Prova Intermedia di Matematica Generale del 9/11/2015 Compito A2

1) $\lim\limits_{x\to 0}\frac{(1-x)^e-1}{\operatorname{tg}3x} = \lim\limits_{x\to 0}\frac{(1-x)^e-1}{-x}\cdot\frac{-1}{3}\cdot\frac{3x}{\operatorname{tg}3x} = e\cdot\left(-\frac{1}{3}\right)\cdot 1 = -\frac{1}{3}e\cdot\left(\lim\limits_{x\to 0}\frac{(1+x)^\alpha-1}{x}=\alpha\right).$

$\lim\limits_{x\to-\infty}\frac{1-3^{-x}+3^{x}+\operatorname{sen}x}{2-2^{x}} = \lim\limits_{x\to-\infty}\frac{-3^{-x}}{2} = -\infty.\ \left(\begin{matrix}3^x\to 0;\ \operatorname{sen}x=o(3^{-x});\ 1=o(3^{-x})\\ 2^x\to 0\end{matrix}\right).$

2)

A	B	C	(A⇒B)	non A	(C o non A)	(A⇒B) ⇔ (C o non A)
1	1	1	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	1
0	0	0	1	1	1	1

B	C	(B ⇔ C)
1	1	1
1	0	0
0	1	0
0	0	1

3) $f(x)=3x-2;\ g(x)=3^{1-x}.\ f\left(\frac{3}{g(x)-1}\right)=f\left(\frac{3}{3^{1-x}-1}\right)=\frac{9}{3^{1-x}-1}-2=y \Rightarrow$

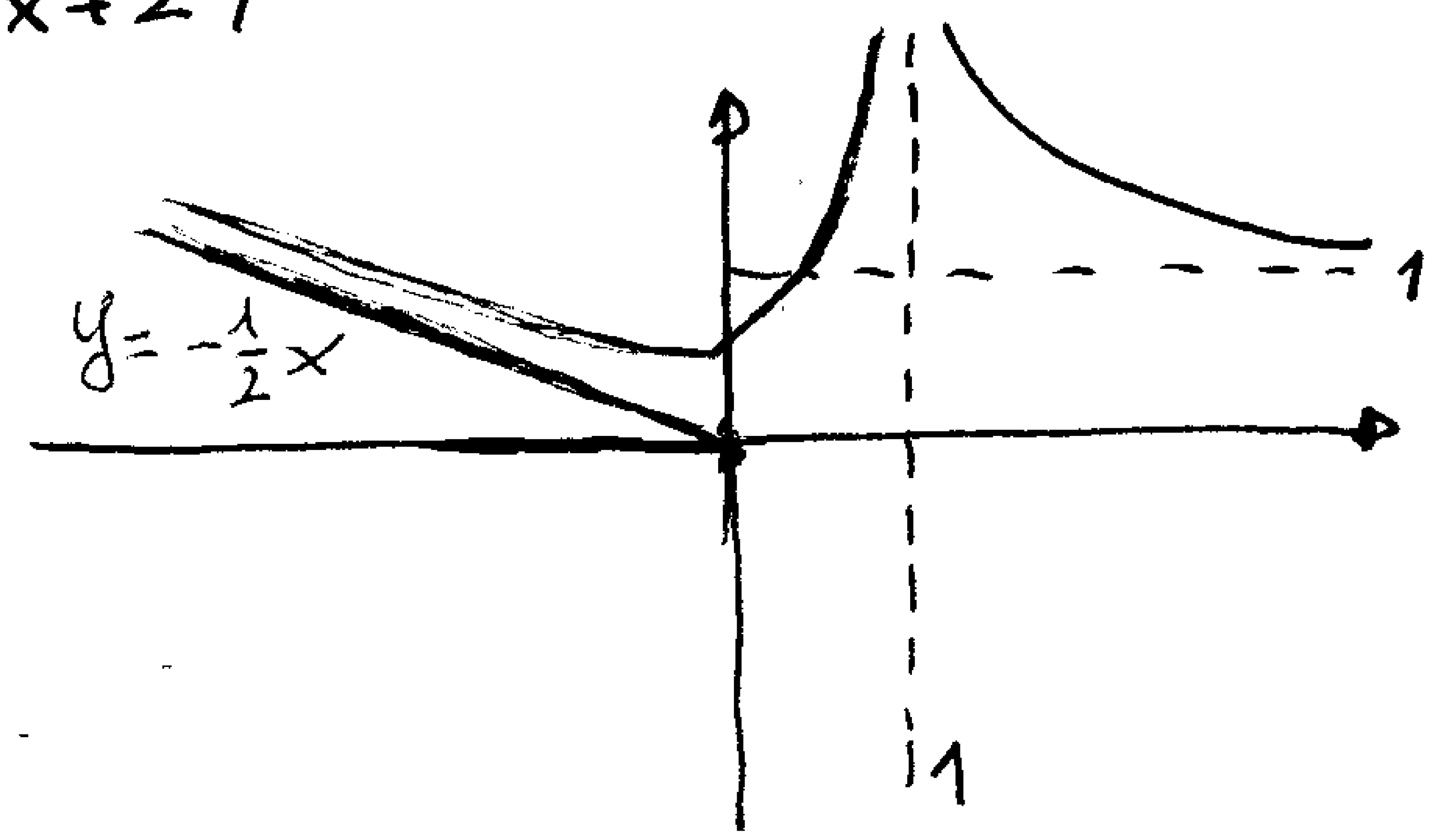
$\Rightarrow \frac{9}{3^{1-x}-1}=y+2 \Rightarrow 3^{1-x}-1=\frac{9}{y+2} \Rightarrow 3^{1-x}=\frac{9}{y+2}+1=\frac{11+y}{y+2} \Rightarrow 1-x=\log_3\left(\frac{11+y}{y+2}\right)\Rightarrow$

