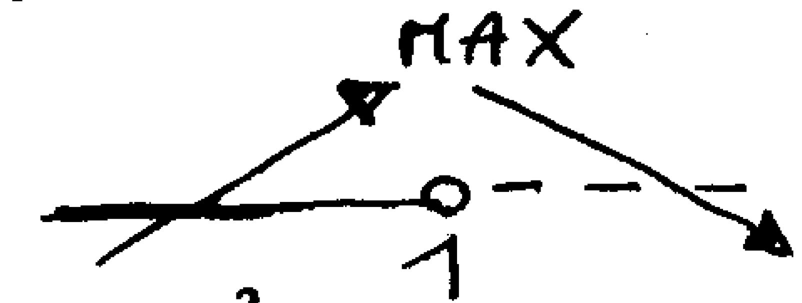
Compito di Matematica Generale del 28/3/2015 $\boxed{MG1}$

1) $f(x)=e^{2x-x^2}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=0^+$; $\lim_{x\to+\infty} f(x)=0^+$. $f(x)>0\ \forall x\in\mathbb{R}$.

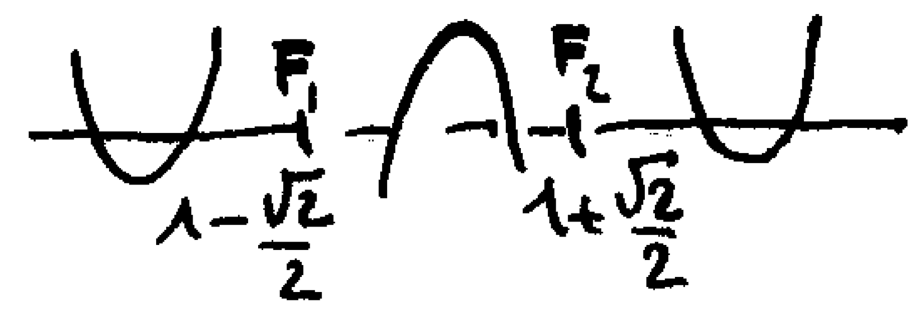
$f'(x)=(2-2x)\,e^{2x-x^2}\geq 0 \Rightarrow 2(1-x)\geq 0 \Rightarrow x\leq 1$



$f(1)=e$

$f''(x)=(-2)\,e^{2x-x^2}+(2-2x)^2\cdot e^{2x-x^2}=(4x^2-8x+2)\,e^{2x-x^2}\geq 0 \Rightarrow 2(2x^2-4x+1)\geq 0\Rightarrow$

$\Rightarrow x=\dfrac{2\pm\sqrt{4-2}}{2}=1\pm\dfrac{\sqrt{2}}{2}\Rightarrow f''(x)\geq 0$ per $x\leq 1-\dfrac{\sqrt{2}}{2}$ oppure $x\geq 1+\dfrac{\sqrt{2}}{2}$



