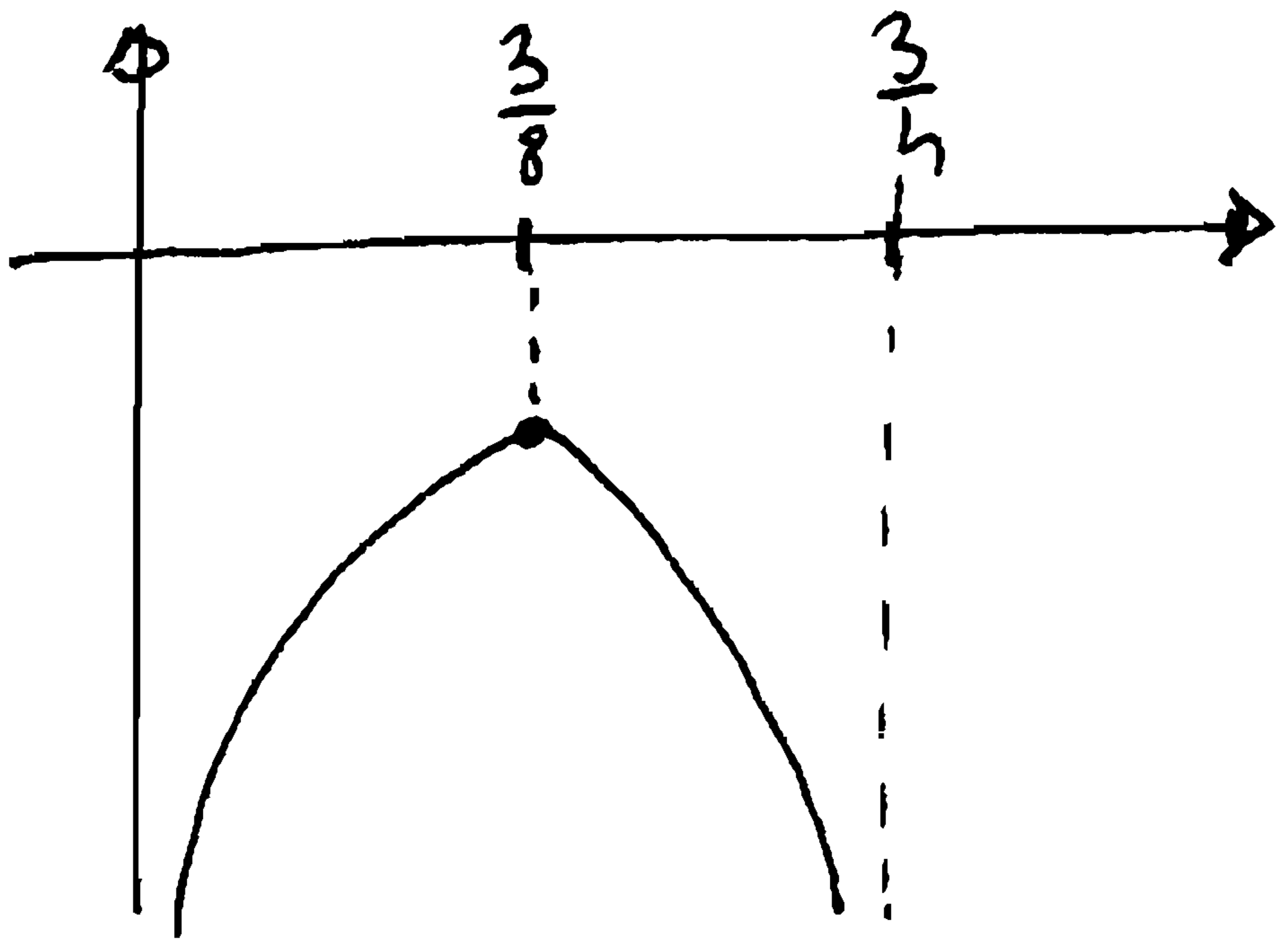
$$= \frac{-24x+32x^2-9-64x^2+48x}{(3x-4x^2)^2} =$$

$$= -\frac{32x^2-24x+9}{(3x-4x^2)^2} \geq 0 \Rightarrow 32x^2-24x+9 \leq 0$$

mai: $\Delta<0$

Funzione sempre concava: $f''(x)<0 \; \forall x$

Grafico



2) $\lim_{x\to 0} \frac{3^{x^2}-\cos x}{2x^2} = \lim_{x\to 0} \frac{3^{x^2}-1}{2\cdot x^2} + \frac{1-\cos x}{2\cdot x^2} = \frac{1}{2}\cdot \lim_{t\to 0} \frac{3^t-1}{t^2} + \frac{1}{2}\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}\left(\log 3 + \frac{1}{2}\right)$

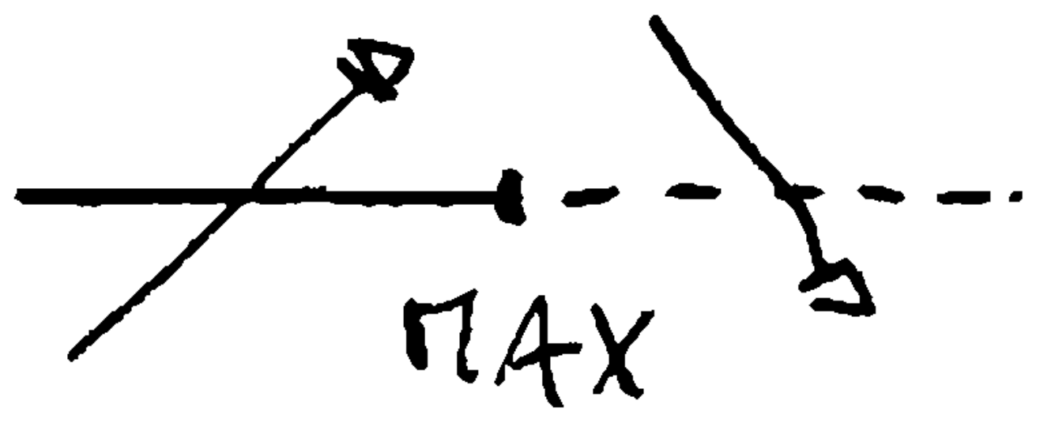
$\lim_{x\to -\infty} \frac{3^x-\log(1-x)}{1+x} = \lim_{x\to -\infty} \frac{-\log(1-x)}{x} = 0^+ \; (3^x \to 0^+;\ \log(1-x) = o(x);\ 1 = o(x))$.

3) $f(x) = 3^x - x$; $g(x) = 2^x - x$.

$f(x) \sim g(x)$ se $\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{3^x-x}{2^x-x} = 1$. Vera se $x\to -\infty$ e se $x \to 0$.

4) $f(x) = 3x - me^{1+2x}$. C.E.: $\mathbb{R}$; $f'(x) = 3-2me^{1+2x} \geq 0 \Rightarrow e^{1+2x} \leq \frac{3}{2m} \Rightarrow$

$\Rightarrow x \leq \frac{1}{2}\cdot\left(\log\frac{3}{2m}-1\right)$



. $f'(1) = 0 \Rightarrow 3-2me^3 = 0 \Rightarrow m = \frac{3}{2}e^{-3}$.

5) $f(x) = e^{3x-1}$; $g(x) = e^{2x+1}$. Per avere tangenti parallele: $f'(x_0) = g'(x_0) \Rightarrow$

$\Rightarrow 3e^{3x-1} = 2e^{2x+1} \Rightarrow \frac{e^{3x-1}}{e^{2x+1}} = e^{x-2} = \frac{2}{3} \Rightarrow x-2 = \log\frac{2}{3} \Rightarrow x_0 = 2+\log\frac{2}{3}$.

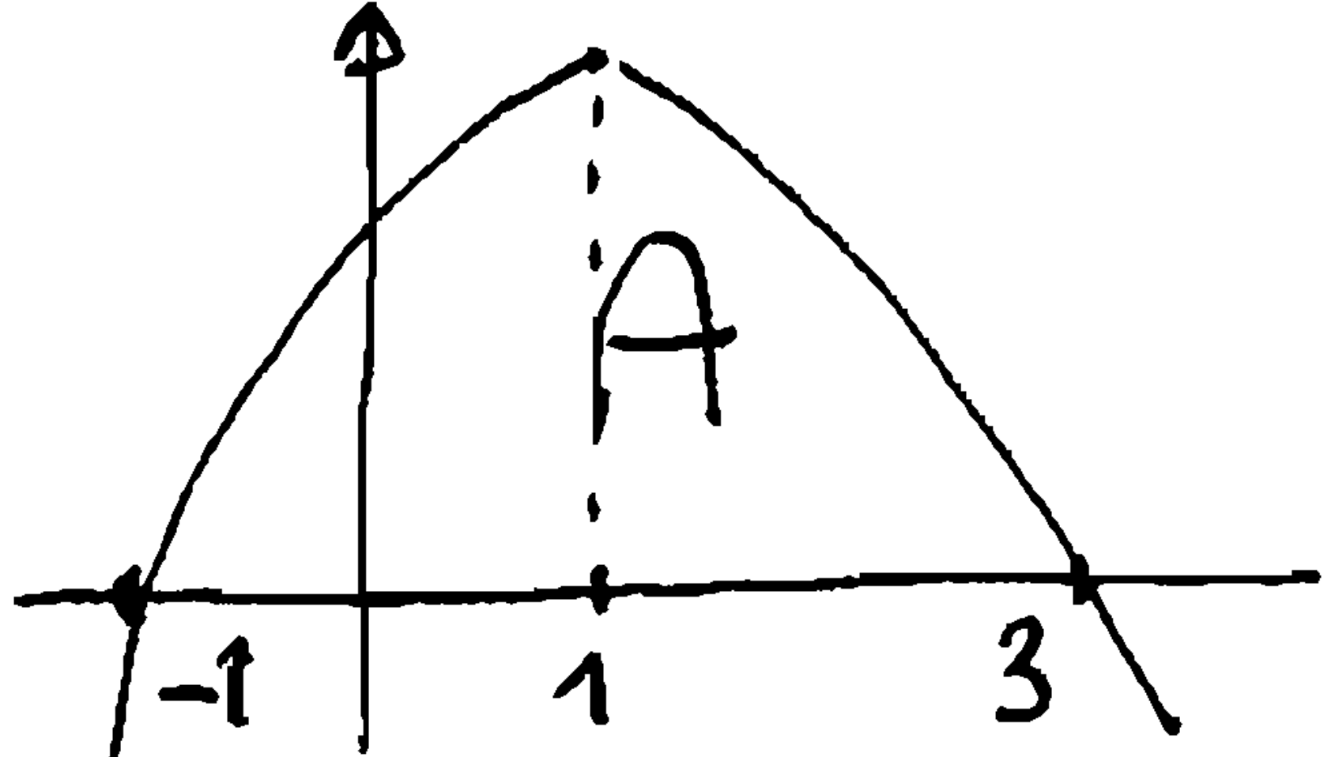
6) $f(x) = \sqrt[3]{3x+2}$; $f(g(x)) = \log x$.

