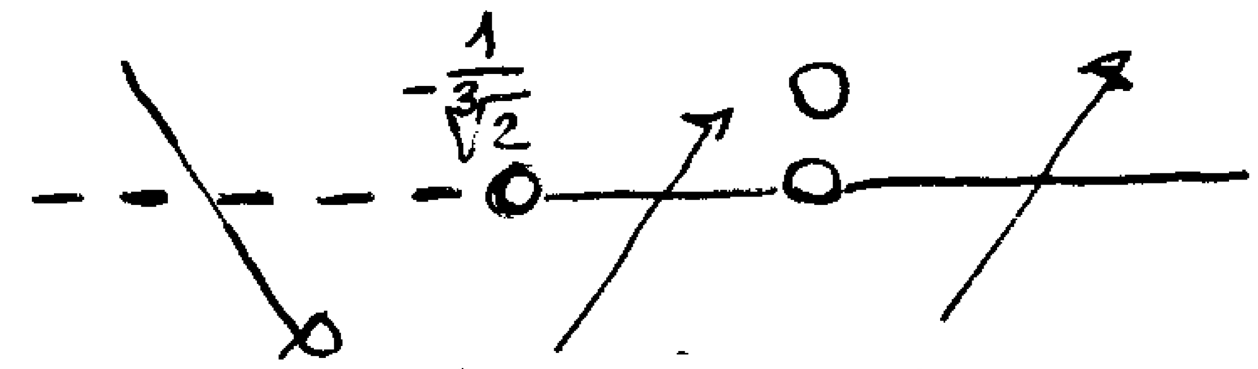
Compito di Matematica Generale del 22/9/2015 $\boxed{MG1}$

1) $f(x) = x^2 - \frac{1}{x}$. C.E.: $x \neq 0$. $\lim_{x\to\pm\infty} f(x) = +\infty$; $\lim_{x\to 0^-} f(x) = +\infty$; $\lim_{x\to 0^+} f(x) = -\infty$.

$f(x) \geq 0 : x^2 \geq \frac{1}{x}$ vera $\forall x<0$ e $\forall x>1$. $f(1)=0$.

$f'(x) = 2x + \frac{1}{x^2} = \frac{2x^3+1}{x^2} \geq 0 \Rightarrow 2x^3 \geq -1 \Rightarrow x \geq -\frac{1}{\sqrt[3]{2}}$



$f''(x) = 2 - 2\frac{1}{x^3} = 2\left(1-\frac{1}{x^3}\right) = 2\left(\frac{x^3-1}{x^3}\right) \geq 0$

