Compito di Matematica Generale del 10/10/2015 $\boxed{MG1}$

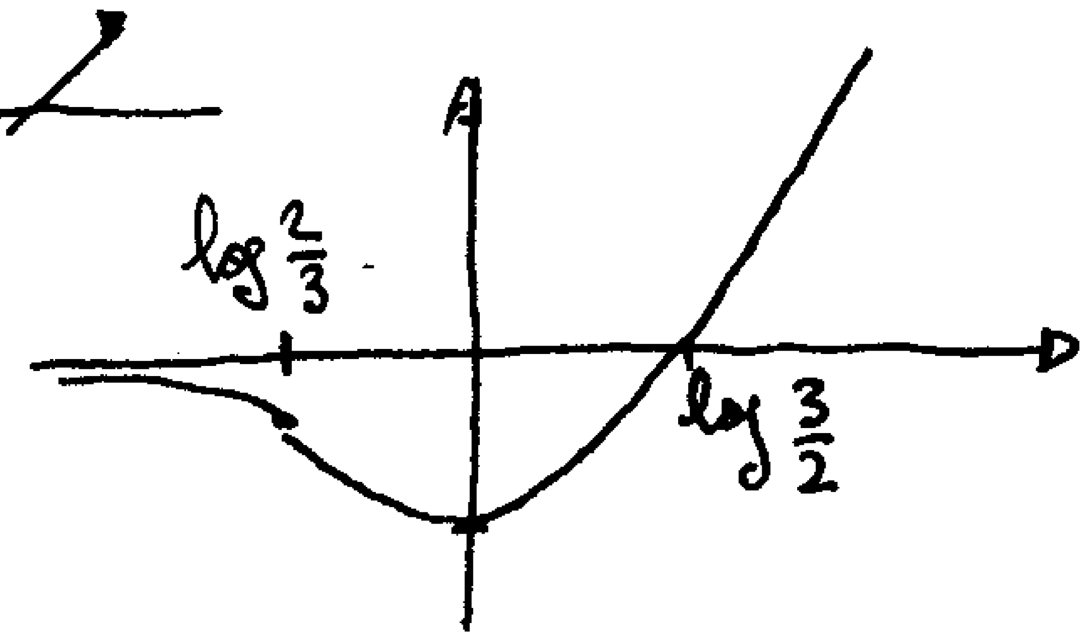
1) $f(x)=2e^{3x}-3e^{2x}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=0^-$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$: $2e^{3x}\geq 3e^{2x} \Rightarrow e^x\geq \frac{3}{2} \Rightarrow x\geq \log\frac{3}{2}>0$

$f'(x)=6e^{3x}-6e^{2x}\geq 0 \Rightarrow e^{3x}\geq e^{2x} \Rightarrow e^x\geq 1 \Rightarrow x\geq 0$

$f''(x)=18e^{3x}-12e^{2x}\geq 0 \Rightarrow e^x\geq \frac{2}{3} \Rightarrow x\geq \log\frac{2}{3}<0$

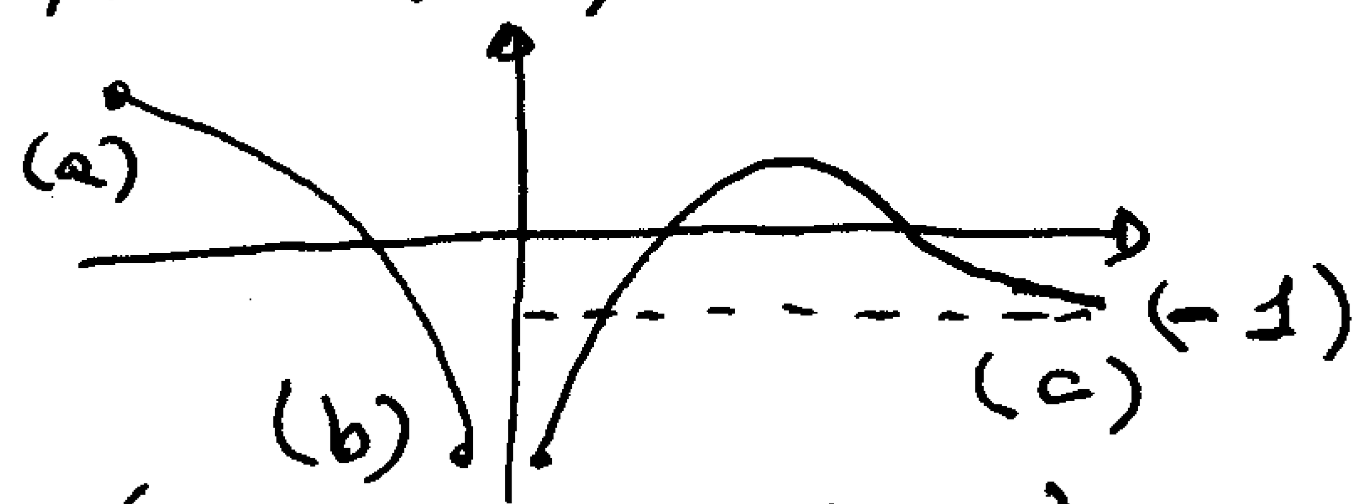
$f(0)=-1$; $f(\log\frac{2}{3})=-\frac{20}{27}$; $f(\log\frac{3}{2})=0$. Grafico:



2) $\lim\limits_{x\to 0}\frac{\operatorname{sen}3x-\operatorname{sen}2x+\operatorname{sen}x}{2x}=\lim\limits_{x\to 0}\frac{\operatorname{sen}3x}{3x}\cdot\frac{3}{2}-\frac{\operatorname{sen}2x}{2x}+\frac{\operatorname{sen}x}{x}\cdot\frac{1}{2}=\frac{3}{2}-1+\frac{1}{2}=2-1=1.$

$\lim\limits_{x\to+\infty}\frac{2^{-x}-x^2}{x-3^x}=\lim\limits_{x\to+\infty}\frac{-x^2}{-3^x}=0^+$ $\left(2^{-x}\to 0;\ x=o(3^x);\ x^2=o(3^x)\right)$.