$\Rightarrow x=1-\log_3\left(\frac{y+11}{y+2}\right)$. Inversa $F^{-1}(x)=1-\log_3\left(\frac{x+11}{x+2}\right)$.

4) $\forall\varepsilon\ \exists\delta(\varepsilon): 0<|x-1|<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon: \lim\limits_{x\to 1}f(x)=+\infty$

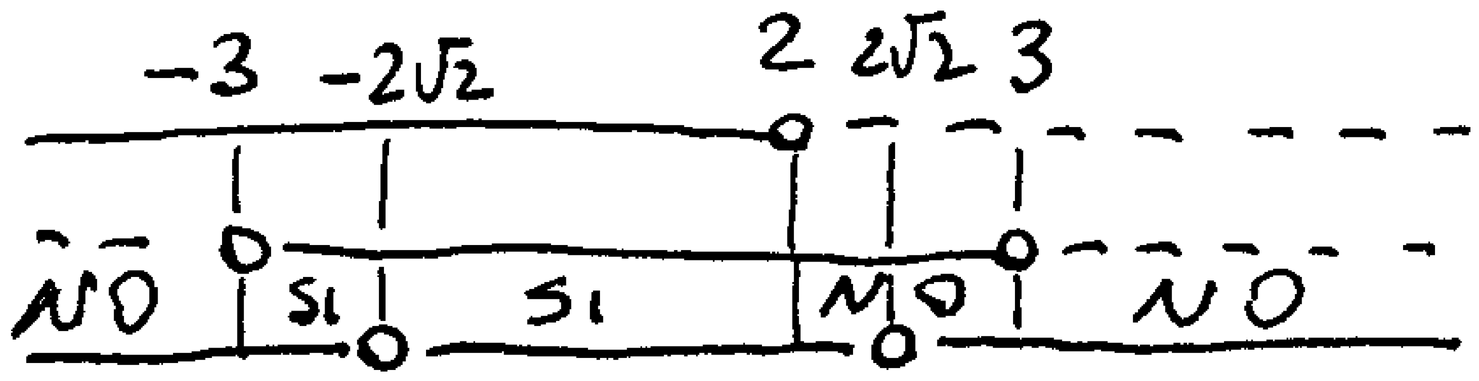
$\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow 1<f(x)<1+\varepsilon: \lim\limits_{x\to+\infty}f(x)=1^{+}$

con Asintoto Obliquo a sinistra $y=-2x$.



