Compito di Matematica Generale del 4/7/2015 $\boxed{MG1}$

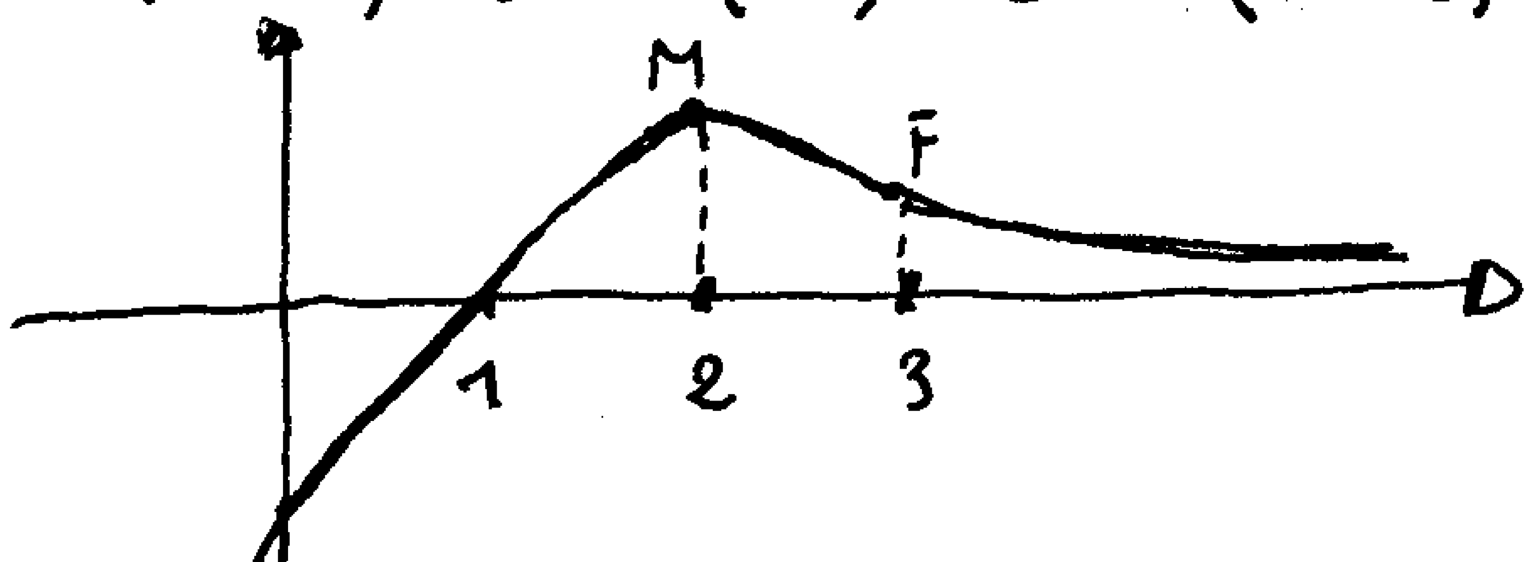
1) $f(x)=(x-1)\cdot e^{3-x}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=-\infty$; $\lim\limits_{x\to+\infty} f(x)=0^+$. $f(0)=-e^3$.

$f(x)\geq 0:\ x-1\geq 0 \Rightarrow x\geq 1$ (−) 1 (+)

$f'(x)=1\cdot e^{3-x}+(x-1)\,e^{3-x}\cdot(-1)=e^{3-x}\cdot(2-x)\geq 0 : 2-x\geq 0 \Rightarrow x\leq 2$ 2 MAX

$f''(x)=-e^{3-x}\cdot(2-x)+e^{3-x}\cdot(-1)=e^{3-x}\cdot(x-3)\geq 0 : x-3\geq 0 \Rightarrow x\geq 3$ 3

Grafico:



2) $\lim\limits_{x\to 0}\dfrac{e^{5x}-e^{3x}}{x}=\lim\limits_{x\to 0}\dfrac{e^{5x}-1}{5x}\cdot 5-\dfrac{e^{3x}-1}{3x}\cdot 3=1\cdot 5-1\cdot 3=2$;

$\lim\limits_{x\to-\infty}\dfrac{e^x-\log(1-x)}{2^{1-x}}=\lim\limits_{x\to-\infty}\dfrac{-\log(1-x)}{2^{1-x}}=0^-\ \left(e^x\to 0;\ \log(1-x)=o(2^{1-x})\right)$.

3) $A\cdot B=\begin{Vmatrix}1 & m & -2\\ k & 2 & -1\end{Vmatrix}\cdot\begin{Vmatrix}1 & -1\\ -1 & 1\\ 1 & -1\end{Vmatrix}=\begin{Vmatrix}1-m-2 & -1+m+2\\ k-2-1 & -k+2+1\end{Vmatrix}=\begin{Vmatrix}-m-1 & m+1\\ k-3 & 3-k\end{Vmatrix}=\begin{Vmatrix}1 & -1\\ 0 & 0\end{Vmatrix}\Rightarrow\begin{cases}m=-2\\ k=3\end{cases}$.

4) $f(x)=\dfrac{\log(1+kx)}{1+kx}$. Se $k>0$: C.E.: $1+kx>0\Rightarrow x>-\dfrac{1}{k}$.

