Compito di Matematica Generale del 4/9/2015 $\boxed{\text{MG 1}}$

1) $f(x) = \log(2x - x^2)$. C.E: $2x - x^2 = x(2-x) > 0 \Rightarrow 0 < x < 2$.

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 2^-} f(x) = -\infty$. $f(x) \geq 0 \Rightarrow 2x - x^2 \geq 1 \Rightarrow x^2 - 2x + 1 = (x-1)^2 \leq 0$

quindi $f(x) \leq 0 \ \forall x \in$ C.E. $f(1) = 0$.

$f'(x) = \dfrac{2-2x}{2x-x^2} \geq 0 \Rightarrow x \leq 1$ $x = 1$ punto di massimo.

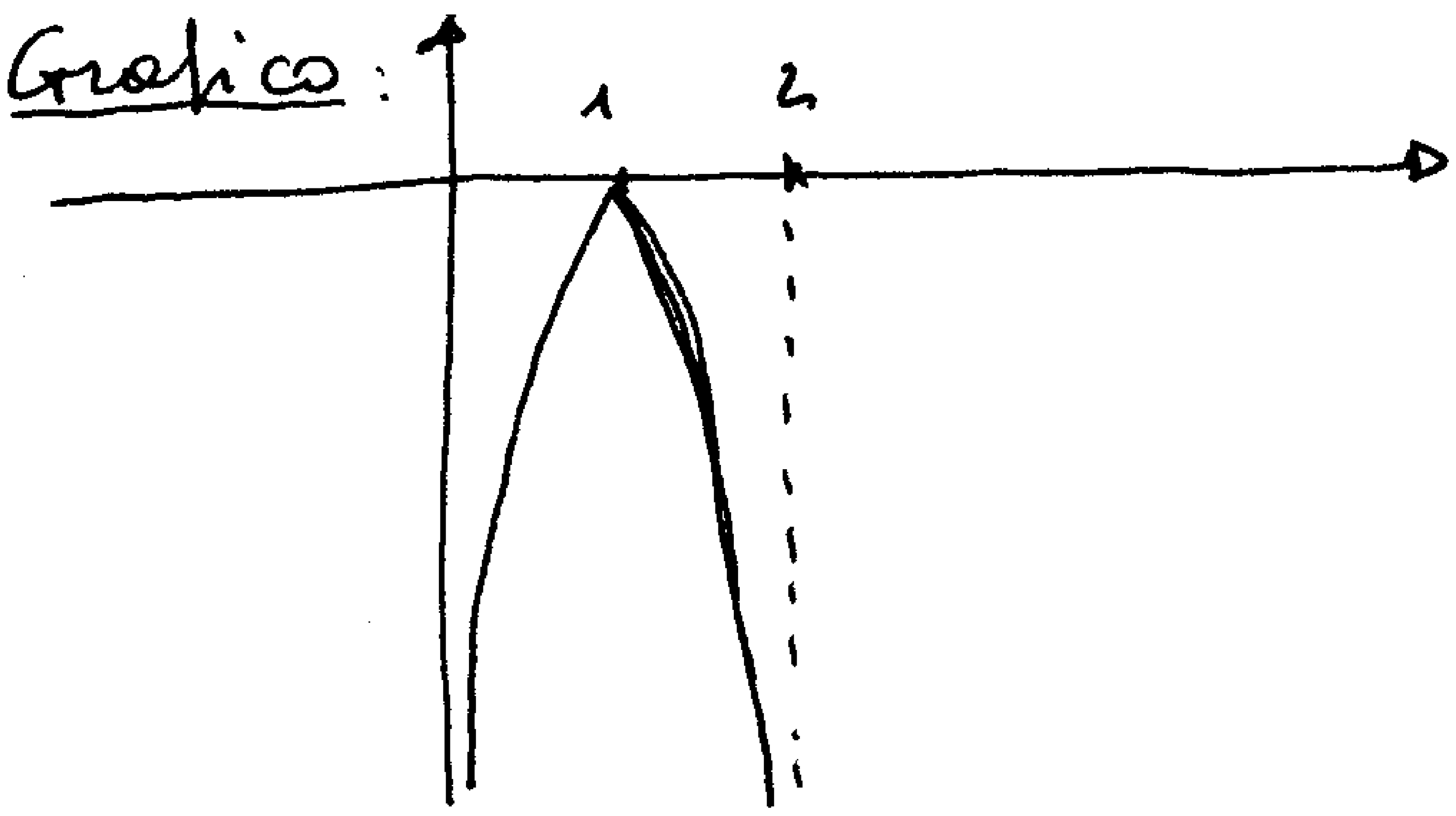
$f''(x) = \dfrac{-2(2x-x^2) - (2-2x)(2-2x)}{(2x-x^2)^2} =$

$= f''(x) = \dfrac{2}{(2x-x^2)^2} \cdot (x^2 - 2x - 2 + 2x + 2x - 2x^2) \geq 0 \Rightarrow$

$\Rightarrow x^2 - 2x + 2 \leq 0$ mai soddisfatta in

quanto $\Delta < 0$.