3) $\lim\limits_{x\to-\infty} f(x)=+\infty$ (a) $\lim\limits_{x\to 0} f(x)=-\infty$ (b) $\lim\limits_{x\to+\infty} f(x)=-1^+$ (c)



4) $e^x=o(3+\operatorname{sen}x) \Rightarrow \lim\limits_{x\to x_0}\frac{e^x}{3+\operatorname{sen}x}=0$ Vera per $x\to-\infty$ ($3+\operatorname{sen}x$ è limitata).

5) $\int_1^K 1-2x\,dx=\left(x-x^2\right)\Big|_1^K=K-K^2-(1-1)=\frac{1}{4}\Rightarrow K^2-K+\frac{1}{4}=0\Rightarrow 4K^2-4x+1=0\Rightarrow$

$\Rightarrow (2K-1)^2=0 \Rightarrow K=\frac{1}{2}$.

6) $f(x;y)=e^{2x+y-x^2-y^2}$.

$$\begin{cases} f'_x=(2-2x)e^{2x+y-x^2-y^2}=0\Rightarrow 2-2x=0\Rightarrow x=1\\ f'_y=(+1-2y)e^{2x+y-x^2-y^2}=0\Rightarrow 1-2y=0\Rightarrow y=\frac{1}{2}\end{cases}$$ Punto Stazionario $P_0=(1;\frac{1}{2})$.

$$H(x;y)=\begin{Vmatrix}(-2+(2-2x)^2)\cdot e^{2x+y-x^2-y^2} & (2-2x)(1-2y)e^{2x+y-x^2-y^2}\\ (2-2x)(1-2y)e^{2x+y-x^2-y^2} & (-2+(1-2y)^2)e^{2x+y-x^2-y^2}\end{Vmatrix}.$$

$$H(1;\tfrac{1}{2})=\begin{Vmatrix}-2\cdot e^{\frac{5}{4}} & 0\\ 0 & -2\cdot e^{\frac{5}{4}}\end{Vmatrix}\Rightarrow\begin{cases}-2\cdot e^{\frac{5}{4}}<0\\ 4\cdot e^{\frac{5}{2}}>0\end{cases}$$: Punto di Massimo.

7) $f(x) = \begin{cases} 2^{x-3} & : x < K \\ 2^{3-2x} & : K \leq x \end{cases}$. Per la continuità deve essere:

$$\lim_{x \to K^-} f(x) = 2^{K-3} = \lim_{x \to K^+} f(x) = 2^{3-2K} \Rightarrow 2^{K-3} = 2^{3-2K} \Rightarrow 2^{K-3-3+2K} = 1 \Rightarrow$$

$$\Rightarrow 2^{3K-6} = 1 \Rightarrow 3K - 6 = 0 \Rightarrow K = 2.$$

8) $f(x) = e^{1-x}$; $g(x) = e^x$. $F(x) = f(g(x)) = e^{1-g(x)} = e^{1-e^x}$.

$F'(x) = e^{1-e^x} \cdot (-e^x) < 0 \ \forall x \in \mathbb{R}$. Funzione invertibile su tutto $\mathbb{R}$.

$\lim_{x \to -\infty} F(x) = e$; $\lim_{x \to +\infty} F(x) = 0^+$. $F(x) : \mathbb{R} \to]0; e[$; $F^{-1}(x) :]0; e[\to \mathbb{R}$.

$F(x) = e^{1-e^x} = y \Rightarrow 1 - e^x = \log y \Rightarrow e^x = 1 - \log y \Rightarrow x = \log(1 - \log y)$.

Inversa: $F^{-1}(x) = \log(1 - \log x)$.

9) $(A+B) \circ X = \left(\begin{Vmatrix} 1 & K \\ K & -1 \end{Vmatrix} + \begin{Vmatrix} 1 & 2 \\ -1 & 0 \end{Vmatrix} \right) \circ \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 2 & 2+K \\ K-1 & -1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 2+2+K \\ K-1-1 \end{Vmatrix} = \begin{Vmatrix} K+4 \\ K-2 \end{Vmatrix} = V.$

$$\|V\| = \sqrt{(K+4)^2 + (K-2)^2} = \sqrt{K^2 + 8K + 16 + K^2 - 4K + 4} = \sqrt{2K^2 + 4K + 20} = 3\sqrt{2} \Rightarrow$$

$$\Rightarrow 2K^2 + 4K + 20 = 18 \Rightarrow K^2 + 2K + 1 = (K+1)^2 = 0 \Rightarrow K = -1.$$

10) $f(x) = x^2 - 2x + 3$. Se $y = 2x - K$ è retta tangente, deve essere

$f'(x_0) = 2 \Rightarrow 2x_0 - 2 = 2 \Rightarrow 2x_0 = 4 \Rightarrow x_0 = 2$.

Dovrà poi essere $y(2) = f(2) \Rightarrow 2 \cdot 2 - K = 4 - 4 + 3 = 3$ e quindi la retta è tangente al grafico della funzione se $4 - K = 3 \Rightarrow K = 1$.