$f'(x)=\dfrac{\frac{k}{1+kx}\cdot(1+kx)-k\cdot\log(1+kx)}{(1+kx)^2}=\dfrac{k(1-\log(1+kx))}{(1+kx)^2}\geq 0\Rightarrow \log(1+kx)\leq 1\Rightarrow$

$\Rightarrow 1+kx\leq e\Rightarrow kx\leq e-1\Rightarrow x\leq\dfrac{e-1}{k}=1$ se $k=e-1$: M 1

5) $\displaystyle\int_0^{\pi}\frac{x}{\pi}-\operatorname{sen}x\,dx=\left(\frac{1}{\pi}\cdot\frac{x^2}{2}+\cos x\right|_0^{\pi}=\left(\frac{1}{\pi}\cdot\frac{\pi^2}{2}+\cos\pi\right)-\left(\frac{1}{\pi}\cdot 0+\cos 0\right)=\frac{\pi}{2}-1-1=\frac{\pi}{2}-2$.

6) $f(x;y)=x^3-3x+y^2-2y\Rightarrow\begin{cases}f'_x=3x^2-3=0\\ f'_y=2y-2=0\end{cases}\Rightarrow\begin{cases}3(x^2-1)=0\\ 2(y-1)=0\end{cases}\Rightarrow\begin{cases}x=\pm 1\\ y=1\end{cases}$.

Punti Stazionari: $P_1=(1;1)$ e $P_2=(-1;1)$. $H(x;y)=\begin{Vmatrix}6x & 0\\ 0 & 2\end{Vmatrix}$.

$H(P_1)=\begin{Vmatrix}6 & 0\\ 0 & 2\end{Vmatrix}:\begin{cases}6>0,\ 2>0\\ 12>0\end{cases}$: Minimo; $H(P_2)=\begin{Vmatrix}-6 & 0\\ 0 & 2\end{Vmatrix}:\begin{cases}-6<0\\ 2>0\end{cases}$: Sella.

7) $f(x)=3x-1$; $g(x)=e^{x}$. $f(g(f(x)))=f(g(3x-1))=f(e^{3x-1})=$

$= 3\cdot e^{3x-1}-1=F(x)$. $F'(x)=3\cdot e^{3x-1}\cdot 3=9e^{3x-1}>0\ \forall x\in\mathbb{R}$.

La funzione $F(x)$ è strettamente crescente e quindi invertibile su tutto $\mathbb{R}$.

$3\cdot e^{3x-1}-1=y \Rightarrow e^{3x-1}=\frac{1}{3}(y+1) \Rightarrow 3x-1=\log\left(\frac{1}{3}(y+1)\right) \Rightarrow x=\frac{1}{3}\left[\log\left(\frac{1}{3}(y+1)\right)+1\right]$.

Funzione Inversa: $y=\frac{1}{3}\left[\log\left(\frac{1}{3}(x+1)\right)+1\right]$. $\lim\limits_{x\to-\infty}F(x)=-1$; $\lim\limits_{x\to+\infty}F(x)=+\infty$.

$F(x):\mathbb{R}\to\,]-1;+\infty[$; $F^{-1}(x):\,]-1;+\infty[\,\to\mathbb{R}$.

8) $f(x)=e^{2x+1}$; $g(x)=e^{3-x}$. Per avere tangenti perpendicolari: $f'(x_0)=-\dfrac{1}{g'(x_0)} \Rightarrow$

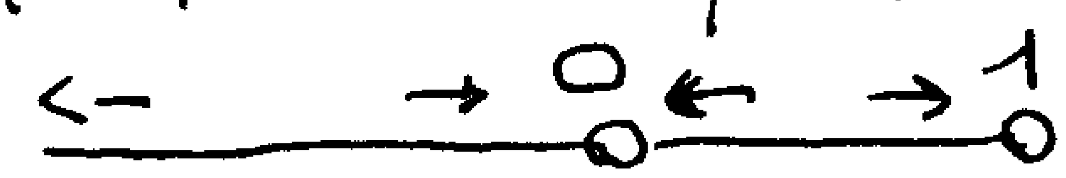
$\Rightarrow 2e^{2x+1}=-\dfrac{1}{-e^{3-x}} \Rightarrow e^{2x+1+3-x}=\frac{1}{2} \Rightarrow e^{x+4}=\frac{1}{2} \Rightarrow x+4=\log\frac{1}{2} \Rightarrow x_0=-\log 2-4$.

9)

A	B	A ⇒ B	A ⇔ B	non (A ⇔ B)	(A ⇒ B) e (non (A ⇔ B))
1	1	1	1	0	0
1	0	0	0	1	0
0	1	1	0	1	1
0	0	1	1	0	0

10) $f(x)=\dfrac{\log(3-3^{x})}{x}$. C.E.: $\begin{cases}3-3^{x}>0\\ x\neq 0\end{cases} \Rightarrow \begin{cases}3^{x}<3\\ x\neq 0\end{cases} \Rightarrow \begin{cases}x<1\\ x\neq 0\end{cases}$.

C.E. $=\,]-\infty;0[\,\cup\,]0;1[$:



Limiti nei punti di frontiera di C.E.: $x\to-\infty$; $x\to0^{-}$; $x\to0^{+}$; $x\to1^{-}$.

$\lim\limits_{x\to-\infty}f(x)=\left(\dfrac{\to\log(3-0)}{\to-\infty}\right)=0^{-}$; $\lim\limits_{x\to1^{-}}f(x)=\left(\dfrac{\to\log(\to0^{+})}{\to1}\right)=-\infty$;

$\lim\limits_{x\to0^{-}}f(x)=\left(\dfrac{\to\log(3-1)}{\to0^{-}}\right)=-\infty$; $\lim\limits_{x\to0^{+}}f(x)=\left(\dfrac{\to\log(3-1)}{\to0^{+}}\right)=+\infty$.