5) $f(x)=\frac{\log(9-3^x)}{\log(9-x^2)}$. c.e.: $\begin{cases}9-3^x>0\\ 9-x^2>0\\ 9-x^2\neq 1\end{cases}\Rightarrow\begin{cases}3^x<9\\ x^2<9\\ x^2\neq 8\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}x<2\\ -3<x<3\\ x\neq\pm2\sqrt{2}\end{cases}$



c.e. : $]-3;-2\sqrt{2}[\ \cup\ ]-2\sqrt{2};2[$.

Prova Intermedia di Matematica Generale del 9/11/2015 Compito B2

1) $\lim\limits_{x\to 0}\dfrac{\log(1-x^2)}{1-\cos 2x}=\lim\limits_{x\to 0}\dfrac{\log(1+(-x^2))}{(-x^2)}\cdot(-1)\cdot\dfrac{4x^2}{1-\cos 2x}\cdot\dfrac{1}{4}=1\cdot(-1)\cdot 2\cdot\dfrac{1}{4}=-\dfrac{1}{2}.$

$\lim\limits_{x\to 0}\dfrac{\sqrt[5]{x}+\sqrt[3]{x}+\operatorname{sen}x}{2x-\sqrt[3]{x}}=\lim\limits_{x\to 0}\dfrac{\sqrt[5]{x}}{-\sqrt[3]{x}}=\lim\limits_{x\to 0}-\dfrac{1}{\sqrt[15]{x^2}}=-\infty\quad\left(\begin{array}{c}\sqrt[3]{x}=o(\sqrt[5]{x});\ \operatorname{sen}x=o(\sqrt[5]{x})\\ 2x=o(\sqrt[3]{x})\end{array}\right)$

2)

A	B	C	(A<=>B)	non C	(non C e A)	(A<=>B) => (non C e A)
1	1	0	1	1	1	1
1	0	0	0	1	1	1
0	1	1	0	0	0	1
0	0	1	1	0	0	0

A	C	A<=>C
1	1	1
1	0	0
0	1	0
0	0	1

3) $f(x)=x-3;\ g(x)=\log_3(x-1).\quad f\left(\dfrac{2}{g(x)+1}\right)=f\left(\dfrac{2}{\log_3(x-1)+1}\right)=\dfrac{2}{\log_3(x-1)+1}-3=y\Rightarrow$