Grafico:



2) $\lim\limits_{x\to 0} \dfrac{1 - \cos x - \operatorname{sen}^2 x}{x^2} = \lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} - \dfrac{\operatorname{sen}^2 x}{x^2} = \dfrac{1}{2} - 1 = -\dfrac{1}{2}$.

$\lim\limits_{x\to +\infty} \dfrac{2^x - 3^x + \operatorname{sen} x}{2^x} = \lim\limits_{x\to+\infty} -\dfrac{3^x}{2^x} = \lim\limits_{x\to+\infty} -\left(\dfrac{3}{2}\right)^x = -\infty$. $\left(2^x = o(3^x);\ \operatorname{sen} x = o(3^x)\right)$.

3) $A \cdot X = \begin{Vmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \\ z \end{Vmatrix} = \begin{Vmatrix} x + 2y + z \\ x + z \end{Vmatrix} = \begin{Vmatrix} 2 \\ 0 \end{Vmatrix} \Rightarrow \begin{cases} x + 2y + z = 2 \\ x + z = 0 \end{cases} \Rightarrow \begin{cases} \cancel{x} + 2y - \cancel{x} = 2 \\ z = -x \end{cases} \Rightarrow \begin{cases} y = 1 \\ z = -x \end{cases}$.

$X = (x; 1; -x)$. $\|X\| = \sqrt{x^2 + 1 + x^2} = 3 \Rightarrow 2x^2 + 1 = 9 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2; y = 1; z = \mp 2$.

4)

	A	B	C	(BoC)	[M: A⇒(BoC)]	(AeC)	[N: B⇒(AeC)]	MoN	
—	1	1	1	1	1	1	1	1	— vera
	1	1	0	1	1	0	0	1	
	1	0	1	1	1	1	1	1	
—	1	0	0	0	0	0	1	1	— vera
—	0	1	1	1	1	0	0	1	— vera
	0	1	0	1	1	0	0	1	
	0	0	1	1	1	0	1	1	
—	0	0	0	0	1	0	1	1	— vera

Si tolgono le righe I; $\overline{\text{IV}}$; $\overline{\text{V}}$ e $\overline{\text{VIII}}$ dove B<=>C è vera.

5) $\displaystyle\int_{\pi}^{2\pi} \frac{\pi}{x} + \cos x\, dx = \Big(\pi \log x + \operatorname{sen} x\Big|_{\pi}^{2\pi} = \pi \log 2\pi + \operatorname{sen} 2\pi - \pi \log \pi - \operatorname{sen} \pi = \pi\left(\log 2 + \log \pi - \log \pi\right) = \pi \log 2$.

MG2

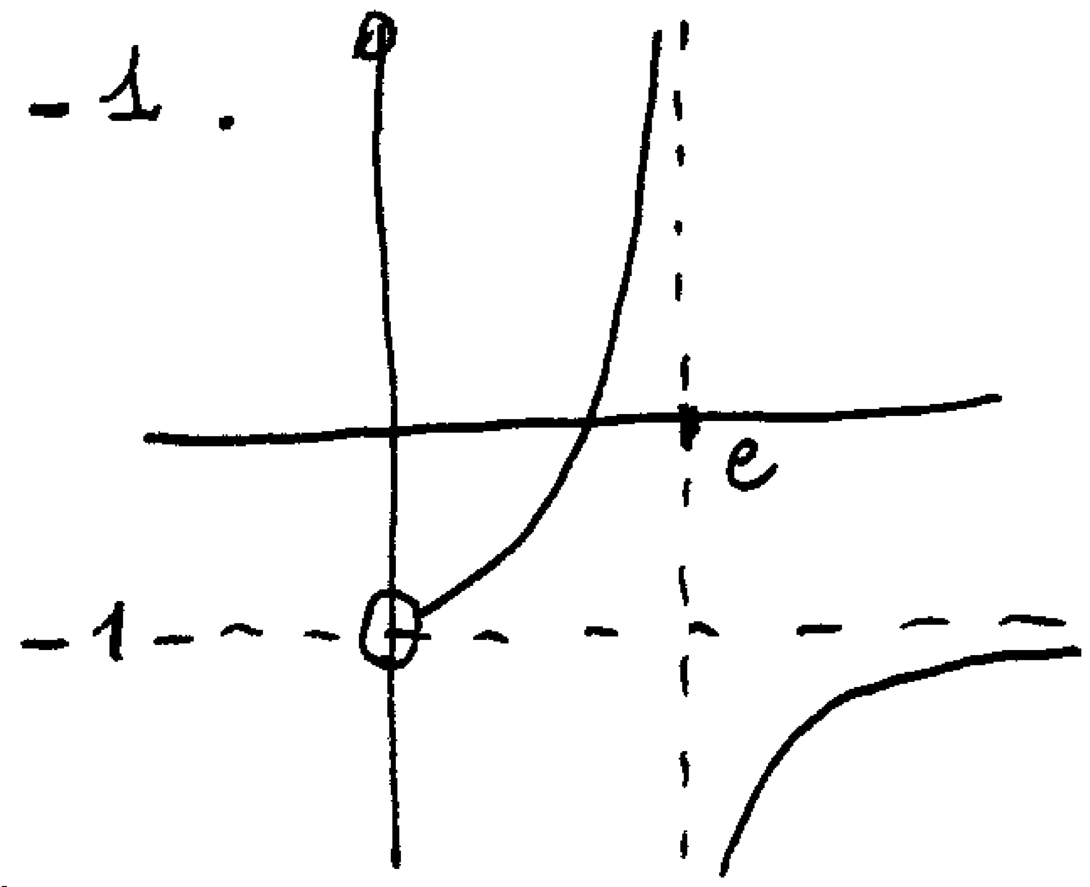
6) $x_1 \cdot x_2 = (x;y;y)\cdot(1-x;\, x-y;\, -1) = x - x^2 + xy - y^2 - y = f(x;y)$.

