Prova Intermedia di Matematica Generale del 9/11/2015 Compito A1

1) $\lim\limits_{x\to 0}\dfrac{(1+x)^{\pi}-1}{\operatorname{sen} 2x} = \lim\limits_{x\to 0}\dfrac{(1+x)^{\pi}-1}{x}\cdot\dfrac{2x}{\operatorname{sen} 2x}\cdot\dfrac{1}{2} = \pi\cdot 1\cdot\dfrac{1}{2} = \dfrac{\pi}{2}\cdot\left(\lim\limits_{x\to 0}\dfrac{(1+x)^{\alpha}-1}{x}=\alpha\right).$

$\lim\limits_{x\to +\infty}\left(\dfrac{1-3x}{2-x}\right)^{1-x} = (\to 3)^{(\to -\infty)} = 0^{+}.$

2)

A	B	C	D	(A ⇒ B)	non D	(C ⇒ non D)	(A ⇒ B) ⇔ (C ⇒ non D)
1	1	1	0	1	1	1	1
1	1	0	0	1	1	1	1
0	1	1	0	1	1	1	1
0	1	0	0	1	1	1	1

B vera
D falsa

3) $f(x)=3x-2;\ g(x)=3^{1-x}.\ f(g(f(x))) = f(g(3x-2)) = f\left(3^{1-(3x-2)}\right) =$

$= f\left(3^{3-3x}\right) = 3\cdot 3^{3-3x}-2 = y \Rightarrow 3\cdot 3^{3-3x} = y+2 \Rightarrow 3^{3-3x} = \frac{1}{3}(y+2) \Rightarrow$

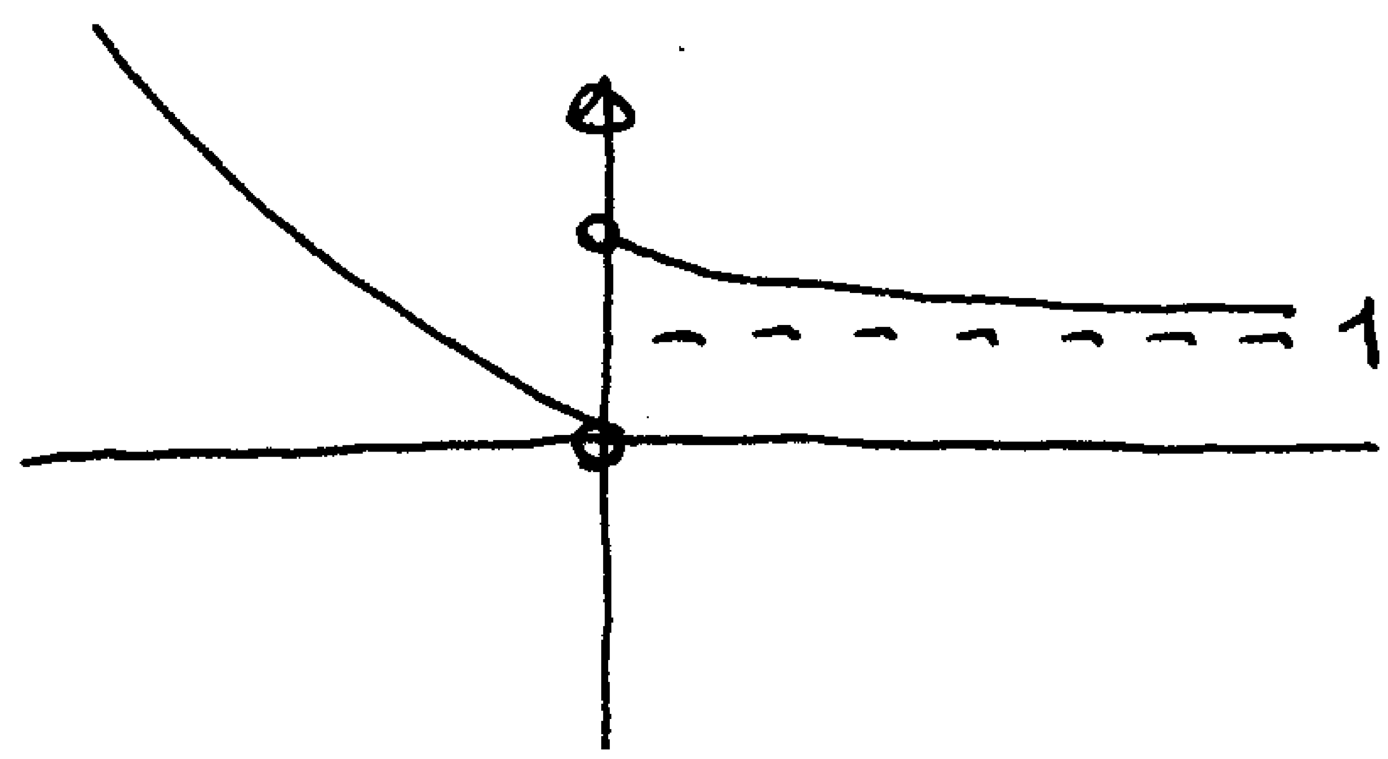
$\Rightarrow 3-3x = \log_3\left(\frac{1}{3}(y+2)\right) \Rightarrow 3x = 3-\log_3\left(\frac{1}{3}(y+2)\right) \Rightarrow x = 1-\frac{1}{3}\log_3\left(\frac{1}{3}(y+2)\right).$