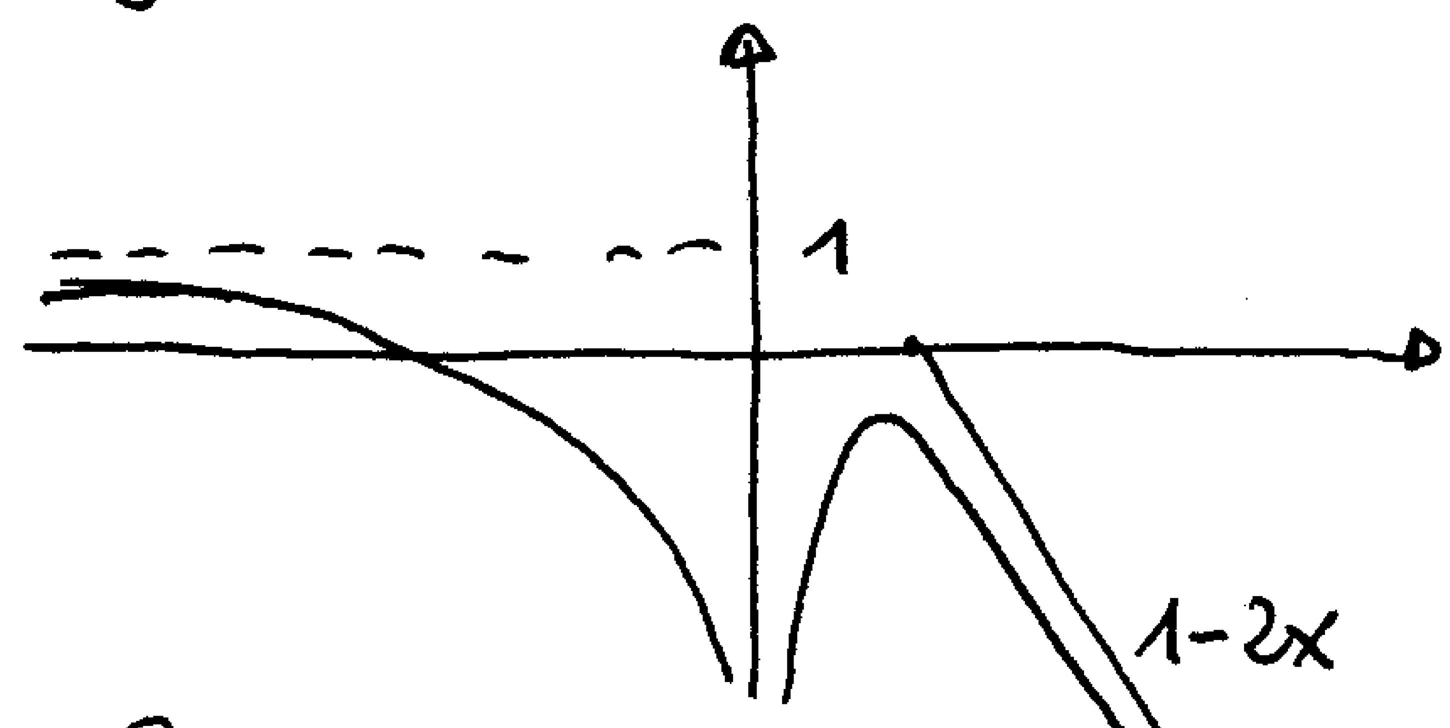
$\Rightarrow\dfrac{2}{\log_3(x-1)+1}=y+3\Rightarrow\log_3(x-1)+1=\dfrac{2}{y+3}\Rightarrow\log_3(x-1)=\dfrac{2}{y+3}-1=\dfrac{-y-1}{y+3}\Rightarrow$

$\Rightarrow x-1=3^{\frac{-y-1}{y+3}}\Rightarrow x=3^{\frac{-y-1}{y+3}}+1.$ Inversa $F^{-1}(x)=y=3^{\frac{-x-1}{x+3}}+1.$

4) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow 1-\varepsilon<f(x)<1:\ \lim\limits_{x\to-\infty}f(x)=1^-$

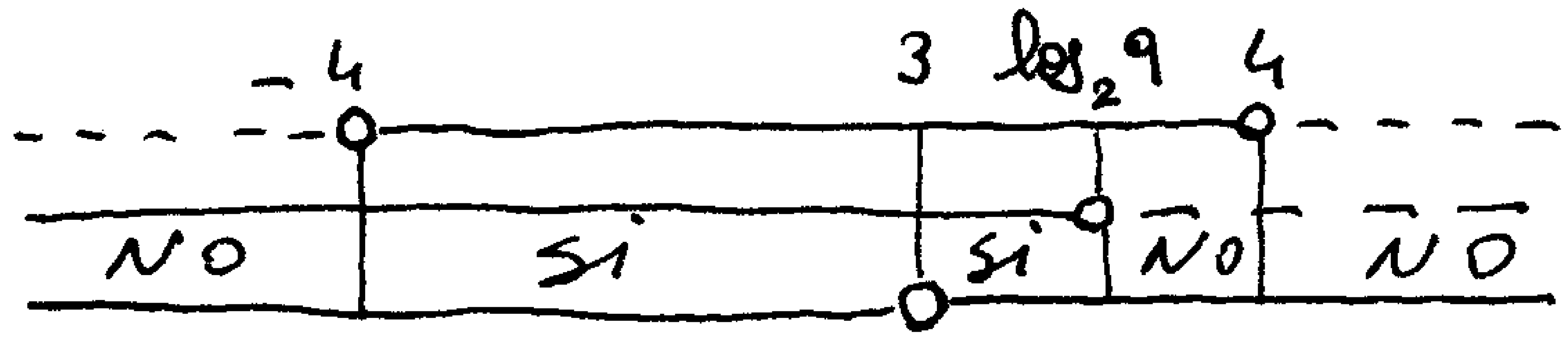
$\forall\varepsilon\ \exists\delta(\varepsilon): 0<|x|<\delta(\varepsilon)\Rightarrow f(x)<\varepsilon:\ \lim\limits_{x\to 0}f(x)=-\infty$

con asintoto obliquo a destra $y=1-2x$.