$\begin{cases} f'_x = 1-2x+y = 0 \\ f'_y = x-2y-1 = 0 \end{cases} \Rightarrow \begin{cases} y = 2x-1 \\ x-4x+2-1=0 \end{cases} \Rightarrow \begin{cases} 3x = 1 \\ y = 2x-1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{3} \\ y = -\frac{1}{3} \end{cases}$.

$H(x;y) = \left\|\begin{matrix} -2 & 1 \\ 1 & -2 \end{matrix}\right\| = H\left(\frac{1}{3};-\frac{1}{3}\right)$. $\begin{cases} -2<0;\, -2<0 \\ 4-1 = 3 > 0 \end{cases}$: Punto di Massimo.

7) $f(x) = \dfrac{1+\log x}{1-\log x}$. C.E.: $\begin{cases} x>0 \\ \log x \neq 1 \end{cases} \Rightarrow \begin{cases} x>0 \\ x \neq e \end{cases}$. $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to +\infty} f(x) = -1$.

$\lim\limits_{x\to e^-} f(x) = +\infty$; $\lim\limits_{x\to e^+} f(x) = -\infty$. $f'(x) = \dfrac{2}{x\cdot(1-\log x)^2} > 0\ \forall x \in$ C.E.



Discontinuità di III sp. in 0^+; di II sp. in $x = e$.

Funzione sempre invertibile. $y = \dfrac{1+\log x}{1-\log x} \Rightarrow y - y\log x = 1 + \log x \Rightarrow$

$\Rightarrow \log x = \dfrac{y-1}{1+y} \Rightarrow x = e^{\frac{y-1}{y+1}} \Rightarrow$ Inversa : $y = e^{\frac{x-1}{x+1}}$.

8) Rette perpendicolari: $m_1 = -\dfrac{1}{m_2} \Rightarrow f'(x_0) = -\dfrac{1}{g'(x_0)} \Rightarrow 2e^{2x+1} = -\dfrac{1}{-e^{3-x}} \Rightarrow$

$\Rightarrow e^{2x+1+3-x} = \frac{1}{2} \Rightarrow e^{x+4} = \frac{1}{2} \Rightarrow x+4 = \log\frac{1}{2} \Rightarrow x = -4 - \log 2$.

9) $f(x;y;z) = (y-1)\log z + x^{z-y}$. ; $P_0 = (1;1;2)$

$f'_x = 0 + (z-y)\cdot x^{z-y-1}\Big|_{P_0} = (2-1)\cdot 1^0 = 1$.

$f'_y = \log z - x^{z-y}\cdot \log x\Big|_{P_0} = \log 2 - 1\cdot 0 = \log 2$.

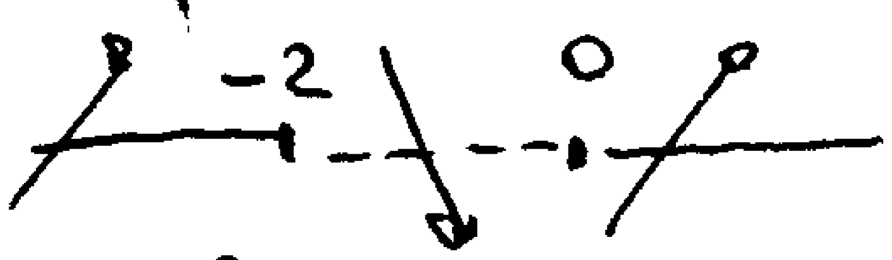
$f'_z = (y-1)\cdot\frac{1}{z} + x^{z-y}\cdot\log x\Big|_{P_0} = 0 + 1\cdot 0 = 0$.

$\nabla f(P_0)\circ V = 0$

in quanto i due vettori sono perpendicolari.

10) $f(x) = x^2\cdot e^x$. $f(x) \in C(\mathbb{R})$. $f'(x) = 2xe^x + x^2e^x = x\cdot e^x\cdot(2+x) \geq 0 \Rightarrow$

$f'(x) \geq 0$ per $x \leq -2$ oppure $x \geq 0$



$f(-2) = \dfrac{4}{e^2}$; $f(1) = e$. Dato che

$f(1) > f(-2)$ in $x = 1$ abbiamo il Massimo Assoluto. In $x = 0$ il minimo Assoluto.