Inversa: $F^{-1}(x) = y = 1-\frac{1}{3}\log_3\left(\frac{1}{3}(x+2)\right).$

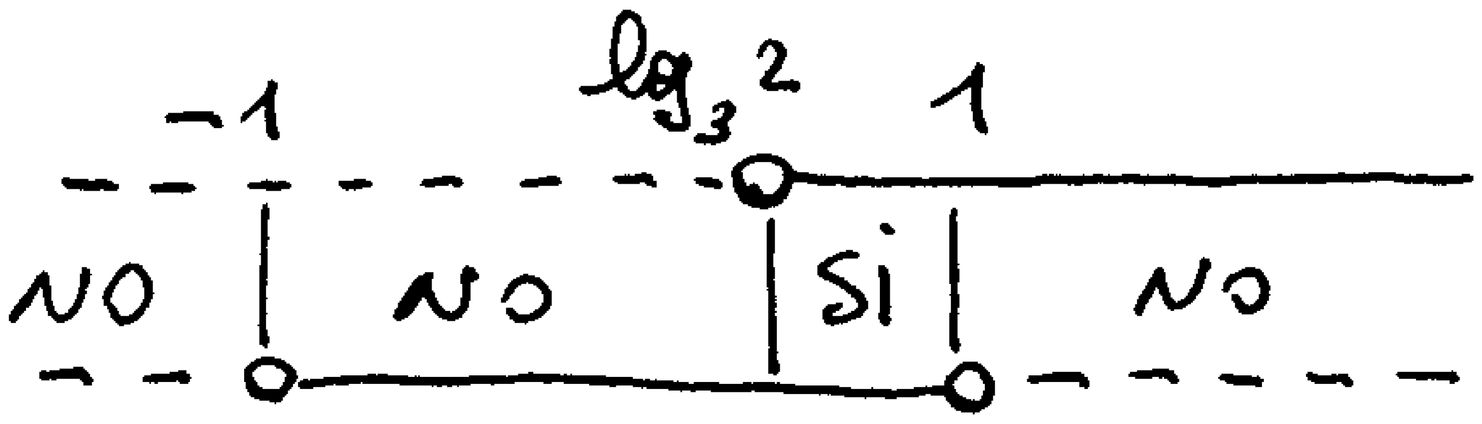
4) $\forall\varepsilon\ \exists\,\delta(\varepsilon):\ x<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon:\ \lim\limits_{x\to -\infty} f(x) = +\infty;$

$\forall\varepsilon>0\ \exists\,\delta(\varepsilon):\ x>\delta(\varepsilon)\Rightarrow 1<f(x)<1+\varepsilon:\ \lim\limits_{x\to +\infty} f(x) = 1^{+}$

con discontinuità di I specie in $x=0$



5) $f(x) = \dfrac{\log(3^{x}-2)}{\sqrt{1-x^{2}}}$. e.e.: $\begin{cases}3^{x}-2>0\\ 1-x^{2}>0\end{cases} \Rightarrow \begin{cases}3^{x}>2\\ x^{2}<1\end{cases} \Rightarrow \begin{cases}x>\log_3 2\\ -1<x<1\end{cases}$



. e.e.: $]\log_3 2;1[$.