$\boxed{\text{MGA2}}$

$f(g(x)) = \sqrt[3]{3g(x)+2} = \log x \Rightarrow 3g(x)+2 = \log^3 x \Rightarrow g(x) = \frac{1}{3}\left(\log^3 x - 2\right).$

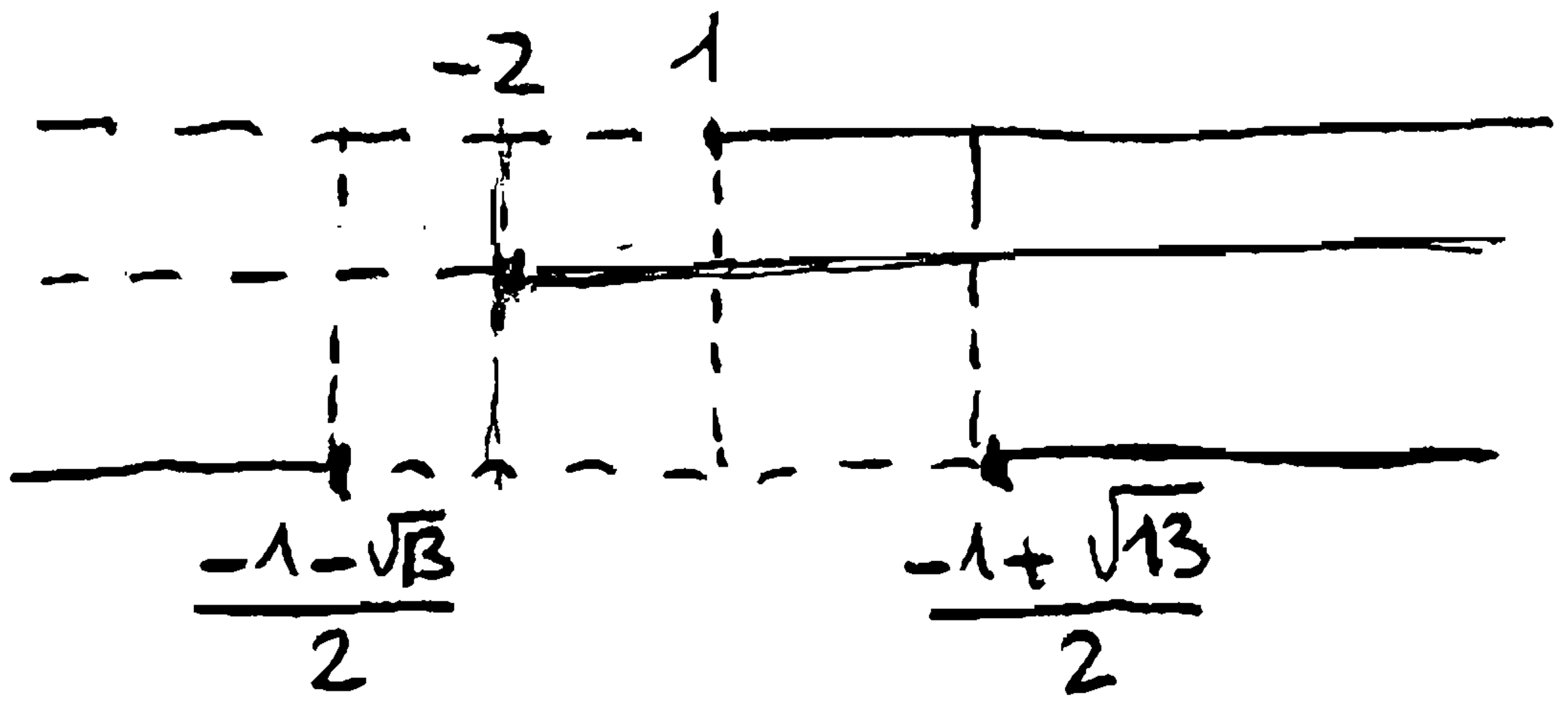
7) $f(x) = 3+2x-x^2$ (Parabola) $f(x)=0$ per $x^2-2x-3=(x+1)(x-3)=0$ per $x=-1$ e $x=3$.

$f'(x) = 2-2x = 0$ per $x = 1$ (Vertice). Area $= \int_0^3 f(x)\,dx = \int_0^3 3+2x-x^2\,dx =$



$= \left(3x+x^2-\frac{1}{3}x^3\right|_0^3 = (9+9-9)-(0+0-0) = 9.$

8) $H = \begin{Vmatrix} k-1 & -1 \\ -1 & k+2 \end{Vmatrix}$ : $f''_{xx} = k-1 > 0$ per $k>1$
$f''_{yy} = k+2 > 0$ per $k>-2$



$|H_2| = (k-1)(k+2)-1 = k^2+k-3 > 0$ : $k < \frac{-1-\sqrt{13}}{2} \cup k > \frac{-1+\sqrt{13}}{2}$

$k = \frac{-1\pm\sqrt{1+12}}{2} = \frac{-1\pm\sqrt{13}}{2}$

Se $-\infty < k < \frac{-1-\sqrt{13}}{2}$ : $f''_{xx} < 0$ ; $f''_{yy} < 0$; $|H_2| > 0$ : Punto di Massimo

Se $\frac{-1+\sqrt{13}}{2} < k < +\infty$ : $f''_{xx} > 0$ ; $f''_{yy} > 0$; $|H_2| > 0$ : Punto di Minimo.

Se $\frac{-1-\sqrt{13}}{2} < k < \frac{-1+\sqrt{13}}{2}$ : $|H_2| < 0$ : Punto di Sella.

9) $A \cdot B \cdot V = \begin{Vmatrix} k & 1 & -2 \\ 1 & 1 & k \end{Vmatrix} \cdot \begin{Vmatrix} 1 & -1 \\ 0 & k \\ k & 2 \end{Vmatrix} \cdot \begin{Vmatrix} 2 \\ 1 \end{Vmatrix} = \begin{Vmatrix} -k & -4 \\ 1+k^2 & 3k-1 \end{Vmatrix} \cdot \begin{Vmatrix} 2 \\ 1 \end{Vmatrix} = \begin{Vmatrix} -2k-4 \\ 2k^2+3k+1 \end{Vmatrix} = \begin{Vmatrix} -2 \\ 0 \end{Vmatrix} \Rightarrow$

$\Rightarrow -2k-4 = -2 \Rightarrow 2k = -2 \Rightarrow k = -1$; Se $k=-1$ : $2\cdot 1 - 3 + 1 = 0 \Rightarrow k = -1$.

10) $f(x;y) = xy - y + \log(x-y)$

$\frac{\partial f}{\partial x} = y - 0 + \frac{1}{x-y} = y + \frac{1}{x-y}$ quindi la proposizione A è FALSA

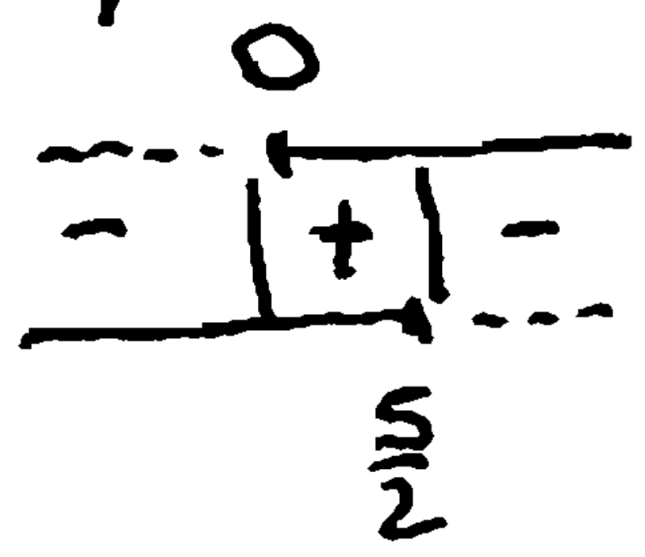
$\frac{\partial f}{\partial y} = x - 1 - \frac{1}{x-y}$ quindi la proposizione B è VERA.

| A | B | non B | non B ⇒ A |
|---|---|---|---|
| 0 | 1 | 0 | 1 |

: la proposizione non B ⇒ A è VERA.

# Compito di Matematica Generale del 17/2/2015 Compito B

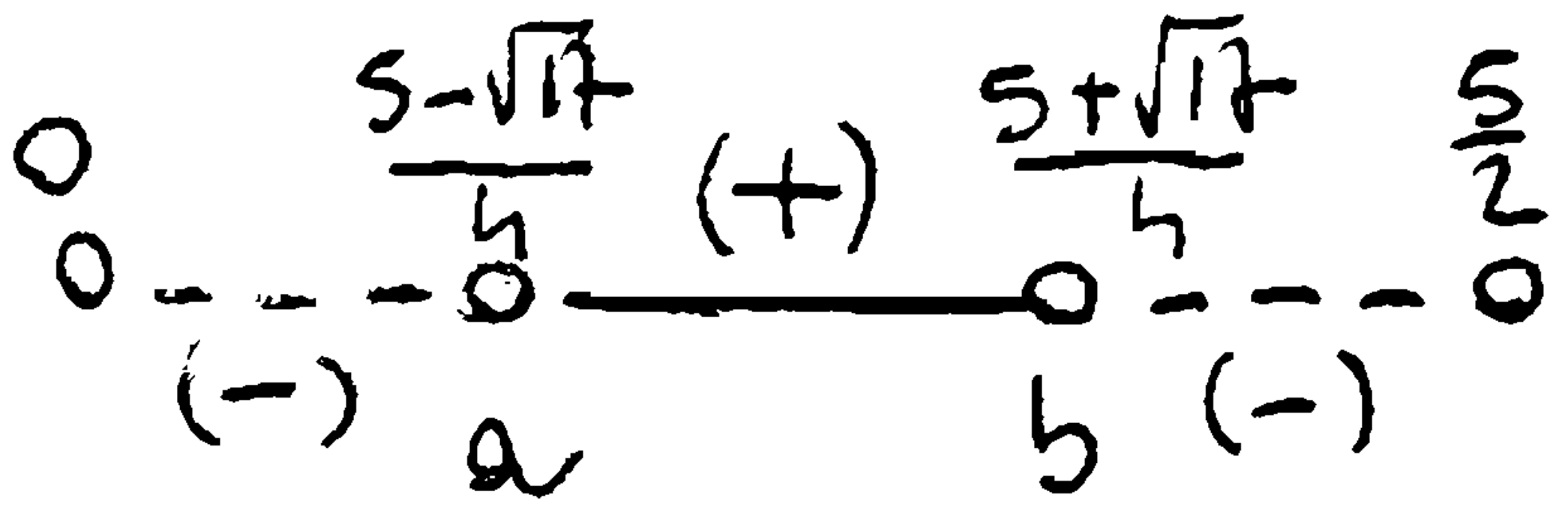
$\boxed{MGB1}$

1) $f(x)=\log(5x-2x^2)$. C.E.: $5x-2x^2=x\cdot(5-2x)>0 \begin{cases} x>0 \\ x<\frac{5}{2} \end{cases}$ 