$x^3-1 \geq 0 : x^3 \geq 1 \Rightarrow x \geq 1$

$x^3 > 0 \Rightarrow x > 0$

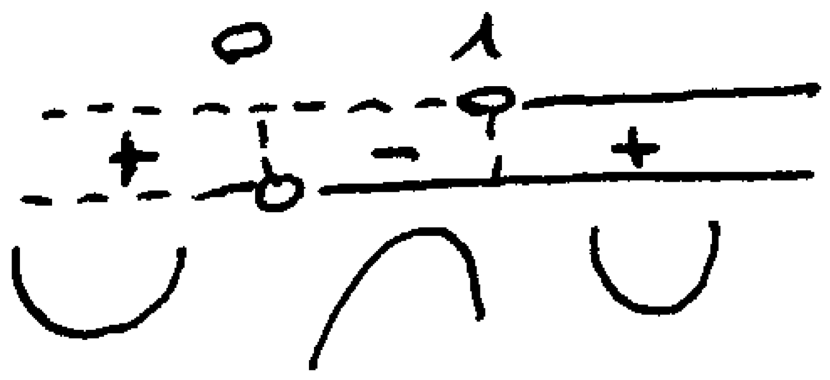
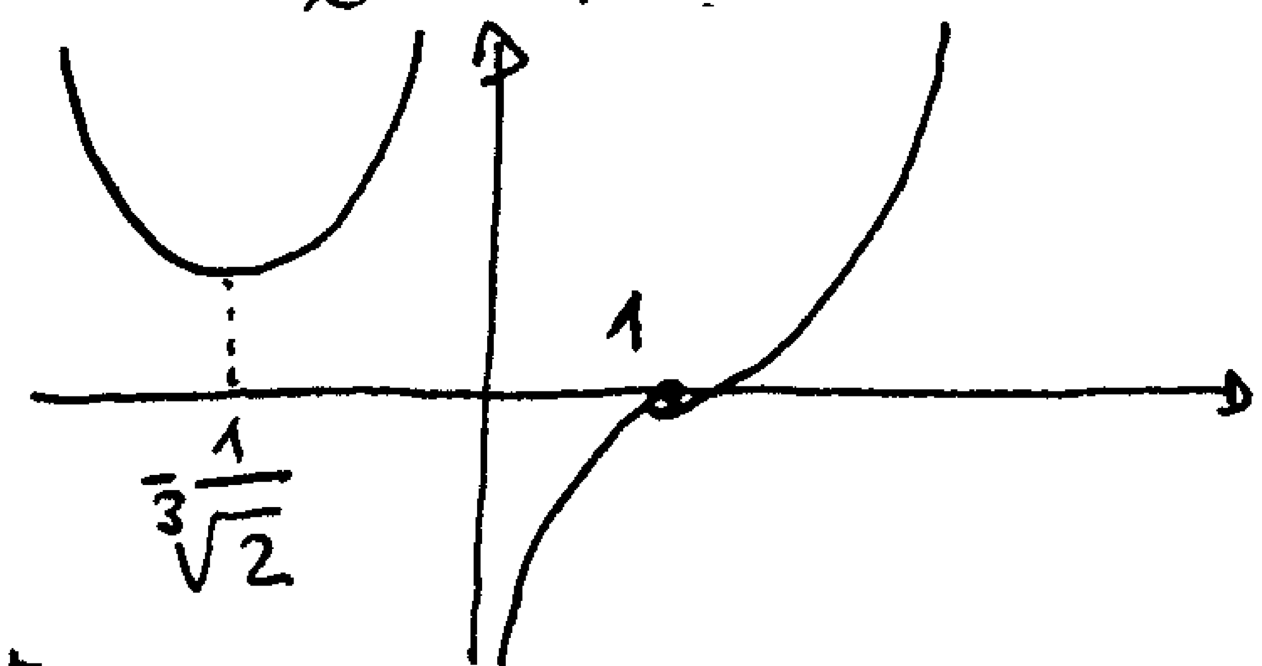


Grafico:

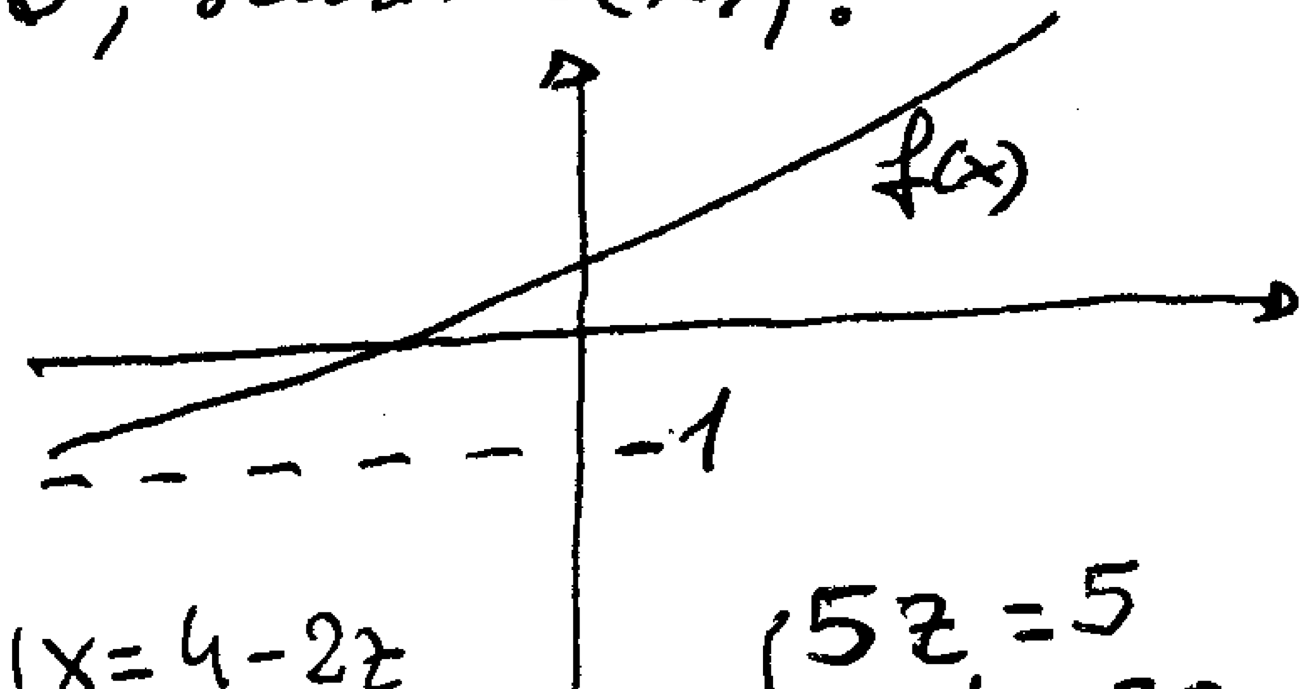


2) $\lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2} = \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \frac{1-\cos x}{x^2} = \lim_{t\to 0}\frac{e^t-1}{t} + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$.