5) $f(x)=\dfrac{\log(16-x^2)}{\log(9-2^x)}.\quad$ c.e.: $\begin{cases}16-x^2>0\\ 9-2^x>0\\ 9-2^x\neq 1\end{cases}\Rightarrow\begin{cases}x^2<16\\ 2^x<9\\ 2^x\neq 8\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}-4<x<4\\ x<\log_2 9\\ x\neq 3\end{cases}$



c.e.: $]-4;3[\ \cup\ ]3;\log_2 9[.$

Prova Intermedia di Matematica Generale del 9/11/2015 Compito C2

1) $\lim_{x\to 0}\frac{3^{-x}-3^{x}+\operatorname{sen}x}{3x}=\frac{1}{3}\left[\lim_{x\to 0}\frac{3^{-x}-1}{(-x)}(-1)-\frac{3^{x}-1}{x}+\frac{\operatorname{sen}x}{x}\right]=\frac{1}{3}\left(-\log 3-\log 3+1\right)=\frac{1}{3}(1-\log 9).$

$\lim_{x\to -\infty}\frac{10x-2^{-x}+4^{x}}{2+\operatorname{sen}x+2^{x}}=\lim_{x\to -\infty}\frac{-2^{-x}}{2+\operatorname{sen}x}=-\infty$ $\left(2^{-x}\to+\infty;\ 4^{x}\to 0;\ 10x=o(2^{-x});\ 2+\operatorname{sen}x \text{ è limitata}\right).$

2)

A	B	C	non B	(A e non B)	(C ⇒ A)	(A e non B) ⇔ (C ⇒ A)
1	1	1	0	0	1	0
1	1	0	0	0	1	0
0	0	1	1	0	0	1
0	0	0	1	0	1	0

A	B	B ⇔ A
1	1	1
1	0	0
0	1	0
0	0	1

3) $f(x)=3x+3;\ g(x)=2^{x-2}.\ f\left(\frac{2}{1-g(x)}\right)=f\left(\frac{2}{1-2^{x-2}}\right)=\frac{6}{1-2^{x-2}}+3=y\Rightarrow$