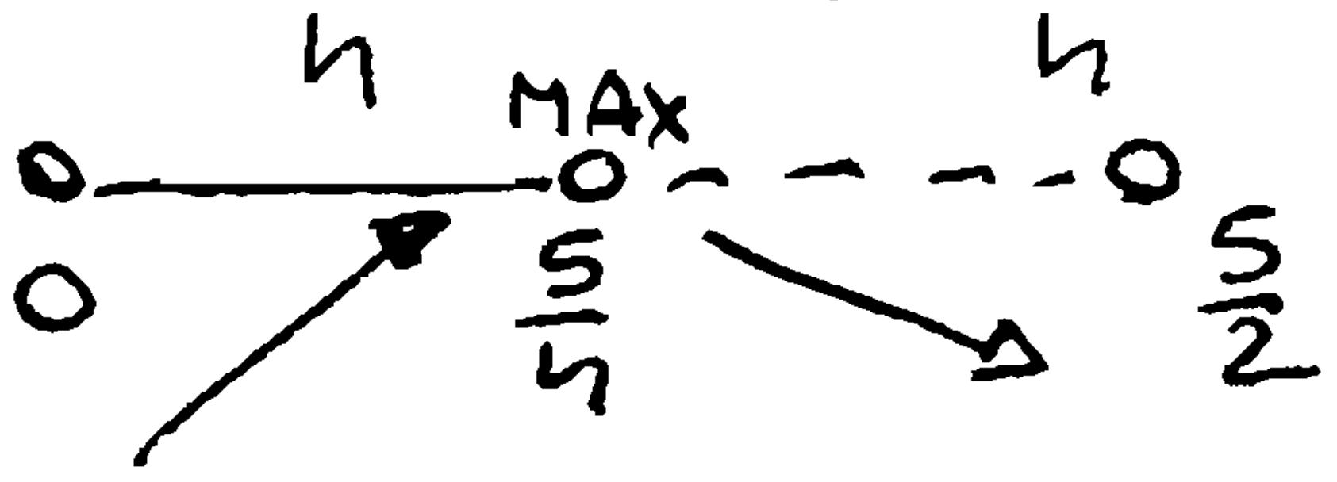
C.E. $=]0;\frac{5}{2}[$.

$\lim_{x\to0^+} f(x)=-\infty$; $\lim_{x\to\frac{5}{2}^-} f(x)=-\infty$.

$f(x)\geq0 \Rightarrow 5x-2x^2>1 \Rightarrow 2x^2-5x+1<0 \Rightarrow \frac{5-\sqrt{17}}{4}<x<\frac{5+\sqrt{17}}{4}$



$f'(x)=\frac{5-4x}{5x-2x^2}\geq0 \Rightarrow 5-4x\geq0 \Rightarrow x\leq\frac{5}{4}$

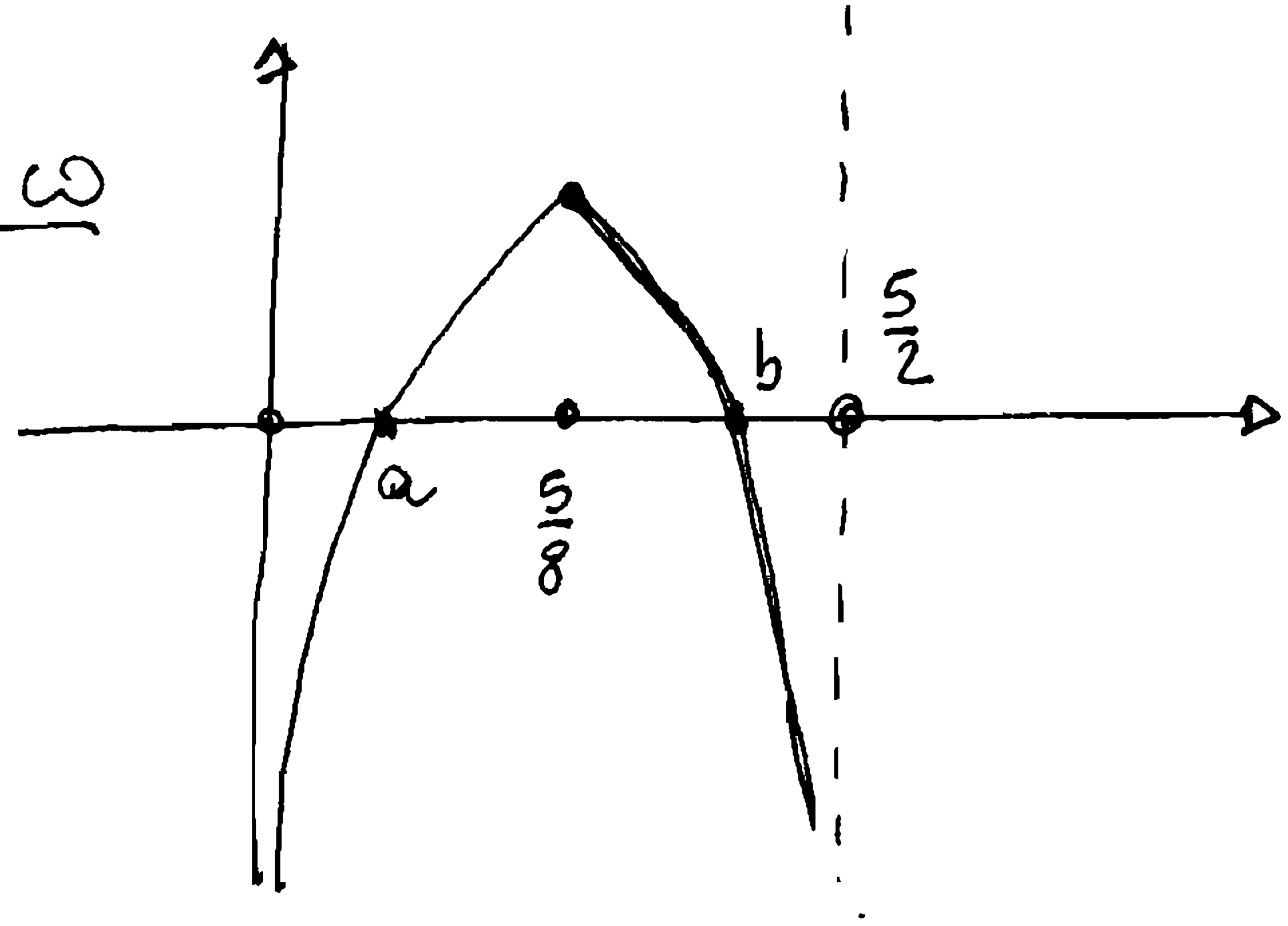


$f''(x)=\frac{-4(5x-2x^2)-(5-4x)(5-4x)}{(5x-2x^2)^2}=$

$=\frac{-20x+8x^2-25-16x^2+40x}{(5x-2x^2)^2}=$

$=-\frac{8x^2-20x+25}{(5x-2x^2)^2}\geq0 \Rightarrow 8x^2-20x+25\leq0$
mai: $\Delta<0$

Grafico



Funzione sempre concava: $f''(x)<0\ \forall x$.

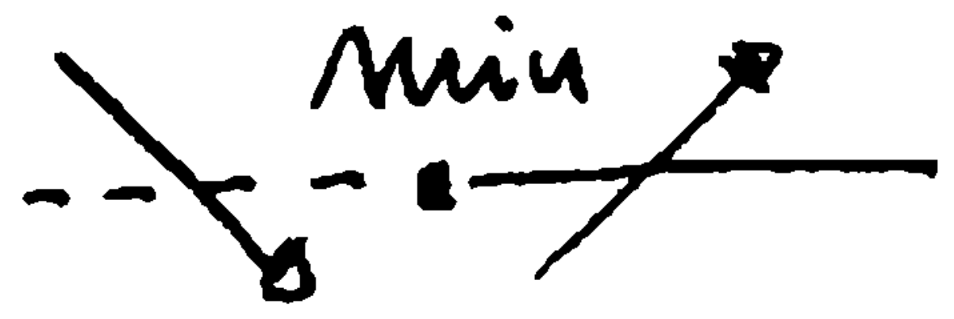
2) $\lim_{x\to0}\frac{e^{x^2}-\cos x}{3x^2}=\lim_{x\to0}\frac{e^{x^2}-1}{3\cdot x^2}+\frac{1-\cos x}{3x^2}=\frac{1}{3}\lim_{t\to0}\frac{e^t-1}{t}+\frac{1}{3}\lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{3}\cdot1+\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{3}\cdot\frac{3}{2}=\frac{1}{2}$.

$\lim_{x\to-\infty}\frac{x-2^x}{\log(2-x)}=\lim_{x\to-\infty}\frac{x}{\log(2-x)}=-\infty \left(2^x\to0;\ \log(2-x)=o(x)\right)$.

3) $f(x)=2^x-x$ ; $g(x)=4^x-x$.

$f(x)\sim g(x)$ se $\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{2^x-x}{4^x-x}=1$. Vera se $x\to-\infty$ e se $x\to0$.

4) $f(x)=me^{1+2x}-3x$. C.E.: $\mathbb{R}$. $f'(x)=2me^{1+2x}-3\geq0 \Rightarrow e^{1+2x}\geq\frac{3}{2m} \Rightarrow$

$\Rightarrow x\geq\frac{1}{2}\left(\log\frac{3}{2m}-1\right)$ 

. $f'(1)=0 \Rightarrow 2me^3-3=0 \Rightarrow m=\frac{3}{2}e^{-3}$.

5) $f(x)=e^{2x-1}$ ; $g(x)=e^{3x+2}$. Per avere tangenti parallele: $f'(x_0)=g'(x_0) \Rightarrow$