Prova Intermedia di Matematica Generale del 9/11/2015 Compito B1

1) $\lim\limits_{x\to 0} \dfrac{\cos 3x - 1}{e^{x^2}-1} = \lim\limits_{x\to 0}\left(-\dfrac{1-\cos 3x}{9x^2}\right)\cdot 9\cdot\dfrac{x^2}{e^{x^2}-1} = -\dfrac{1}{2}\cdot 9\cdot 1 = -\dfrac{9}{2}.$

$\lim\limits_{x\to+\infty}\left(\dfrac{2+2x}{3+2x}\right)^{x+1} \to (\to 1)^{\to+\infty} \Rightarrow \lim\limits_{x\to+\infty}\left(\dfrac{3+2x-1}{3+2x}\right)^{x+1} = \lim\limits_{x\to+\infty}\left[\left(1+\dfrac{(-1)}{3+2x}\right)^{3+2x}\right]^{\frac{x+1}{3+2x}} =$

$= \left[\lim\limits_{t\to+\infty}\left(1+\dfrac{(-1)}{t}\right)^{t}\right]^{\lim\limits_{x\to+\infty}\frac{x+1}{3+2x}} = \left(e^{-1}\right)^{\frac{1}{2}} = e^{-\frac{1}{2}} = \dfrac{1}{\sqrt{e}}.$

2)

A	B	C	D	non A	(non A ⇒ C)	(B ⇔ D)	(non A ⇒ C) ⇒ (B ⇔ D)
1	1	1	0	0	1	0	0
1	0	1	0	0	1	1	1
0	1	1	0	1	1	0	0
0	0	1	0	1	1	1	1

C vera
D falsa

3) $f(x) = 2x+3;\ g(x) = \log_2(x+1).\ f(g(f(x))) = f(g(2x+3)) = f(\log_2(2x+3+1)) =$

$= f(\log_2(2x+4)) = 2\log_2(2x+4)+3 = y \Rightarrow 2\log_2(2x+4) = y-3 \Rightarrow \log_2(2x+4) = \frac{1}{2}(y-3) \Rightarrow$

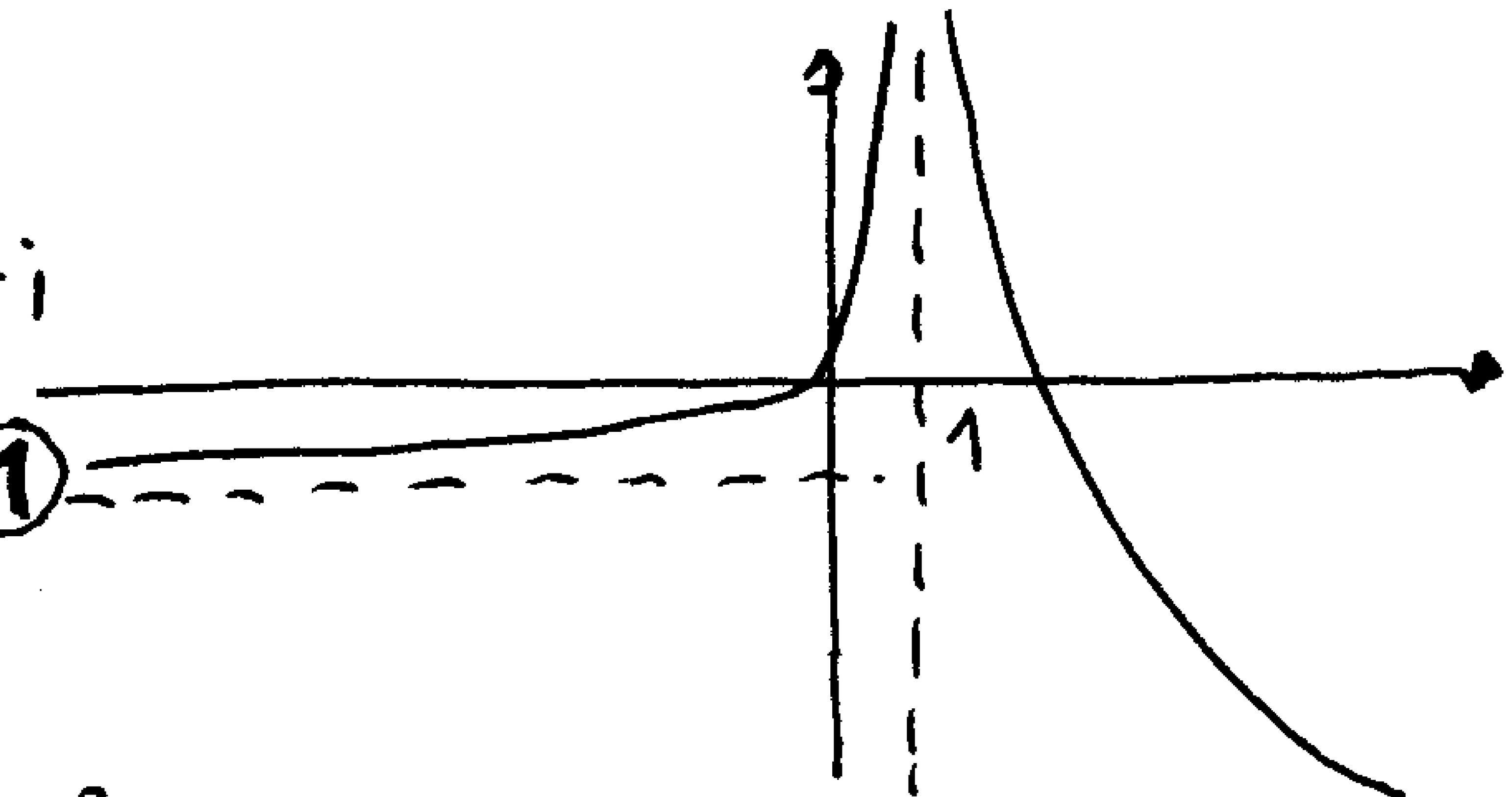
$\Rightarrow 2x+4 = 2^{\frac{1}{2}(y-3)} \Rightarrow 2x = 2^{\frac{1}{2}(y-3)} - 4 \Rightarrow x = \frac{1}{2}2^{\frac{1}{2}(y-3)} - 2.$