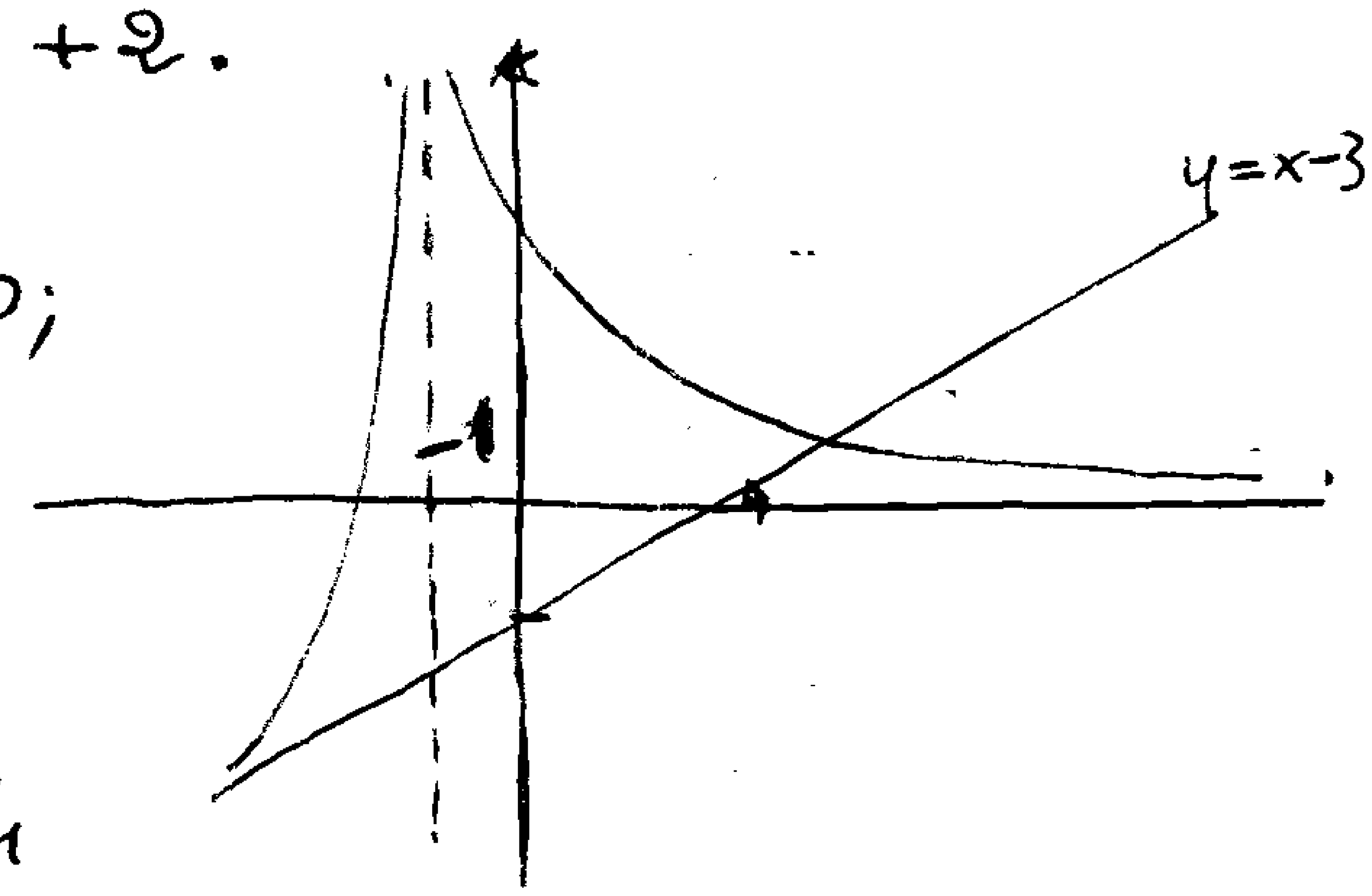
$\Rightarrow\frac{6}{1-2^{x-2}}=y-3\Rightarrow 1-2^{x-2}=\frac{6}{y-3}\Rightarrow 2^{x-2}=1-\frac{6}{y-3}=\frac{y-9}{y-3}\Rightarrow x-2=\log_2\frac{y-9}{y-3}\Rightarrow$

$\Rightarrow x=\log_2\frac{y-9}{y-3}+2.$ Inversa $F^{-1}(x)=y=\log_2\frac{x-9}{x-3}+2.$

4) $\forall\varepsilon\ \exists\delta(\varepsilon): 0<|x+1|<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon: \lim_{x\to -1}f(x)=+\infty;$