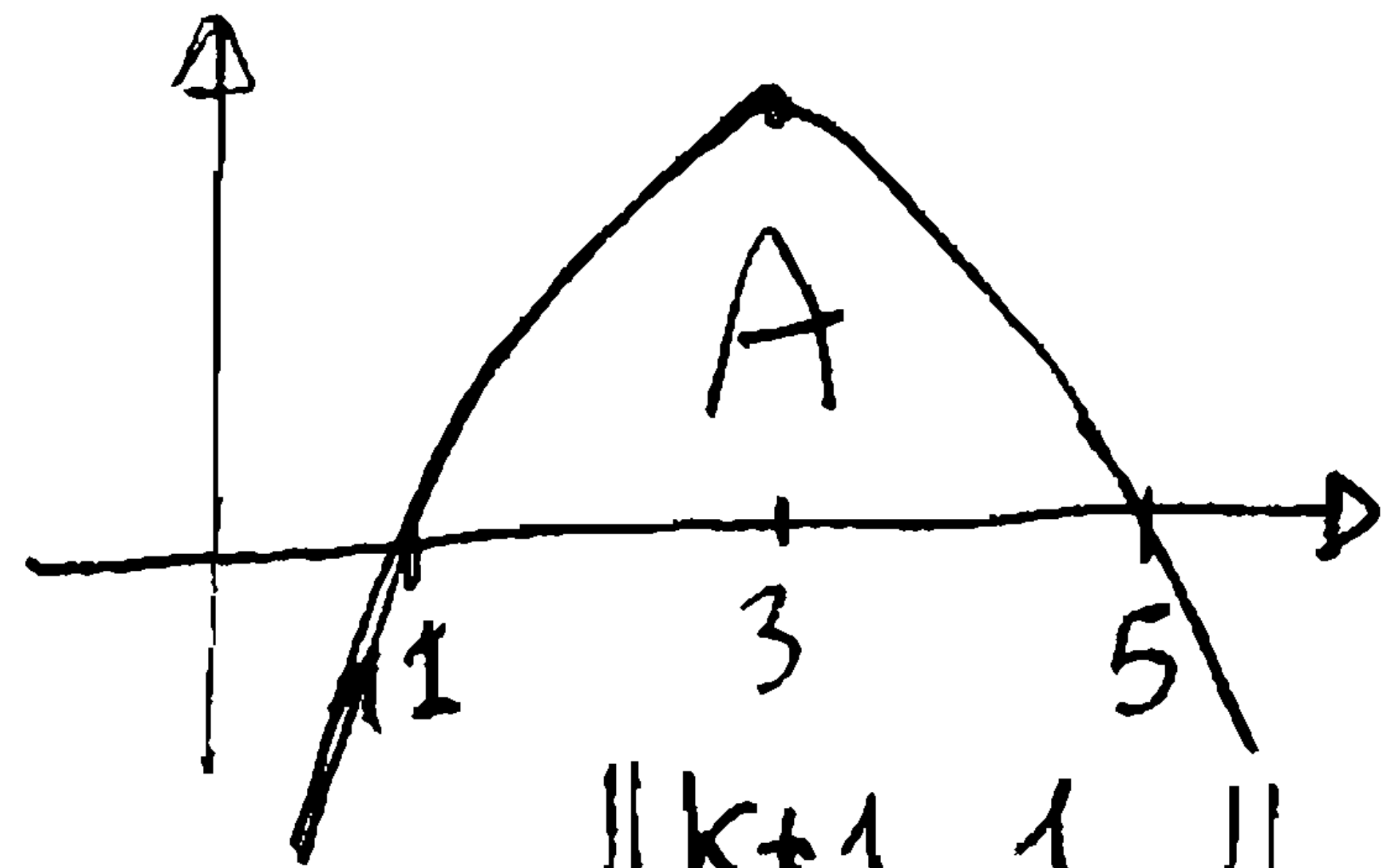
$\Rightarrow 2e^{2x-1}=3e^{3x+2} \Rightarrow \frac{e^{3x+2}}{e^{2x-1}}=\frac{2}{3} \Rightarrow e^{x+3}=\frac{2}{3} \Rightarrow x+3=\log\frac{2}{3} \Rightarrow x_0=\log\frac{2}{3}-3$.

6) $f(x)=\sqrt[3]{2x-1}$ ; $f(g(x))=e^x$. $f(g(x))=\sqrt[3]{2g(x)-1}=e^x \Rightarrow$

$\Rightarrow 2g(x)-1=(e^x)^3 \Rightarrow 2g(x)=e^{3x}+1 \Rightarrow g(x)=\frac{1}{2}\left(e^{3x}+1\right).$

7) $f(x)=6x-x^2-5$ (Parabola). $f(x)=0$ per $x^2-6x+5=(x-1)(x-5)=0$ per $x=1$ e $x=5$.

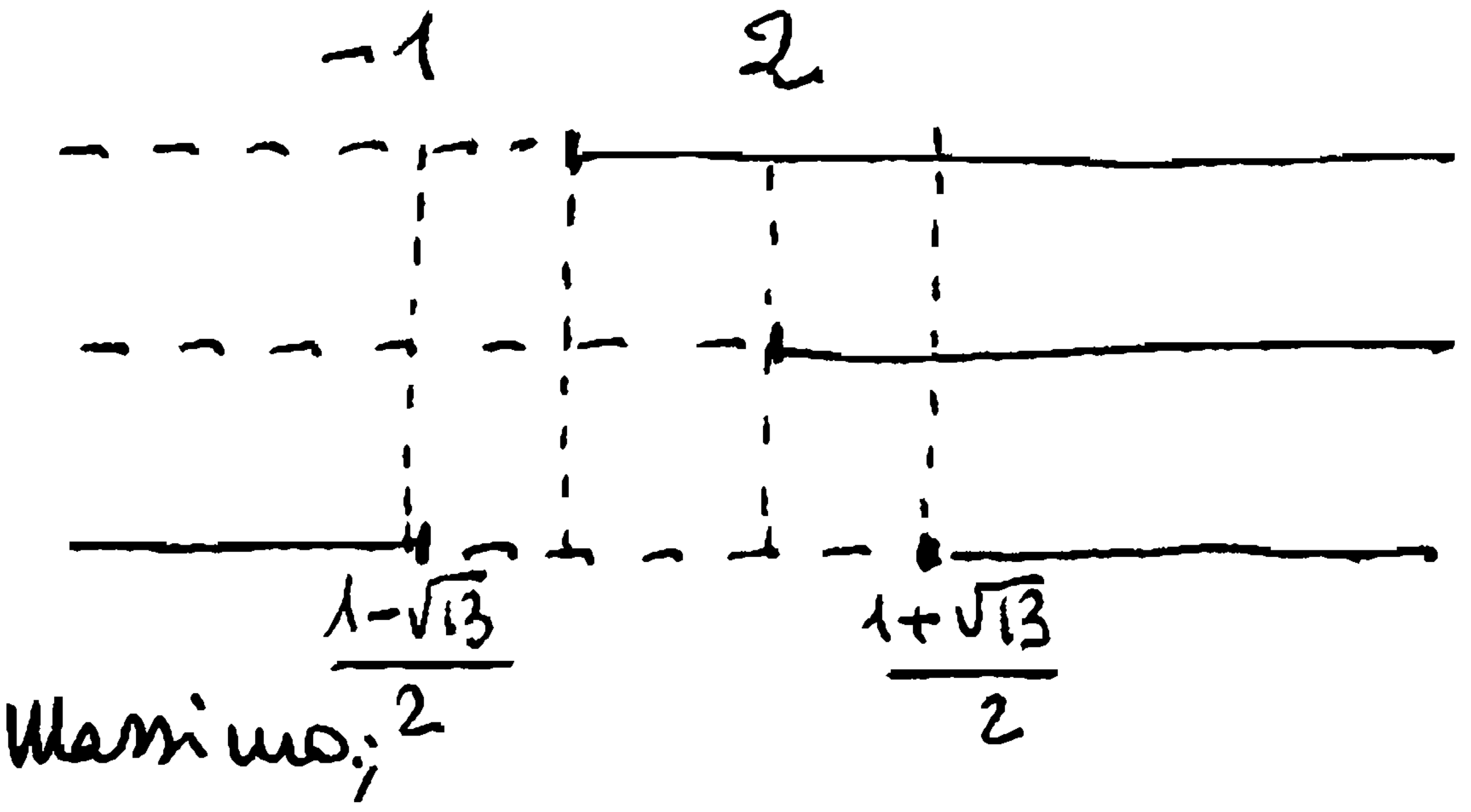
$f'(x)=6-2x=0$ per $x=3$ (Vertice). Area $=\int_1^5 f(x)\,dx=\int_1^5 6x-x^2-5\,dx=$



$=\left(3x^2-\frac{x^3}{3}-5x\right|_1^5=\left(75-\frac{125}{3}-25\right)-\left(3-\frac{1}{3}-5\right)=\frac{25}{3}+\frac{7}{3}=\frac{32}{3}.$

8) $H=\begin{Vmatrix} k+1 & 1 \\ 1 & k-2 \end{Vmatrix}$ : $f''_{xx}=k+1>0$ per $k>-1$

$f''_{yy}=k-2>0$ per $k>2$



$|H_2|=(k+1)(k-2)-1=k^2-k-3>0$ : $k<\frac{1-\sqrt{13}}{2} \cup k>\frac{1+\sqrt{13}}{2}$

Se $-\infty<k<\frac{1-\sqrt{13}}{2}$ : $f''_{xx}<0$; $f''_{yy}<0$; $|H_2|>0$ : punto di Massimo;

Se $\frac{1+\sqrt{13}}{2}<k<+\infty$ : $f''_{xx}>0$; $f''_{yy}>0$; $|H_2|>0$ : punto di minimo;

Se $\frac{1-\sqrt{13}}{2}<k<\frac{1+\sqrt{13}}{2}$ : $|H_2|<0$ : Punto di Sella.

9) $A\cdot B\cdot V=\begin{Vmatrix} 2 & -1 & k \\ k & 0 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 & 2 \\ k & 1 \\ 1 & k \end{Vmatrix}\cdot\begin{Vmatrix} -1 \\ 1 \end{Vmatrix}=\begin{Vmatrix} 2 & 3+k^2 \\ k+1 & 3k \end{Vmatrix}\cdot\begin{Vmatrix} -1 \\ 1 \end{Vmatrix}=\begin{Vmatrix} k^2+1 \\ 2k-1 \end{Vmatrix}=\begin{Vmatrix} 2 \\ 1 \end{Vmatrix} \Rightarrow$

$\Rightarrow 2k-1=1 \Rightarrow 2k=2 \Rightarrow k=1$ ; Se $k=1$ : $1^2+1=2 \Rightarrow k=1$.

10) $f(x;y)=x^2+xy-e^{x-y}$.

$\frac{\partial f}{\partial x}=2x+y-e^{x-y}$ quindi la proposizione A è VERA.

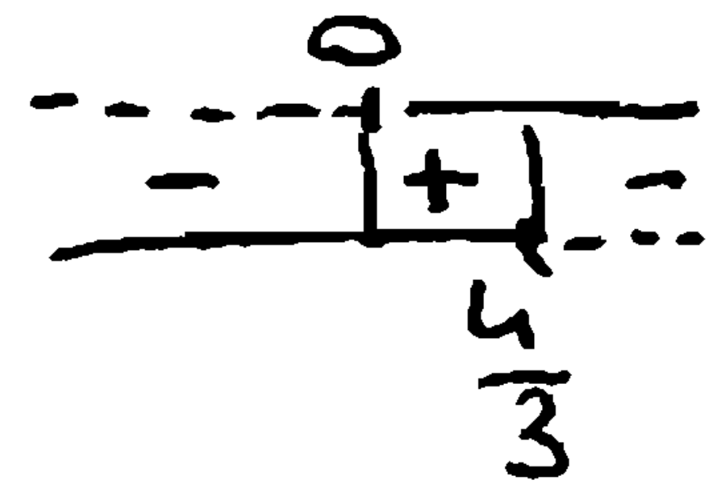
$\frac{\partial f}{\partial y}=0+x+e^{x-y}$ quindi la proposizione B è FALSA.

| A | B | non B | A ⇒ non B |
|---|---|---|---|
| 1 | 0 | 1 | 1 |

: la proposizione A ⇒ non B è VERA.