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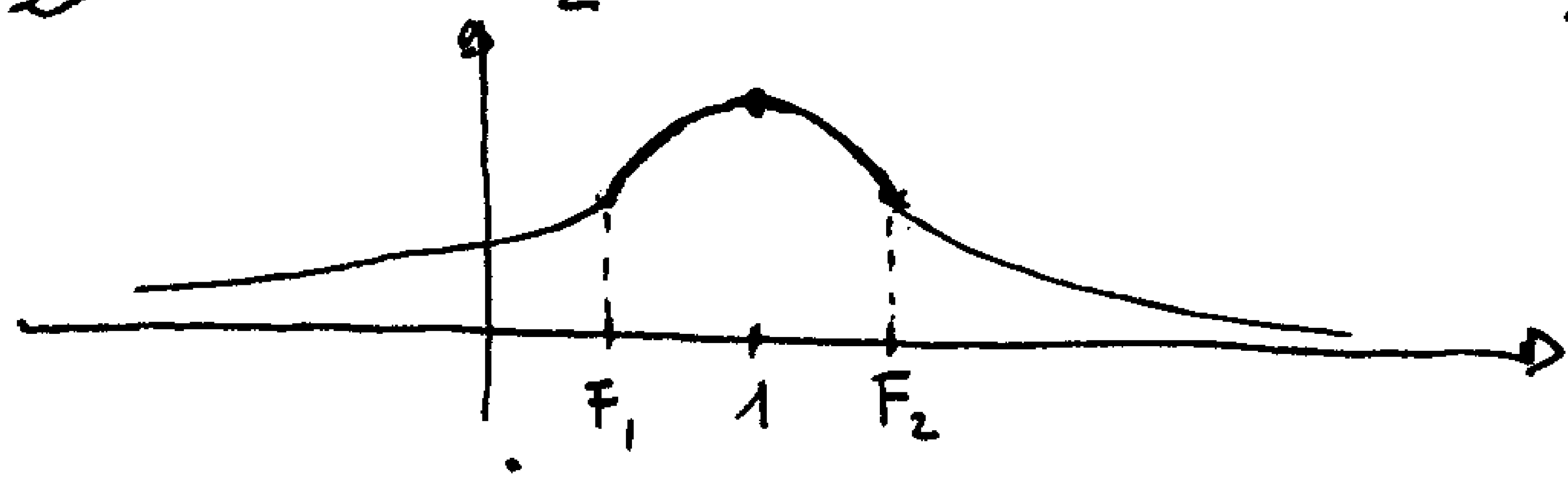


2) $\lim_{x\to 0}\dfrac{\operatorname{sen}x-\operatorname{sen}x^2+\operatorname{sen}3x}{x}=\lim_{x\to 0}\dfrac{\operatorname{sen}x}{x}-\dfrac{\operatorname{sen}x^2}{x^2}\cdot x+\dfrac{\operatorname{sen}3x}{3x}\cdot 3=1-1\cdot 0+1\cdot 3=4.$

$\lim_{x\to+\infty}\dfrac{e^{x^2}-e^x}{e^{2x}}=\lim_{x\to+\infty} e^{x^2-2x}-\dfrac{1}{e^x}=+\infty-0=+\infty.$

3) $f(x)=3x-5$; $g(x)=e^x$. $F(x)=f(g(f(x)))=f(g(3x-5))=f(e^{3x-5})=3\cdot e^{3x-5}-5.$

$F'(x)=3\cdot 3e^{3x-5}-0>0\ \forall x\in\mathbb{R}$: funzione strettamente crescente.

$\lim_{x\to-\infty}F(x)=-5$; $\lim_{x\to+\infty}F(x)=+\infty$: $f:\mathbb{R}\to\,]-5;+\infty[$; $f^{-1}:\,]-5;+\infty[\,\to\mathbb{R}$.

$y=3e^{3x-5}-5\Rightarrow e^{3x-5}=\frac{1}{3}(y+5)\Rightarrow 3x-5=\log\left(\frac{1}{3}(y+5)\right)\Rightarrow x=\frac{1}{3}\left(\log\left(\frac{1}{3}(y+5)\right)+5\right).$

Funzione inversa: $F^{-1}(x)=y=\frac{1}{3}\left(\log\left(\frac{1}{3}(x+5)\right)+5\right).$

4) $f(x)=e^x=f'(x)$. Equazione tangente in x_0: $y-e^{x_0}=e^{x_0}(x-x_0)\Rightarrow y=e^{x_0}(x-x_0)+e^{x_0}\Rightarrow$

$\Rightarrow y=e^{x_0}\cdot x-e^{x_0}(x_0-1)$. Passa per $(0;0)$ se: $e^{x_0}(x_0-1)=0\Rightarrow x_0-1=0\Rightarrow x_0=1.$