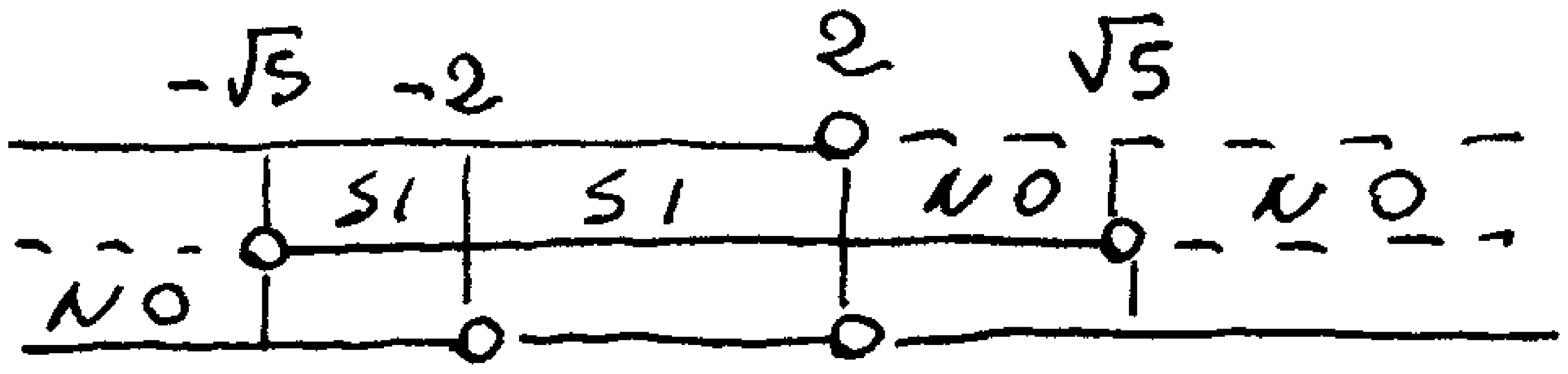
$\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow|f(x)|<\varepsilon: \lim_{x\to+\infty}f(x)=0;$

con asintoto obliquo a sinistra $y=x-3$.



5) $f(x)=\frac{\log(4-2^{x})}{\log(5-x^{2})}$. C.E.: $\begin{cases}4-2^{x}>0\\5-x^{2}>0\\5-x^{2}\neq 1\end{cases}\Rightarrow\begin{cases}2^{x}<4\\x^{2}<5\\x^{2}\neq 4\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}x<2\\-\sqrt{5}<x<\sqrt{5}\\x\neq 2 \text{ e } x\neq -2\end{cases}$



C.E.: $]-\sqrt{5};-2[\ \cup\ ]-2;2[$.

Prova Intermedia di Matematica Generale del 9/11/2015 Compito D2

1) $\lim_{x\to 0} \frac{\cos 2x - \cos 3x}{3x^2} = \frac{1}{3}\left(-\frac{1-\cos 2x}{4x^2}\cdot 4 + \frac{1-\cos 3x}{9x^2}\cdot 9\right) = \frac{1}{3}\left(-\frac{4}{2}+\frac{9}{2}\right) = \frac{5}{6}.$

$\lim_{x\to 0} \frac{x^2+\sqrt[7]{x}+\sin^2 x}{2-\sqrt[7]{x}} = \lim_{x\to 0}\frac{\sqrt[7]{x}}{2} = 0 \quad \left(\begin{array}{l} x^2 = o(\sqrt[7]{x})\,;\ \sin^2 x = o(\sqrt[7]{x}) \\ \sqrt[7]{x} = o(2) \end{array}\right).$

2)

A	B	C	$(A \circ B)$	non C	$(A \Rightarrow \text{non } C)$	$(A \circ B) \Leftrightarrow (A \Rightarrow \text{non } C)$
1	1	0	1	1	1	1
0	1	0	1	1	1	1
1	0	1	1	0	0	0
0	0	1	0	0	1	0

B	C	$B \Leftrightarrow C$
1	1	1
1	0	0
0	1	0
0	0	1

3) $f(x) = 1-2x$; $g(x) = 1+\log_3 x$. $f\left(\frac{-1}{1+g(x)}\right) = f\left(\frac{-1}{1+1+\log_3 x}\right) = f\left(\frac{-1}{\log_3 x+2}\right) =$