# Compito di Matematica Generale del 17/2/2015 Compito C $\boxed{\text{MGC1}}$

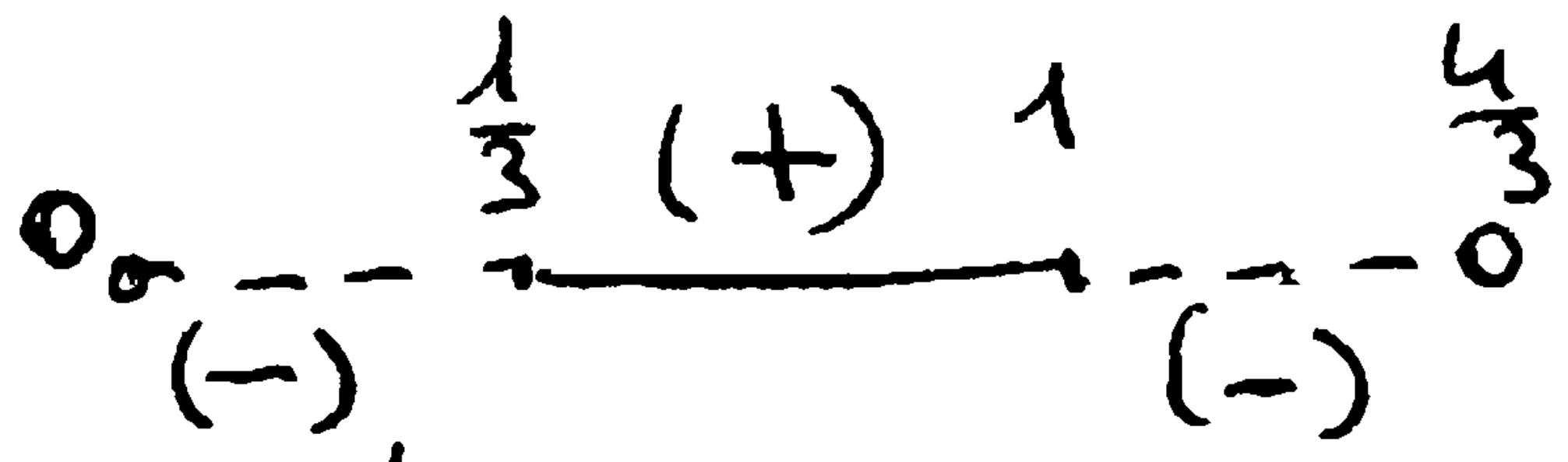
1) $f(x)=\log(4x-3x^2)$. C.E.: $4x-3x^2=x(4-3x)>0 \begin{cases} x>0 \\ x<\frac{4}{3} \end{cases}$



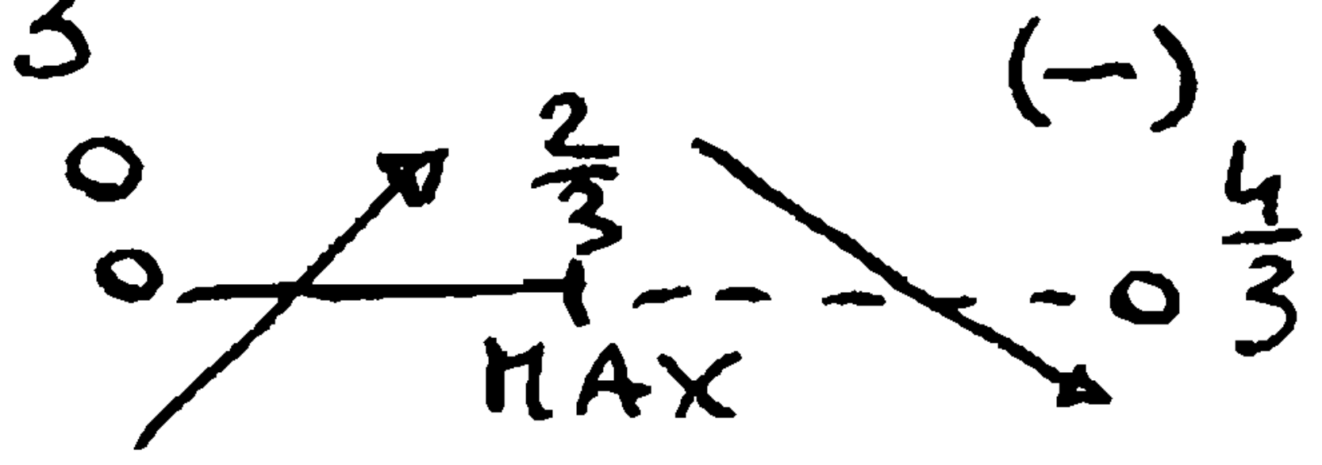
C.E. $=]0;\frac{4}{3}[$.

$\lim_{x\to0^+} f(x)=-\infty$; $\lim_{x\to\frac{4}{3}^-} f(x)=-\infty$.

$f(x)\geq0 \Rightarrow 4x-3x^2>1 \Rightarrow 3x^2-4x+1<0 \Rightarrow \frac{1}{3}<x<1$



$f'(x)=\frac{4-6x}{4x-3x^2}\geq0 \Rightarrow 4-6x\geq0 \Rightarrow x\leq\frac{2}{3}$



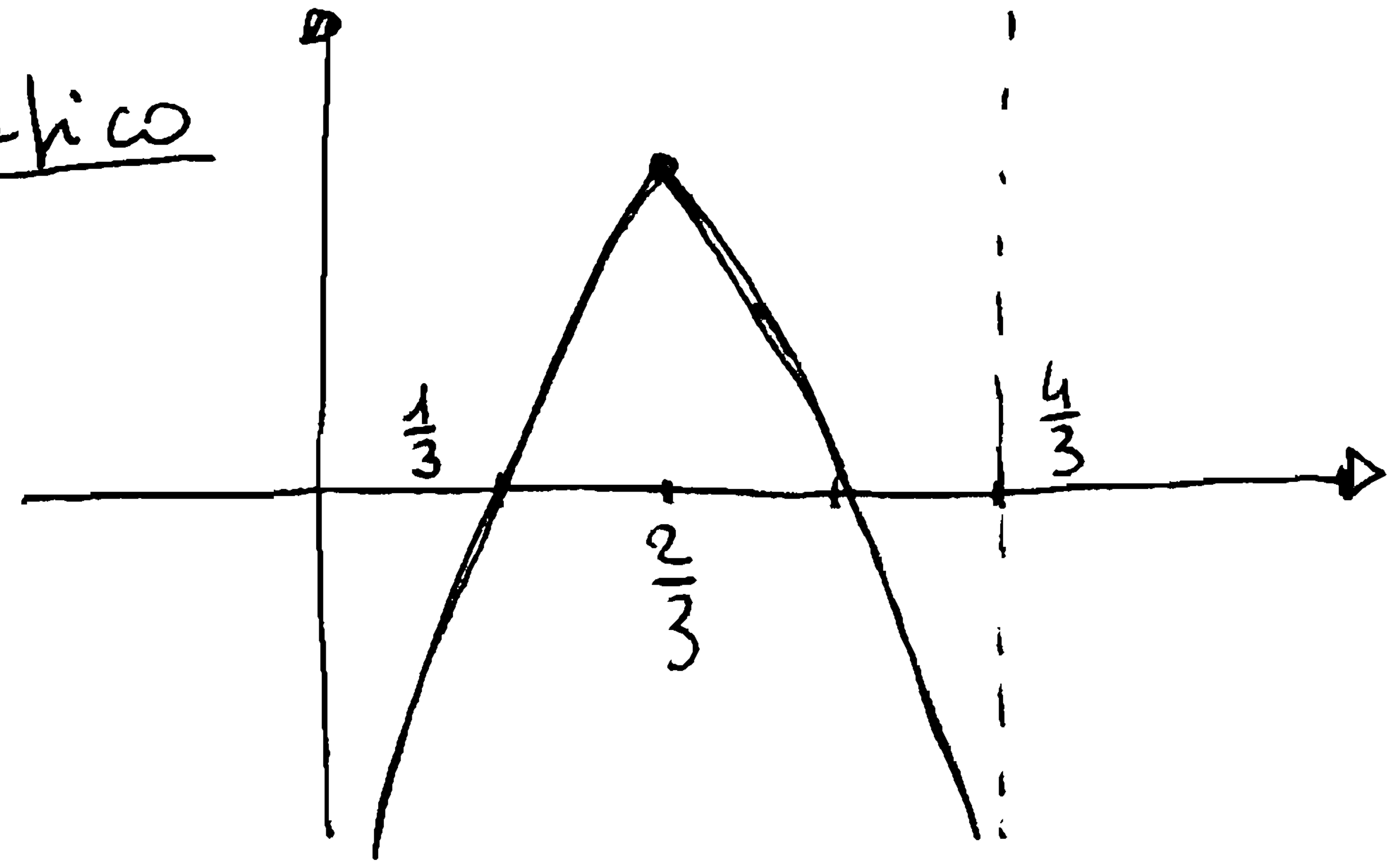
$f''(x)=\frac{-6(4x-3x^2)-(4-6x)(4-6x)}{(4x-3x^2)^2}=$

$=\frac{-24x+18x^2-16-36x^2+48x}{(4x-3x^2)^2}=$

$=-\frac{18x^2-24x+16}{(4x-3x^2)^2}\geq0 \Rightarrow 18x^2-24x+16<0$ mai: $\Delta<0$

Funzione Sempre Concava: $f''(x)<0\ \forall x$.

Grafico



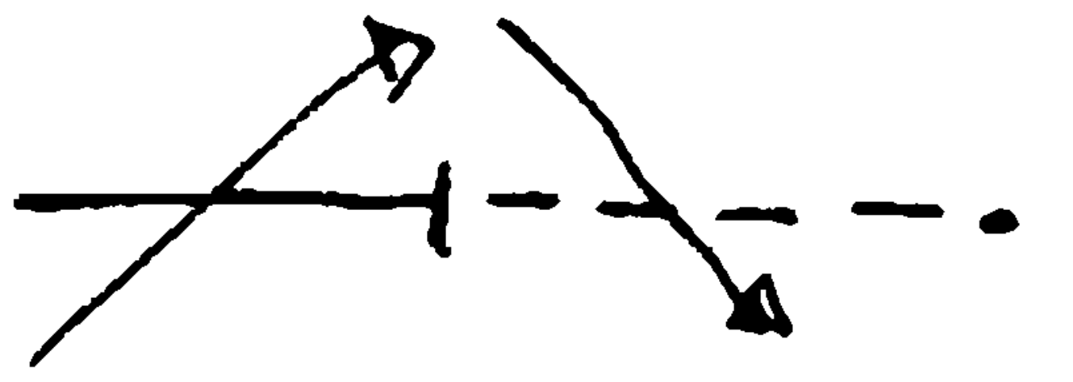
2) $\lim_{x\to0}\frac{2^{x^2}-\cos x}{3x^2}=\lim_{x\to0}\frac{2^{x^2}-1}{3\cdot x^2}+\frac{1-\cos x}{3\cdot x^2}=\frac{1}{3}\lim_{t\to0}\frac{2^t-1}{t}+\frac{1}{3}\lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{3}\log2+\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{3}\left(\log2+\frac{1}{2}\right)$.

$\lim_{x\to-\infty}\frac{\log(1-x)-e^x}{1-x}=\lim_{x\to-\infty}\frac{\log(1-x)}{-x}=0^+$ $\left(e^x\to0;\ \log(1-x)=o(x);\ 1=o(x)\right)$.

3) $f(x)=4^x-x$; $g(x)=3^x-x$.

$f(x)\sim g(x)$ se $\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{4^x-x}{3^x-x}=1$. Vera se $x\to-\infty$ e se $x\to0$.