Inversa $F^{-1}(x) = \frac{1}{2}\cdot 2^{\frac{1}{2}(x-3)} - 2.$

4) $\forall \varepsilon>0\ \exists\,\delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow |f(x)+1|<\varepsilon : \lim\limits_{x\to-\infty} f(x) = -1;$

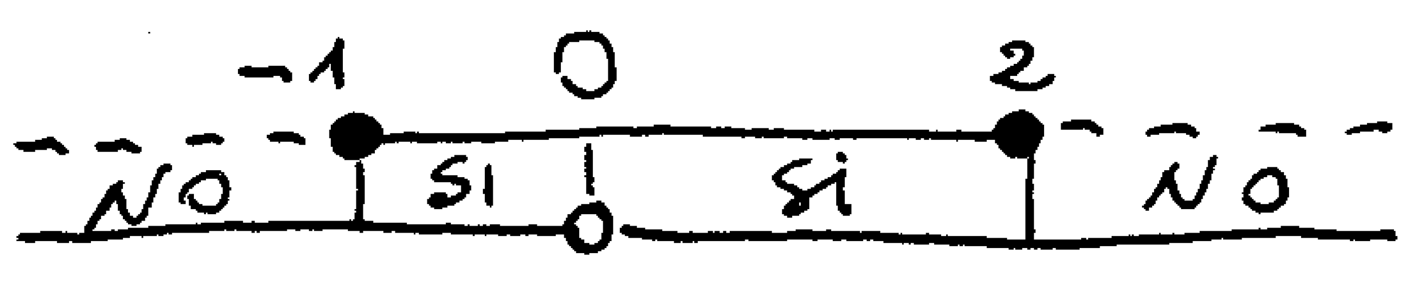
$\forall\varepsilon\ \exists\,\delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow f(x)<\varepsilon : \lim\limits_{x\to+\infty} f(x) = -\infty;$

con discontinuità $\underline{\text{II}}$ specie infinita in $x=1$.



