Compito di Matematica Generale del 3/6/2015 $\boxed{MG1}$

1) $f(x)=\dfrac{\log 2x}{x}$. C.E.: $x>0$. $\lim\limits_{x\to 0^+} f(x) = -\infty$; $\lim\limits_{x\to +\infty} f(x) = 0^+$.

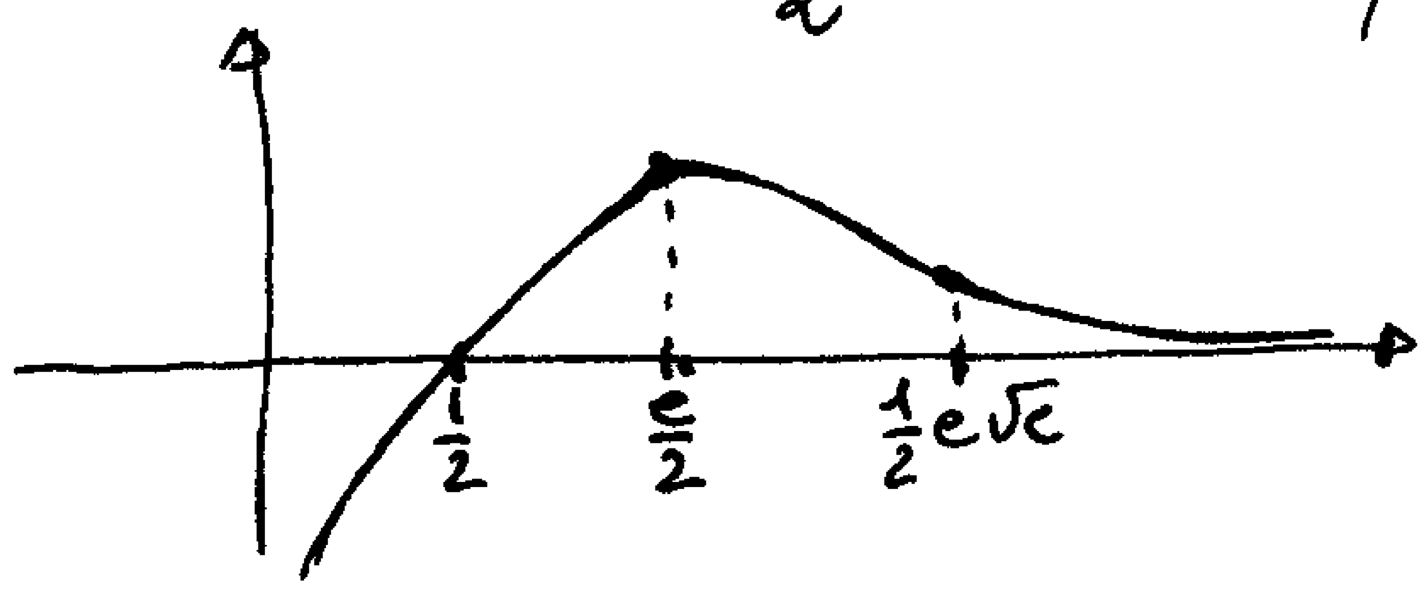
$f(x)\geq 0 \Rightarrow \log 2x \geq 0 \Rightarrow 2x\geq 1 \Rightarrow x\geq \frac{1}{2}$. $\quad 0 \; (-) \; \frac{1}{2} \; (+)$

$f'(x)=\dfrac{\frac{1}{2x}\cdot 2\cdot x - 1\cdot \log 2x}{x^2} = \dfrac{1-\log 2x}{x^2}\geq 0 \Rightarrow \log 2x\leq 1 \Rightarrow 2x\leq e \Rightarrow x\leq \dfrac{e}{2}$ $\quad 0 \nearrow \frac{e}{2} \searrow$ MAX

$f''(x)=\dfrac{-\frac{1}{2x}\cdot 2\cdot x^2 - 2x(1-\log 2x)}{x^4} = -\dfrac{1}{x^3}\cdot(3-2\log 2x)\geq 0 \Rightarrow 3-2\log 2x\leq 0 \Rightarrow$

$\Rightarrow \log 2x \geq \frac{3}{2} \Rightarrow 2x\geq e^{\frac{3}{2}} \Rightarrow x\geq \frac{1}{2}\cdot e\sqrt{e}$ $\quad 0 \; \frac{1}{2}e\sqrt{e}$ F $\qquad f\left(\frac{e}{2}\right)=\frac{2}{e}$

Grafico:



2) $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 2x}{x^2} = \lim\limits_{x\to 0}\dfrac{\cos x - 1 + 1 - \cos 2x}{x^2} = \lim\limits_{x\to 0}\dfrac{1-\cos 2x}{4x^2}\cdot 4 - \dfrac{1-\cos x}{x^2} = \dfrac{1}{2}\cdot 4 - \dfrac{1}{2} = \dfrac{3}{2}$.

$\lim\limits_{x\to 0^+}\dfrac{x^2-x+\log x}{1-e^x} = \lim\limits_{x\to 0^+}\dfrac{\log x}{1-e^x} = \left(\dfrac{\to -\infty}{\to 0^-}\right) = +\infty \quad \left(x^2\to 0^+; x\to 0^+; \log x\to -\infty;\ 1-e^x\to 0^-\right)$

3) $f(x)=e^{2x}-e^x \in \mathcal{C}(\mathbb{R})$. $f'(x)=2e^{2x}-e^x = e^x(2e^x-1)\geq 0 \Rightarrow 2e^x-1\geq 0 \Rightarrow e^x\geq \frac{1}{2} \Rightarrow$

$\Rightarrow x\geq \log\frac{1}{2} \Rightarrow x\geq -\log 2$ $\quad \searrow -\log 2 \nearrow$. La funzione può essere invertita in $]-\infty; -\log 2]$ oppure in $[-\log 2; +\infty[$.