4) $f(x)=2x-me^{1+3x}$. C.E. $=\mathbb{R}$. $f'(x)=2-3me^{1+3x}\geq0 \Rightarrow e^{1+3x}\leq\frac{2}{3m} \Rightarrow$

$\Rightarrow x\leq\frac{1}{3}\left(\log\frac{2}{3m}-1\right)$  $f'(1)=0 \Rightarrow 2-3me^4=0 \Rightarrow m=\frac{2}{3}\cdot e^{-4}$.

5) $f(x)=e^{3x+1}$; $g(x)=e^{2x+2}$. Per avere tangenti parallele: $f'(x_0)=g'(x_0) \Rightarrow$

$3e^{3x+1}=2e^{2x+2} \Rightarrow \frac{e^{3x+1}}{e^{2x+2}}=\frac{2}{3} \Rightarrow e^{x-1}=\frac{2}{3} \Rightarrow x-1=\log\frac{2}{3} \Rightarrow x_0=1+\log\frac{2}{3}$.

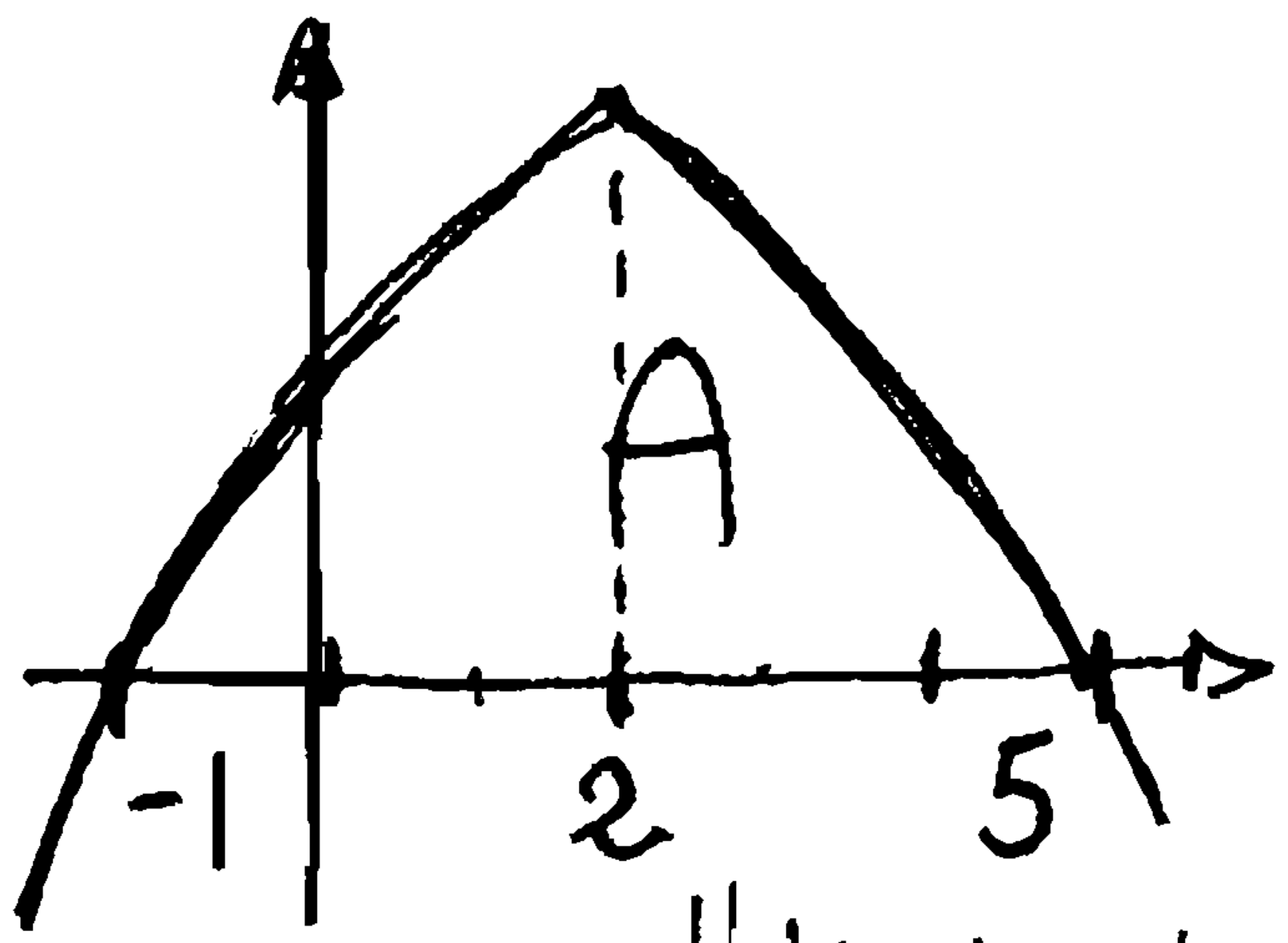
$\boxed{MGC2}$

6) $f(x)=\sqrt[3]{x+3}$ ; $f(g(x))=\operatorname{sen}x$ ; $f(g(x))=\sqrt[3]{g(x)+3}=\operatorname{sen}x \Rightarrow$

$\Rightarrow g(x)+3=\operatorname{sen}^3x \Rightarrow g(x)=\operatorname{sen}^3x-3.$

7) $f(x)=5+4x-x^2$ (Parabola). $f(x)=0$ per $x^2-4x-5=(x+1)(x-5)=0$ per $x=-1$ e $x=5$.

$f'(x)=4-2x=0$ per $x=2$ (Vertice). Area $=\int_0^5 f(x)\,dx=\int_0^5 5+4x-x^2\,dx=$

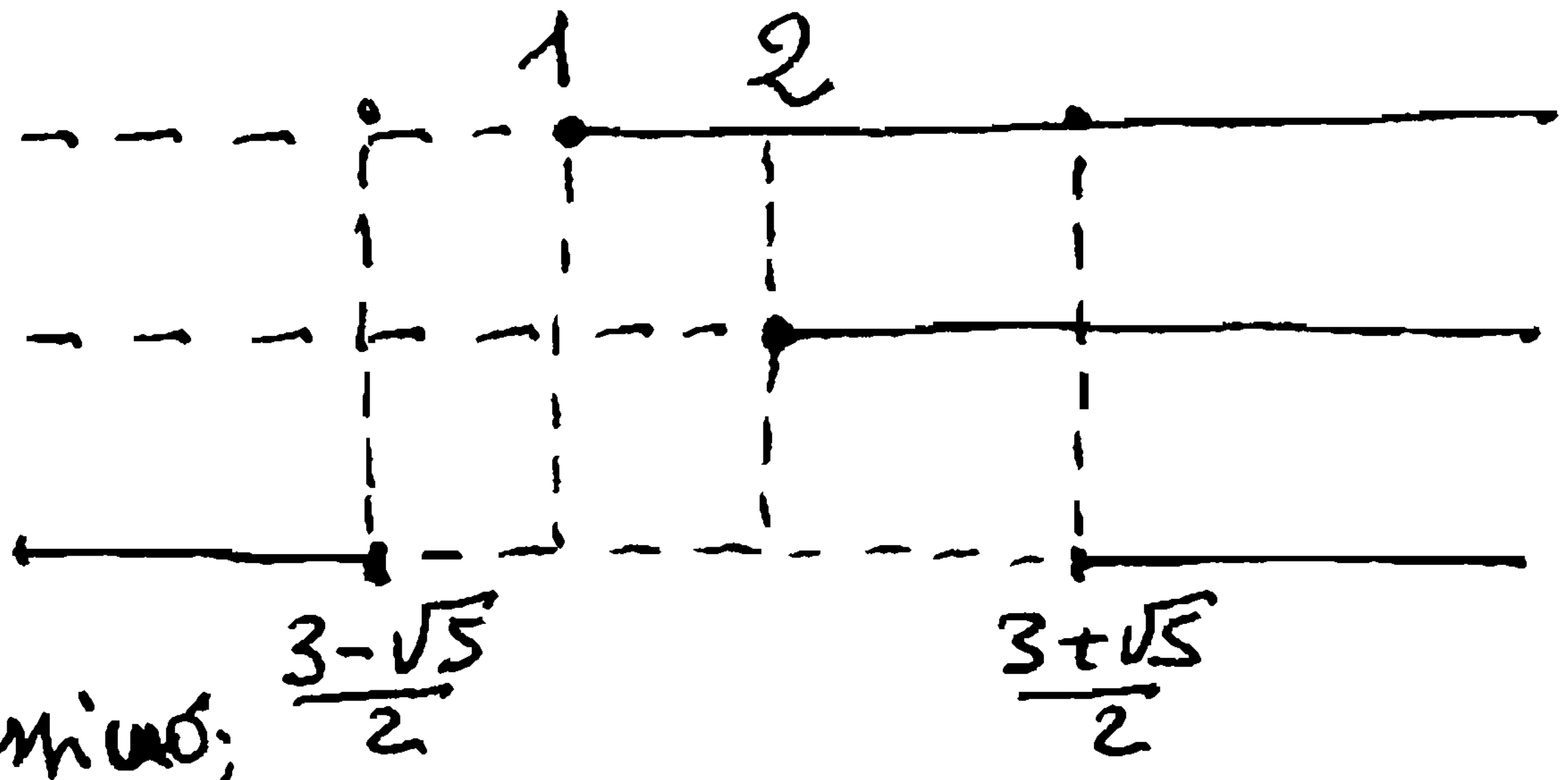


$$=\left(5x+2x^2-\frac{x^3}{3}\right|_0^5=\left(25+50-\frac{125}{3}\right)-(0+0-0)=75-\frac{125}{3}=\frac{100}{3}.$$

8) $H=\begin{Vmatrix} K-1 & 1 \\ 1 & K-2 \end{Vmatrix}$ : $f''_{xx}=K-1>0$ per $K>1$

$f''_{yy}=K-2>0$ per $K>2$

$|H_2|=(K-1)(K-2)-1=K^2-3K+1>0$ : $K<\frac{3-\sqrt5}{2} \cup K>\frac{3+\sqrt5}{2}$



Se $-\infty<K<\frac{3-\sqrt5}{2}$ : $f''_{xx}<0$; $f''_{yy}<0$; $|H_2|>0$ : Punto di Massimo;

Se $\frac{3+\sqrt5}{2}<K<+\infty$ : $f''_{xx}>0$; $f''_{yy}>0$; $|H_2|>0$ : Punto di Minimo;

Se $\frac{3-\sqrt5}{2}<K<\frac{3+\sqrt5}{2}$ : $|H_2|<0$ : Punto di Sella.

9) $A\circ B\circ V=\begin{Vmatrix} K & 1 & 1 \\ 2 & -1 & K \end{Vmatrix}\cdot\begin{Vmatrix} 1 & 1 \\ 1 & K \\ K & -1 \end{Vmatrix}\circ\begin{Vmatrix} 2 \\ -1 \end{Vmatrix}=\begin{Vmatrix} 2K+1 & 2K-1 \\ 1+K^2 & 2-2K \end{Vmatrix}\cdot\begin{Vmatrix} 2 \\ -1 \end{Vmatrix}=\begin{Vmatrix} 2K+3 \\ 2K^2+2K \end{Vmatrix}=\begin{Vmatrix} 1 \\ 0 \end{Vmatrix}\Rightarrow$

$\Rightarrow 2K+3=1 \Rightarrow 2K=-2 \Rightarrow K=-1$ ; Se $K=-1$ : $2(-1)^2+2(-1)=2-2=0 \Rightarrow K=-1.$

10) $f(x;y)=x^{2y}-y\operatorname{sen}x.$

$\frac{\partial f}{\partial x}=2y\cdot x^{2y-1}-y\cos x$ quindi la proposizione A è VERA.