$\lim_{x\to-\infty} \frac{2^x - x + \operatorname{sen} x}{3^x + x} = \lim_{x\to-\infty}\frac{-x}{x} = -1$ $\left(2^x \to 0; 3^x \to 0; \operatorname{sen} x = o(x)\right)$.

3) $\forall \varepsilon\ \exists \delta(\varepsilon) : x > \delta(\varepsilon) \Rightarrow f(x) > \varepsilon : \lim_{x\to+\infty} f(x) = +\infty$

$\forall \varepsilon > 0\ \exists \delta(\varepsilon) : x < \delta(\varepsilon) \Rightarrow |f(x)+1| < \varepsilon : \lim_{x\to-\infty} f(x) = -1$.



4) $A \cdot W = B \Rightarrow \left\|\begin{matrix}1 & 2\\ 2 & -1\end{matrix}\right\| \cdot \left\|\begin{matrix}x & y\\ z & w\end{matrix}\right\| = \left\|\begin{matrix}4 & 7\\ 3 & 4\end{matrix}\right\| \Rightarrow \begin{cases}x+2z=4\\ 2x-z=3\\ y+2w=7\\ 2y-w=4\end{cases} \Rightarrow \begin{cases}x=4-2z\\ 8-4z-z=3\\ y=7-2w\\ 14-4w-w=4\end{cases} \Rightarrow \begin{cases}5z=5\\ x=4-2z\\ 5w=10\\ y=7-2w\end{cases} \Rightarrow$

$\Rightarrow \begin{cases}z=1\\ x=2\\ w=2\\ y=3\end{cases} \Rightarrow W = \left\|\begin{matrix}2 & 3\\ 1 & 2\end{matrix}\right\|$.

5) $\int_{\pi}^{2\pi} \frac{k}{x} - \operatorname{sen} x\, dx = \left(k\log x + \cos x\right|_{\pi}^{2\pi} = k\left(\log 2\pi - \log\pi\right) + \left(\cos 2\pi - \cos\pi\right) =$

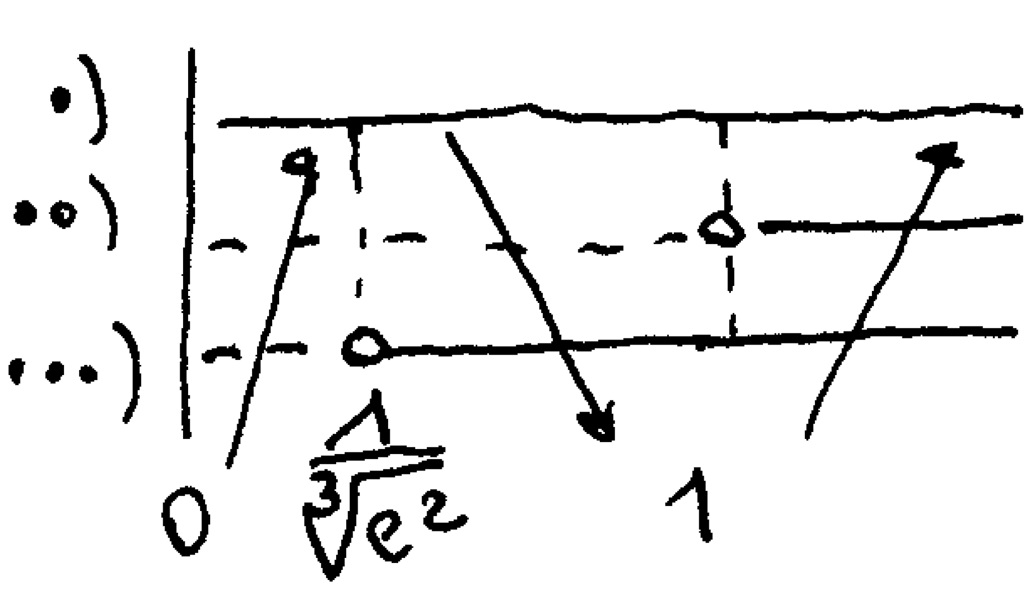
$= k\left(\log 2 + \log\pi - \log\pi\right) + \left(1-(-1)\right) = k\log 2 + 2 = 3 \Rightarrow k\log 2 = 1 \Rightarrow k = \frac{1}{\log 2} = \log_2 e$.