$y=e^{2x}-e^x \Rightarrow e^{2x}-e^x-y=0 \Rightarrow t^2-t-y=0 \Rightarrow t=e^x=\dfrac{1\pm\sqrt{1+4y}}{2} \Rightarrow x=\log\left(\dfrac{1\pm\sqrt{1+4y}}{2}\right)$.

Inversa per $]-\infty; -\log 2]$: $y=\log\left(\dfrac{1-\sqrt{1+4x}}{2}\right)$; Inversa per $[-\log 2; +\infty[$: $y=\log\left(\dfrac{1+\sqrt{1+4x}}{2}\right)$.

4) $f(x)=e^x+x$; $g(x)=x$. $f(x)\sim g(x) \Leftrightarrow \lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}=1 \Rightarrow \lim\limits_{x\to x_0}\dfrac{e^x+x}{x}=1$.

La relazione è vera solo se $x_0=-\infty$: $\lim\limits_{x\to -\infty}\dfrac{e^x+x}{x} = \lim\limits_{x\to -\infty}\dfrac{e^x}{x}+1 = 0+1 = 1$.

5) $f(x)=e^{3x}-2x$; $x+2y=10 \Rightarrow y=-\frac{1}{2}x+5$. Per la perpendicolarità dovrà essere:

$f'(x_0) = -\dfrac{1}{-\frac{1}{2}} \Rightarrow f'(x_0)=2 \Rightarrow 3e^{3x_0}-2=2 \Rightarrow e^{3x_0}=\dfrac{4}{3} \Rightarrow x_0=\dfrac{1}{3}\log\dfrac{4}{3}$.

6) $f(x)=e^{x-1}-\log x$; $f(1)=e^0-\log 1 = 1-0 = 1$;

$f'(x)=e^{x-1}-\frac{1}{x}$; $f'(1)=e^0-1=1-1=0$;

MG2

$f''(x) = e^{x-1} + \frac{1}{x^2}$; $f''(1) = e^0 + 1 = 1+1 = 2$;

$f'''(x) = e^{x-1} + (-2)\cdot x^{-3} = e^{x-1} - \frac{2}{x^3}$; $f'''(1) = 1-2 = -1.$

$P_3(x;1) = 1 + 0\cdot(x-1) + \frac{1}{2}\cdot 2\cdot (x-1)^2 + \frac{1}{6}\cdot(-1)(x-1)^3 = 1 + (x-1)^2 - \frac{1}{6}(x-1)^3.$

7) $\int_1^2 \frac{x^2+x+1}{x}\,dx = \int_1^2 x+1+\frac{1}{x}\,dx = \left(\frac{x^2}{2} + x + \log x\right|_1^2 = \left(\frac{4}{2} + 2 + \log 2\right) - \left(\frac{1}{2} + 1 + \log 1\right) =$

$= 4 + \log 2 - \frac{1}{2} - 1 - 0 = \frac{5}{2} + \log 2.$

8) $f(x;y) = x^2 - xy^2 + 3y + y^4.$

$\begin{cases} f'_x = 2x - y^2 = 0 \\ f'_y = -2xy + 3 + 4y^3 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}y^2 \\ -2\left(\frac{1}{2}y^2\right)\cdot y + 4y^3 = -3 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}y^2 \\ 3y^3 = -3 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}y^2 \\ y^3 = -1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = -1 \end{cases}.$

$H(x;y) = \begin{Vmatrix} 2 & -2y \\ -2y & -2x+12y^2 \end{Vmatrix}$; $H\left(\frac{1}{2};-1\right) = \begin{Vmatrix} 2 & 2 \\ 2 & 11 \end{Vmatrix} \Rightarrow \begin{cases} 2>0;\ 11>0 \\ 2\cdot 11 - 4 > 0 \end{cases}$: Punto di minimo.

9) $V\cdot A\cdot V^T = \begin{Vmatrix} 1 & x \end{Vmatrix} \cdot \begin{Vmatrix} x & 1 \\ 1 & -x \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ x \end{Vmatrix} = \begin{Vmatrix} 1 & x \end{Vmatrix} \cdot \begin{Vmatrix} x+x \\ 1-x^2 \end{Vmatrix} = 2x + x(1-x^2) =$

$= 2x + x - x^3 = 3x - x^3 = x(3-x^2) = 0 \left\langle \begin{matrix} x=0 \\ 3-x^2=0 \end{matrix} \right. \Rightarrow \left\langle \begin{matrix} x=0 \\ x^2=3 \end{matrix} \right. \Rightarrow x=0;\ x=\sqrt{3};\ x=-\sqrt{3}.$

10)

A	B	non A	non A δ B	A $\Rightarrow$ (non A δ B)	[A $\Rightarrow$ (non A δ B)] δ B
1	1	0	1	1	1
1	0	0	0	0	0
0	1	1	1	1	1
0	0	1	1	1	1

Nel caso (Riga II) A vera e B falsa la proposizione risulta falsa, e quindi non è una tautologia.