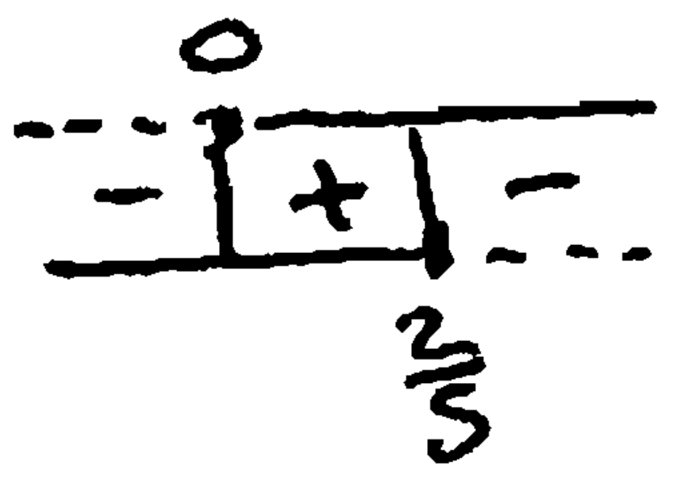
$\frac{\partial f}{\partial y}=x^{2y}\cdot 2\cdot \log x-\operatorname{sen}x$ quindi la proposizione B è VERA.

| A | B | non B | A ⇒ non B |
|---|---|---|---|
| 1 | 1 | 0 | 0 |

: la proposizione A ⇒ non B è FALSA.

# Compito di Matematica Generale del 17/2/2015 Compito D $\boxed{MGD1}$

1) $f(x)=\log(2x-5x^2)$. C.E.: $2x-5x^2=x(2-5x)>0 < \begin{matrix} x>0 \\ x<\frac{2}{5} \end{matrix}$

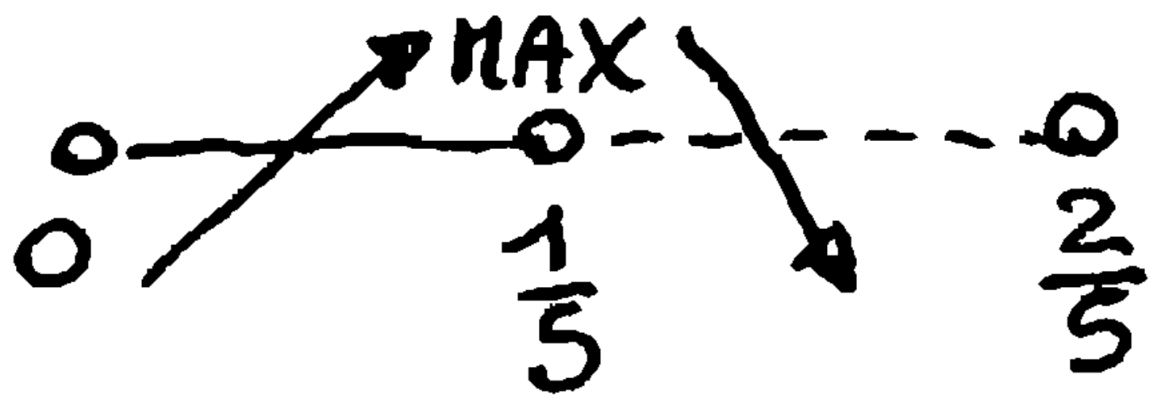


C.E. $= ]0;\frac{2}{5}[$.

$\lim\limits_{x\to 0^+} f(x)=-\infty$; $\lim\limits_{x\to \frac{2}{5}^+} f(x)=-\infty$.

$f(x)\geqslant 0 \Rightarrow 2x-5x^2>1 \Rightarrow 5x^2-2x+1<0$ : $\Delta=1-5<0 \Rightarrow f(x)<0\ \forall x\in$ C.E..

$f'(x)=\dfrac{2-10x}{2x-5x^2}\geqslant 0 \Rightarrow 2-10x\geqslant 0 \Rightarrow x\leqslant \frac{1}{5}$



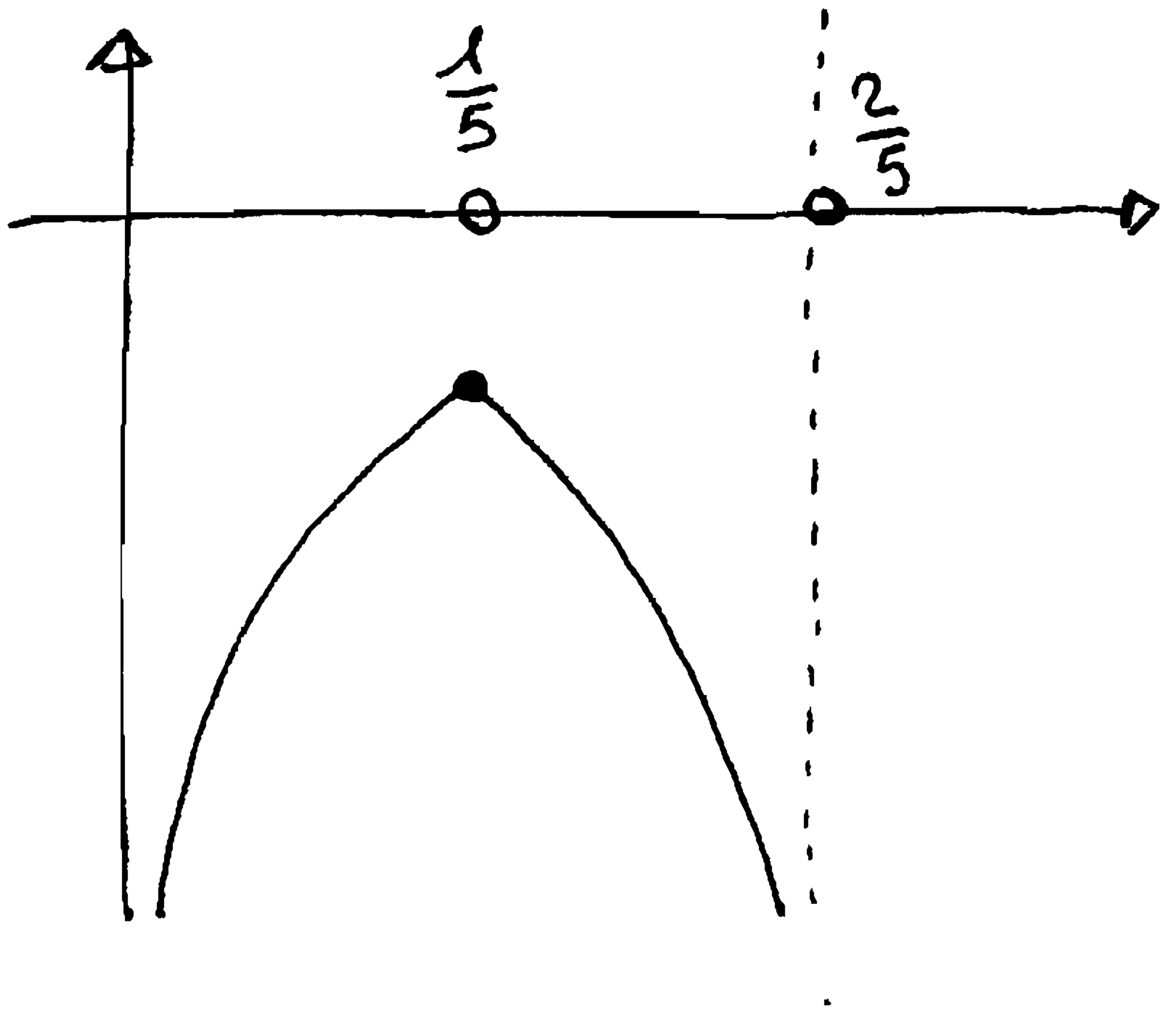
$$f''(x)=\frac{-10(2x-5x^2)-(2-10x)(2-10x)}{(2x-5x^2)^2}=$$

$$=\frac{-20x+50x^2-4-100x^2+40x}{(2x-5x^2)^2}=$$

$$=-\frac{50x^2-20x+4}{(2x-5x^2)^2}\geqslant 0 \Rightarrow 50x^2-20x+4\leqslant 0$$

mai: $\Delta<0$

Grafico:



Funzione sempre concava: $f''(x)<0\ \forall x$.

2) $\lim\limits_{x\to 0}\dfrac{4^{x^2}-\cos x}{2x^2}=\lim\limits_{x\to 0}\dfrac{4^{x^2}-1}{2\cdot x^2}+\dfrac{1-\cos x}{2\cdot x^2}=\dfrac{1}{2}\cdot\lim\limits_{t\to 0}\dfrac{4^t-1}{t}+\dfrac{1}{2}\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}=\dfrac{1}{2}\cdot\log 4+\dfrac{1}{2}\cdot\dfrac{1}{2}=\dfrac{1}{2}\left(\log 4+\dfrac{1}{2}\right)$.

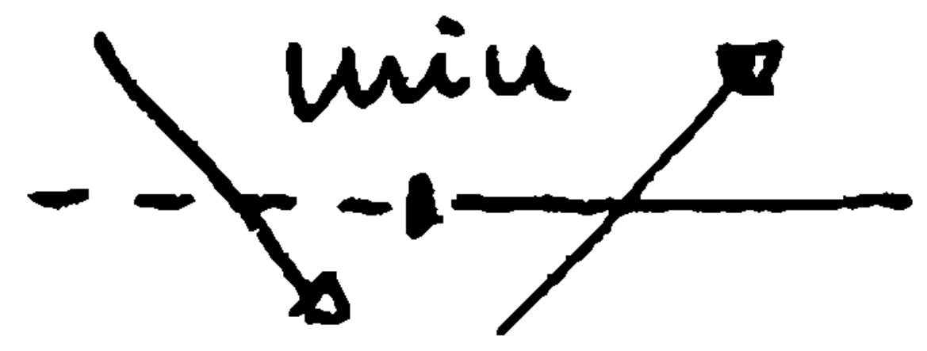
$\lim\limits_{x\to -\infty}\dfrac{x-3^x}{1+\log(2-x)}=\lim\limits_{x\to -\infty}\dfrac{x}{\log(2-x)}=-\infty$. $\left(3^x\to 0;\ \log(2-x)=o(x);\ 1=o\left(\log(2-x)\right)\right)$.

3) $f(x)=2^x-x$; $g(x)=3^x-x$.

$f(x)\sim g(x)$ se $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to x_0}\dfrac{2^x-x}{3^x-x}=1$. Vera se $x\to-\infty$ e se $x\to 0$.