6) $f(x) = x^3 \cdot \log^2 x$. C.E.: $x>0$. $f'(x) = 3x^2\cdot\log^2 x + x^3\cdot 2\cdot\log x\cdot\frac{1}{x} = x^2\cdot\log x\cdot(3\log x + 2) \geq 0$

•) $x^2 > 0\ \forall x$

••) $\log x \geq 0 \Rightarrow x \geq 1$

•••) $3\log x + 2 \geq 0 : \log x \geq -\frac{2}{3} \Rightarrow x \geq e^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{e^2}}$



$\Rightarrow \begin{cases} x = \frac{1}{\sqrt[3]{e^2}} : \text{P. MAX.}\\ x = 1 : \text{P. MIN.}\end{cases}$

MG2

7) $f(x)=\log\left(\frac{1-x}{1+x}\right)$. C.E.: $\frac{1-x}{1+x}>0 \begin{cases} 1-x>0: x<1 \\ 1+x>0: x>-1 \end{cases}$ NO $\big]$ C.E. $\big|$ NO C.E.: $-1<x<1$.

$\lim_{x\to -1^+} f(x)=\log\left(\frac{\to 2}{\to 0^+}\right)=+\infty$; $\lim_{x\to 1^-} f(x)=\log\left(\frac{\to 0^+}{\to 2}\right)=-\infty$. $f:]-1;1[\to \mathbb{R}$.

$f(x)=\log(1-x)-\log(1+x)$; $f'(x)=\frac{-1}{1-x}-\frac{1}{1+x}=-\frac{2}{1-x^2}<0\ \forall x\in$ C.E.: f invertibile in C.E.

$\log\left(\frac{1-x}{1+x}\right)=y \Rightarrow \frac{1-x}{1+x}=e^y \Rightarrow 1-x=e^y+xe^y \Rightarrow x=\frac{1-e^y}{1+e^y} \Rightarrow$ inversa: $y=\frac{1-e^x}{1+e^x}$.

8) $f(x)=e^x$; $g(x)=e^{k-x}$. Per la perpendicolarità: $f'(x)=-\frac{1}{g'(x)} \Rightarrow e^x=+\frac{1}{+e^{k-x}} \Rightarrow$

$\Rightarrow e^{x+k-x}=1 \Rightarrow e^k=1 \Rightarrow k=0 \Rightarrow \mathcal{D}(e^x)=e^x$; $\mathcal{D}(e^{-x})=-e^{-x}=-\frac{1}{e^x}$ vera $\forall x$.

9) $f(x;y;z)=\frac{xy-1}{z}\cdot\log(x-z)$. C.E.: $\begin{cases} x-z>0 \\ z\neq 0 \end{cases} \Rightarrow \begin{cases} z<x \\ z\neq 0 \end{cases}$.

$f'_x=\frac{1}{z}\cdot y\cdot\log(x-z)+\frac{xy-1}{z}\cdot\frac{1}{x-z}\cdot 1$; $f'_x(2;1;1)=1\cdot 1\cdot\log 1+\frac{1}{1}\cdot\frac{1}{1}=0+1=1$;

$f'_y=\frac{1}{z}\cdot x\cdot\log(x-z)+0$; $f'_y(2;1;1)=\frac{2}{1}\cdot\log 1=0$;

$f'_z=-\frac{1}{z^2}\cdot(xy-1)\cdot\log(x-z)+\frac{xy-1}{z}\cdot\frac{-1}{x-z}$; $f'_z(2;1;1)=-1\cdot 1\cdot\log 1+\frac{1}{1}\cdot\frac{-1}{1}=-1$.

$\nabla f(2;1;1)=(1;0;-1)$. $\|\nabla f(P_0)\|=\sqrt{1+0+1}=\sqrt{2}$.

$\nabla f(P_0)\cdot V=\|\nabla f(P_0)\|\cdot\|V\|\cdot\cos 45^\circ=\sqrt{2}\cdot 2\cdot\frac{\sqrt{2}}{2}=2$.

10)

A	B	non B	S: A ⇒ non B	A ⇒ B	T: non (A ⇒ B)	T ⇒ S
1	1	0	0	1	0	1
1	0	1	1	0	1	1
0	1	0	1	1	0	1
0	0	1	1	1	0	1

Se □ : ⇒ la proposizione T □ S diviene una tautologia.

Invece S □ T non può divenire una tautologia.