5) $f(x) = \dfrac{\sqrt{2+x-x^2}}{\log(1+x^2)}.$ C.E.: $\begin{cases} 2+x-x^2 \geq 0 \\ 1+x^2 > 0 \\ 1+x^2 \neq 1 \end{cases} \Rightarrow \begin{cases} x^2-x-2 = (x-2)(x+1) \leq 0 \\ \forall x \\ x \neq 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} -1 \leq x \leq 2 \\ \text{Vera } \forall x \\ x \neq 0 \end{cases}$



C.E.: $[-1;0[\ \cup\ ]0;2].$

Prova Intermedia di Matematica Generale del 9/11/2015 Compito C1

1) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(\operatorname{sen}x)-tg^2x}{e^x-1} = \lim\limits_{x\to 0} \dfrac{\operatorname{sen}(\operatorname{sen}x)}{\operatorname{sen}x}\cdot\dfrac{\operatorname{sen}x}{x}\cdot\dfrac{x}{e^x-1} = 1\cdot 1\cdot 1 = 1.$

$\lim\limits_{x\to+\infty}\left(\dfrac{2x-5}{1+4x}\right)^{x-1} = \left(\to\dfrac{2}{4}\right)^{(\to+\infty)} = \left(\to\dfrac{1}{2}\right)^{(\to+\infty)} = 0^+.$

2)

A	B	C	D	non B	(A ⇒ non B)	(C ⇔ D)	(A ⇒ non B) e (C ⇔ D)
1	1	1	1	0	0	1	0
1	1	0	1	0	0	0	0
1	0	1	1	1	1	1	1
1	0	0	1	1	1	0	0

A vera
D vera

3) $f(x)=3x+4$; $g(x)=\sqrt{2x-1}$. $f(g(f(x)))=f(g(3x+4))=f(\sqrt{2(3x+4)-1}) =$

$= f(\sqrt{6x+7}) = 3\sqrt{6x+7}+4 = y \Rightarrow 3\sqrt{6x+7} = y-4 \Rightarrow \sqrt{6x+7} = \frac{1}{3}(y-4) \Rightarrow$

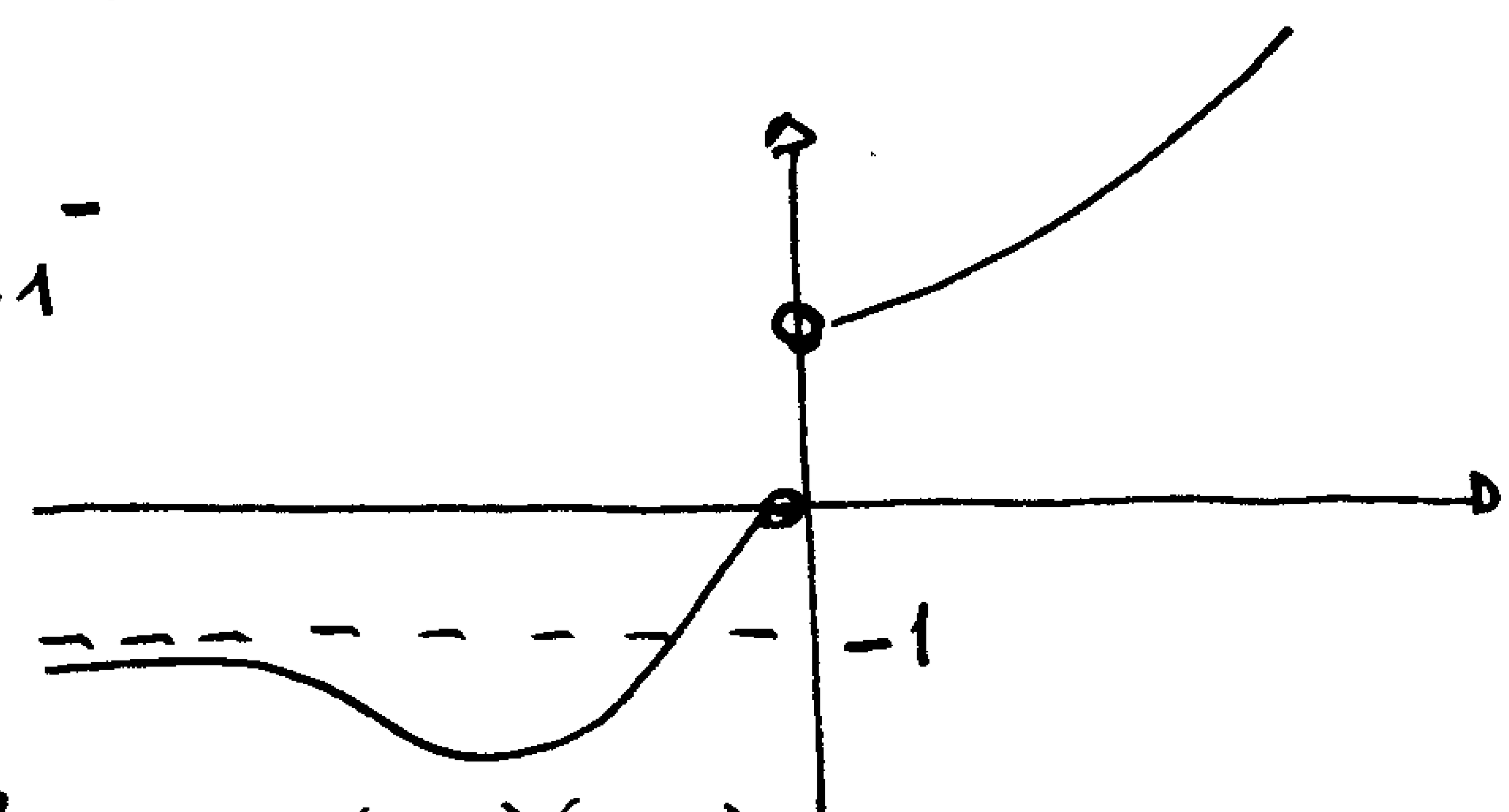
$\Rightarrow 6x+7 = \frac{1}{9}(y-4)^2 \Rightarrow 6x = \frac{1}{9}(y-4)^2 - 7 \Rightarrow x = \frac{1}{54}(y-4)^2 - \frac{7}{6}.$