4) $f(x)=me^{1+3x}-2x$. C.E.: $\mathbb{R}$. $f'(x)=3me^{1+3x}-2\geqslant 0 \Rightarrow e^{1+3x}\geqslant\dfrac{2}{3m} \Rightarrow$

$\Rightarrow x\geqslant \dfrac{1}{3}\left(\log\dfrac{2}{3m}-1\right)$



. $f'(1)=3me^4-2=0 \Rightarrow m=\dfrac{2}{3}\cdot e^{-4}$.

5) $f(x)=e^{2x-2}$; $g(x)=e^{3x+1}$. Per avere tangenti parallele: $f'(x_0)=g'(x_0) \Rightarrow$

$\Rightarrow 2e^{2x-2}=3e^{3x+1} \Rightarrow \dfrac{e^{3x+1}}{e^{2x-2}}=\dfrac{2}{3} \Rightarrow e^{x+3}=\dfrac{2}{3} \Rightarrow x+3=\log\dfrac{2}{3} \Rightarrow x_0=\log\dfrac{2}{3}-3$.

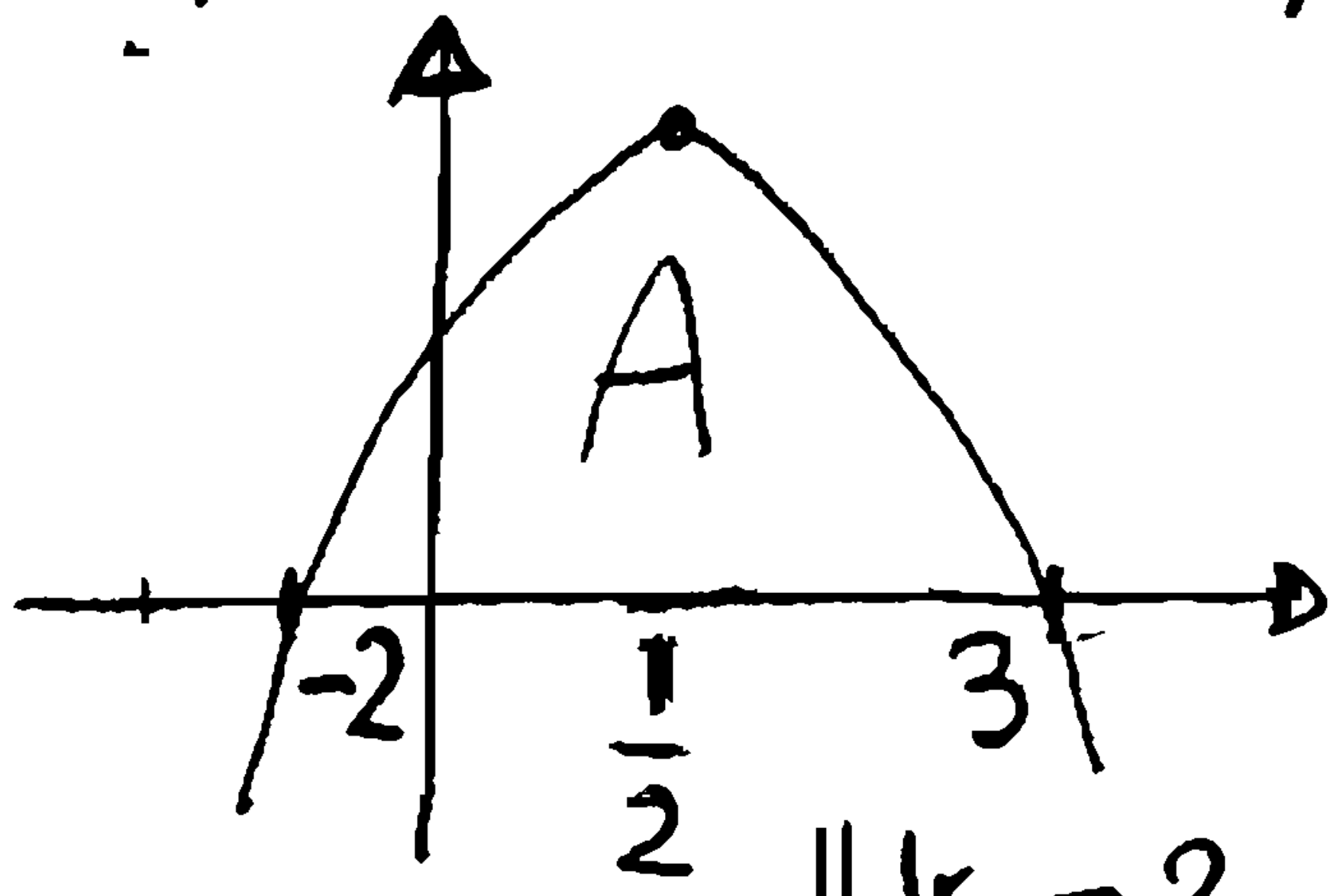
MGD2

6) $f(x)=\sqrt[3]{2x-3}$ ; $f(g(x))=\cos x$ . $f(g(x))=\sqrt[3]{2g(x)-3}=\cos x \Rightarrow$

$\Rightarrow 2g(x)-3=\cos^3 x \Rightarrow 2g(x)=\cos^3 x+3 \Rightarrow g(x)=\frac{1}{2}(\cos^3 x+3)$.

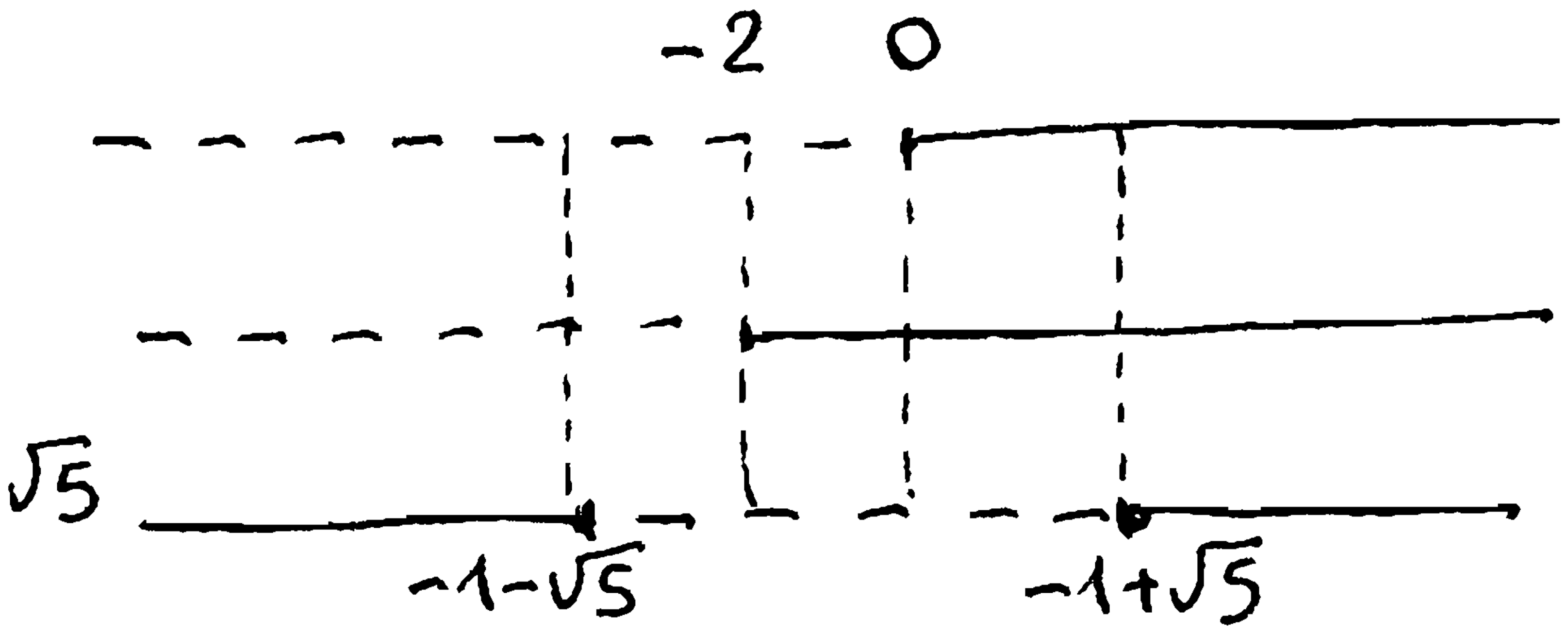
7) $f(x)= x-x^2+6$ (Parabola). $f(x)=0$ per $x^2-x-6=(x+2)(x-3)=0$ per $x=-2$ e $x=3$.

$f'(x)=1-2x=0$ per $x=\frac{1}{2}$ (Vertice). Area$=\int_0^3 f(x)dx=\int_0^3 x-x^2+6\,dx=$



$=\left(\frac{1}{2}x^2-\frac{1}{3}x^3+6x\right|_0^3=\left(\frac{9}{2}-9+18\right)-\left(0-0+0\right)=9+\frac{9}{2}=\frac{27}{2}$.

8) $H=\left\|\begin{matrix} k & -2 \\ -2 & k+2 \end{matrix}\right\|$ : $f''_{xx}=k>0$ per $k>0$
$f''_{yy}=k+2>0$ per $k>-2$



$|H_2|=k\cdot(k+2)-4=k^2+2k-4>0$ : $k<-1-\sqrt{5} \cup k>-1+\sqrt{5}$

Se $-\infty<k<-1-\sqrt{5}$ : $f''_{xx}<0$; $f''_{yy}<0$; $|H_2|>0$ : Punto di Massimo;

Se $-1+\sqrt{5}<k<+\infty$ : $f''_{xx}>0$; $f''_{yy}>0$; $|H_2|>0$ : Punto di minimo;

Se $-1-\sqrt{5}<k<-1+\sqrt{5}$ : $|H_2|<0$ : Punto di Sella.

9) $A\cdot B\cdot V=\left\|\begin{matrix} 1 & 2 & k \\ k-1 & 1 \end{matrix}\right\|\cdot\left\|\begin{matrix} 2 & 1 \\ k & -1 \\ 0 & k \end{matrix}\right\|\cdot\left\|\begin{matrix} -1 \\ 1 \end{matrix}\right\|=\left\|\begin{matrix} 2+2k & k^2-1 \\ k & 2k+1 \end{matrix}\right\|\cdot\left\|\begin{matrix} -1 \\ 1 \end{matrix}\right\|=\left\|\begin{matrix} k^2-2k-3 \\ k+1 \end{matrix}\right\|=\left\|\begin{matrix} -4 \\ 2 \end{matrix}\right\|\Rightarrow$

$\Rightarrow k+1=2 \Rightarrow k=+1$ ; se $k=+1$ : $1^2-2-3=-4 \Rightarrow k=1$.

10) $f(x;y)=x^2 2^y-xy+x$.

$\frac{\partial f}{\partial x}=2x\cdot 2^y-y+1$ : quindi la proposizione A è FALSA.

$\frac{\partial f}{\partial y}=x^2\cdot 2^y\cdot\log 2-x+0$ : quindi la proposizione B è FALSA.

| A | B | non A | non A ⇒ B |
|---|---|---|---|
| 0 | 0 | 1 | 0 |

: la proposizione non A ⇒ B è FALSA.