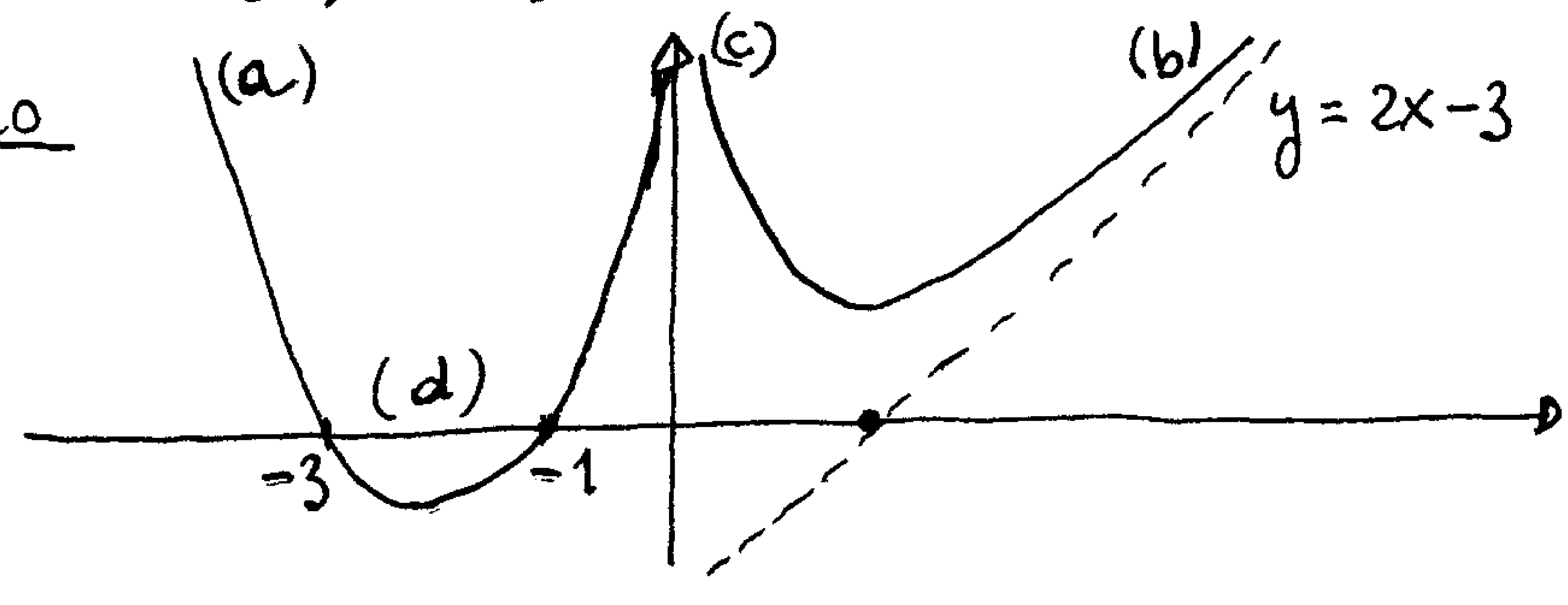
5) a) $\lim_{x\to-\infty}f(x)=+\infty$

b) As. obliquo a destra $y=2x-3$.