Inversa $F^{-1}(x) = \frac{1}{54}\cdot(x-4)^2 - \frac{7}{6}$.

4) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow -1-\varepsilon < f(x) < -1 : \lim\limits_{x\to-\infty} f(x) = -1^-$

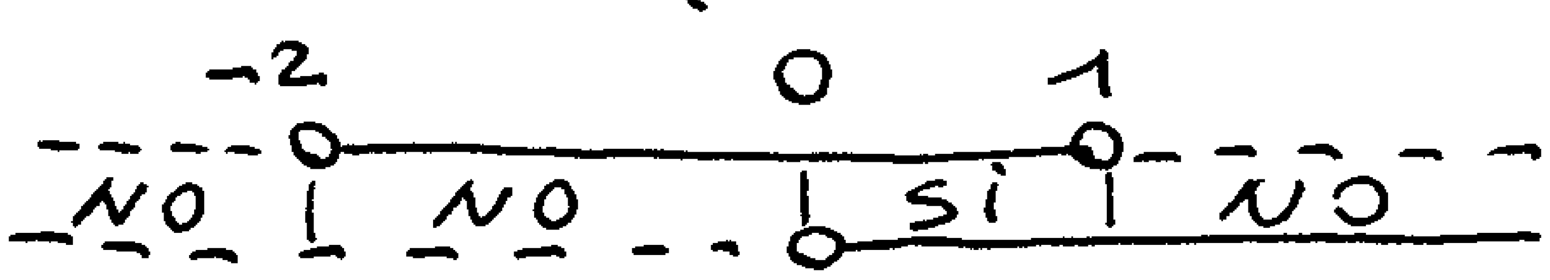
$\forall\varepsilon\ \exists\delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow f(x)>\varepsilon : \lim\limits_{x\to+\infty} f(x) = +\infty;$

con discontinuità di I specie in $x=0$.



5) $f(x) = \dfrac{\log(2-x-x^2)}{\sqrt{x}}$. C.E. $\begin{cases} 2-x-x^2>0 \\ x>0 \end{cases} \Rightarrow \begin{cases} x^2+x-2=(x-1)(x+2)<0 \\ x>0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} -2<x<1 \\ x>0 \end{cases}$



C.E.: $]0;1[$.

Prova Intermedia di Matematica Generale del 9/11/2015 Compito D1

1) $\lim\limits_{x\to 0}\dfrac{\log(1+\operatorname{sen}^2 x)}{1-\cos x} = \lim\limits_{x\to 0}\dfrac{\log(1+\operatorname{sen}^2 x)}{\operatorname{sen}^2 x}\cdot\dfrac{\operatorname{sen}^2 x}{x^2}\cdot\dfrac{x^2}{1-\cos x} = 1\cdot 1\cdot 2 = 2.$

$\lim\limits_{x\to +\infty}\left(\dfrac{3x+7}{3x+5}\right)^{2x} = (\to 1)^{\to +\infty} \Rightarrow \lim\limits_{x\to +\infty}\left(\dfrac{3x+5+2}{3x+5}\right)^{2x} = \lim\limits_{x\to +\infty}\left[\left(1+\dfrac{2}{3x+5}\right)^{3x+5}\right]^{\frac{2x}{3x+5}} =$

$= \lim\limits_{t\to +\infty}\left[\left(1+\dfrac{2}{t}\right)^{t}\right]^{\lim\limits_{x\to+\infty}\frac{2x}{3x+5}} = \left(e^2\right)^{\frac{2}{3}} = e^{\frac{4}{3}} = e\sqrt[3]{e}.$

2)

A B C D	$(A \Leftrightarrow B)$	non C	(non C $\Rightarrow$ D)	$(A\Leftrightarrow B)$ o (non C $\Rightarrow$ D)
1 0 1 1	0	0	1	1
1 0 0 1	0	1	1	1
0 0 1 1	1	0	1	1
0 0 0 1	1	1	1	1

B falsa
D vera

3) $f(x) = 2x-3$; $g(x) = \dfrac{1}{3^x+1}$. $f(g(f(x))) = f(g(2x-3)) = f\left(\dfrac{1}{3^{2x-3}+1}\right) =$

$= \dfrac{2}{3^{2x-3}+1} - 3 = y \Rightarrow y+3 = \dfrac{2}{3^{2x-3}+1} \Rightarrow 3^{2x-3}+1 = \dfrac{2}{y+3} \Rightarrow 3^{2x-3} = \dfrac{2}{y+3} - 1 = \dfrac{-y-1}{y+3} \Rightarrow$

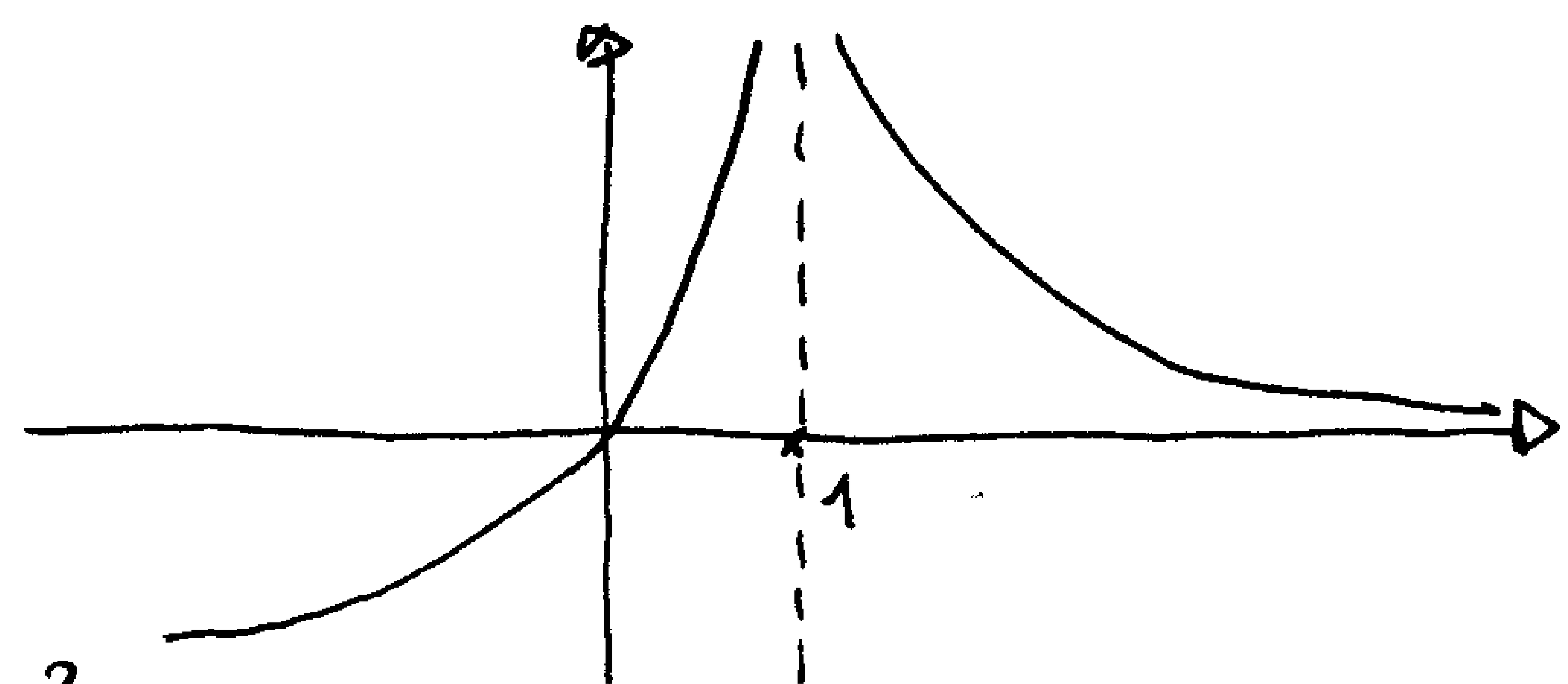
$\Rightarrow 2x-3 = \log_3\left(\dfrac{-y-1}{y+3}\right) \Rightarrow 2x = \log_3\left(\dfrac{-y-1}{y+3}\right)+3 \Rightarrow x = \dfrac{1}{2}\log_3\left(\dfrac{-y-1}{y+3}\right)+\dfrac{3}{2}.$