c) Disc. II Sp. in $x=0$

d) $f(x)<0$ per $x\in[-3;-1]$

Grafico

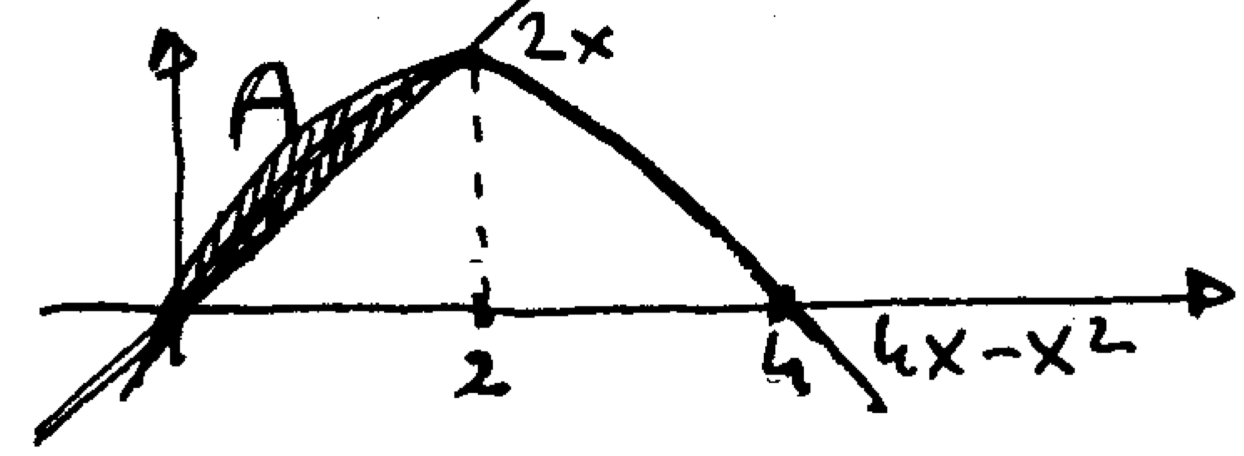


6) $f(x;y;z) = x^2y - y\cdot \operatorname{sen} xz + \log\frac{z}{y} = x^2y - y\cdot\operatorname{sen} xz + \log z - \log y.$

$\nabla f(x;y;z) = \left(2xy - yz\cos xz\,;\ x^2 - \operatorname{sen} xz - \frac{1}{y}\,;\ -xy\cos xz + \frac{1}{z}\right)$. $\nabla f(0;1;1) = (-1;-1;1)$.

$\nabla f(0;1;1) \perp (2;3;k) \Rightarrow (-1;-1;1)\cdot(2;3;k) = -2-3+k = 0 \Rightarrow k = 5.$

7) $4x - x^2 = 2x \Rightarrow x^2 - 2x = x(x-2) = 0 \Rightarrow x = 0$ e $x = 2$.



$\text{Area} = \int_0^2 \left[(4x - x^2) - 2x\right] dx = \int_0^2 2x - x^2\, dx = \left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}.$

8) $f(x) = m\cdot(x-k)\cdot e^{1-x}$. $f'(x) = m\left[1\cdot e^{1-x} + (x-k)e^{1-x}\cdot(-1)\right] = m\cdot e^{1-x}\cdot(1+k-x) \geq 0$

per $x \leq 1+k$:  $\Rightarrow 1+k = -3 \Rightarrow k = -4 \Rightarrow f(x) = m\cdot(x+4)e^{1-x}.$

Se $f(-3) = 5 \Rightarrow m(-3+4)e^{1+3} = 5 \Rightarrow m = 5\cdot e^{-4}.$

9) $A\cdot V = \begin{Vmatrix} 1 & k & -1 \\ 0 & 1 & k \\ k & -2 & m \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1-k-1 \\ 0-1+k \\ k+2+m \end{Vmatrix} = \begin{Vmatrix} -k \\ k-1 \\ k+m+2 \end{Vmatrix} \Rightarrow A\cdot V = (-k;\, k-1;\, k+m+2).$

$A\cdot V \perp (1;1;1) \Rightarrow (-k;\,k-1;\,k+m+2)\cdot(1;1;1) = -k+k-1+k+m+2 = 0 \Rightarrow m = -1-k.$

$A\cdot V = (-k;\,k-1;\,1) \Rightarrow \|A\cdot V\| = \sqrt{2} \Rightarrow k^2 + (k-1)^2 + 1 = 2 \Rightarrow 2k^2 - 2k = 0 \Rightarrow 2k(k-1) = 0.$

Se $k = 0 \Rightarrow m = -1$; Se $k = 1 \Rightarrow m = -2$.

10)

A	B	C	$(A\setminus B)$	$(A\setminus C)$	$[(A\setminus B)\cup(A\setminus C)]$	$(A\cup B)$	$(A\cup C)$	$[(A\cup B)\setminus(A\cup C)]$
1	1	1	0	0	0	1	1	0
1	1	0	0	1	1	1	1	0
1	0	1	1	0	1	1	1	0
1	0	0	1	1	1	1	1	0
0	1	1	0	0	0	1	1	0
0	1	0	0	0	0	1	0	1 → *
0	0	1	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0

Dalla $\overline{\text{VI}}$ Riga vediamo che $(A\cup B)\setminus(A\cup C)$ non è sottoinsieme di $(A\setminus B)\cup(A\setminus C)$. Dalle altre righe ($\overline{\text{II}}$; $\overline{\text{III}}$; $\overline{\text{IV}}$) si vede che non vale neppure il viceversa. Nessuno è sottoinsieme dell'altro.