Inversa $F^{-1}(x) = y = \dfrac{1}{2}\log_3\left(\dfrac{-x-1}{x+3}\right)+\dfrac{3}{2}.$

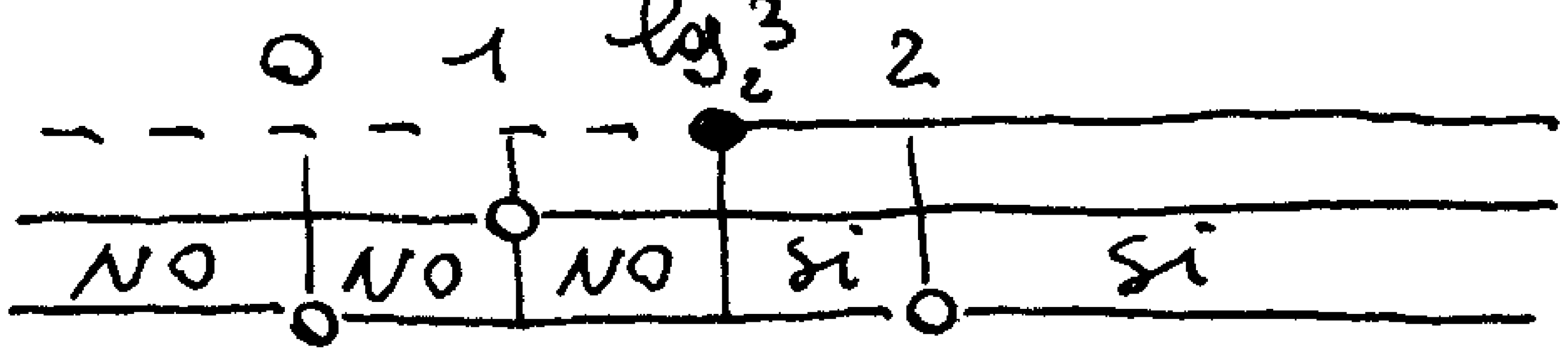
4) $\forall \varepsilon \, \exists \delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow f(x)<\varepsilon : \lim\limits_{x\to -\infty} f(x) = -\infty$;

$\forall \varepsilon>0 \, \exists \delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow |f(x)|<\varepsilon : \lim\limits_{x\to +\infty} f(x) = 0$;

con discontinuità II specie infinita in $x=1$.



5) $f(x) = \dfrac{\sqrt{2^x-3}}{\log(x-1)^2}$. c.e. $\begin{cases} 2^x-3 \geq 0 \\ (x-1)^2 > 0 \\ (x-1)^2 \neq 1 \end{cases} \Rightarrow \begin{cases} x \geq \log_2 3 \\ x \neq 1 \\ x\neq 0 \text{ e } x \neq 2 \end{cases}$



c.e.: $[\log_2 3; 2[\, \cup \,]2; +\infty[.$