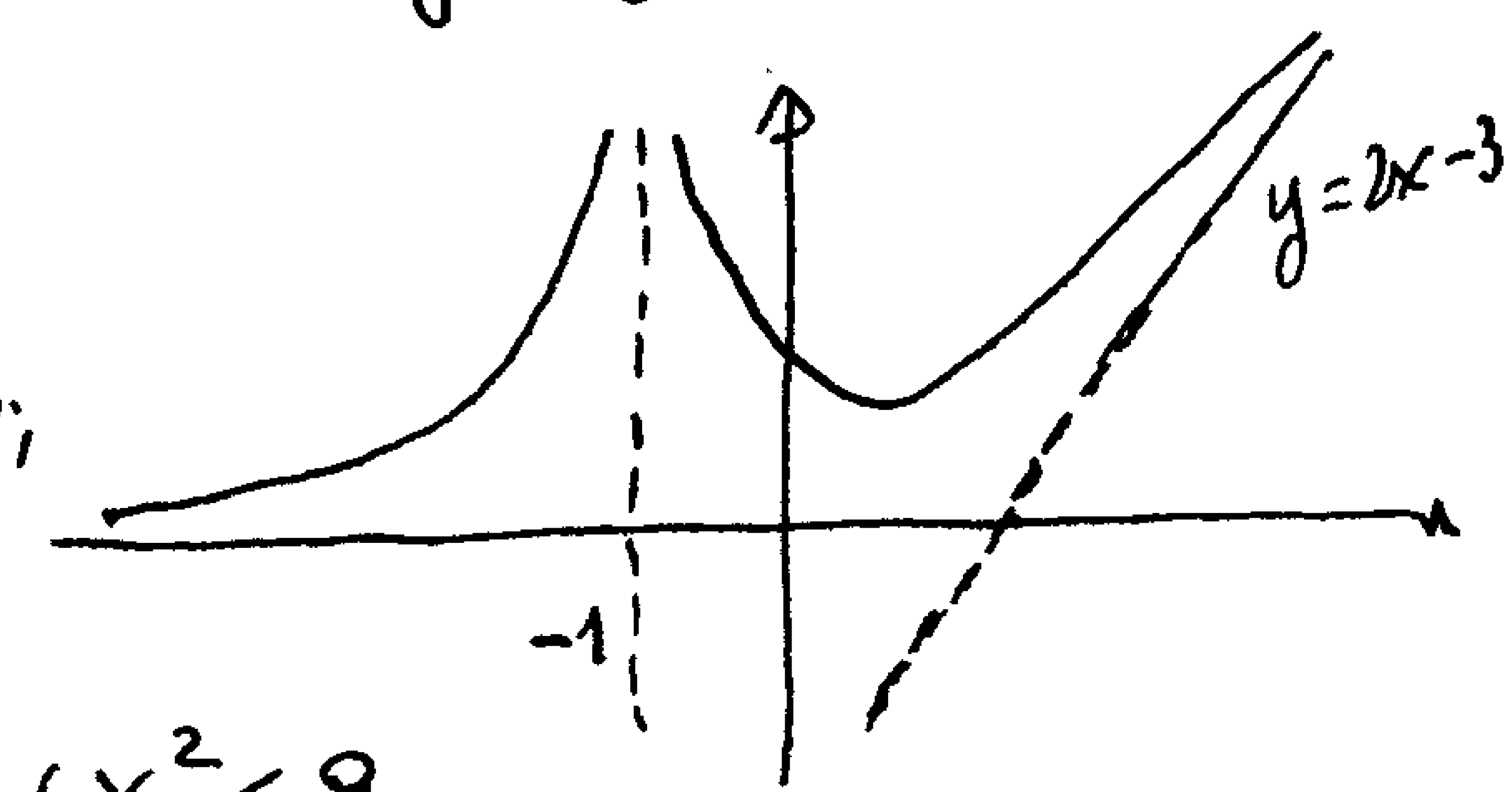
$= 1 - \frac{-2}{\log_3 x+2} = 1+\frac{2}{\log_3 x+2} = y \Rightarrow \frac{2}{\log_3 x+2} = y-1 \Rightarrow \log_3 x+2 = \frac{2}{y-1} \Rightarrow$

$\Rightarrow \log_3 x = \frac{2}{y-1} - 2 = \frac{4-2y}{y-1} \Rightarrow x = 3^{\frac{4-2y}{y-1}}$. Inversa $F^{-1}(x) = y = 3^{\frac{4-2x}{x-1}}$.

4) $\forall \varepsilon>0\ \exists \delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow 0<f(x)<\varepsilon : \lim_{x\to-\infty} f(x) = 0^+$;

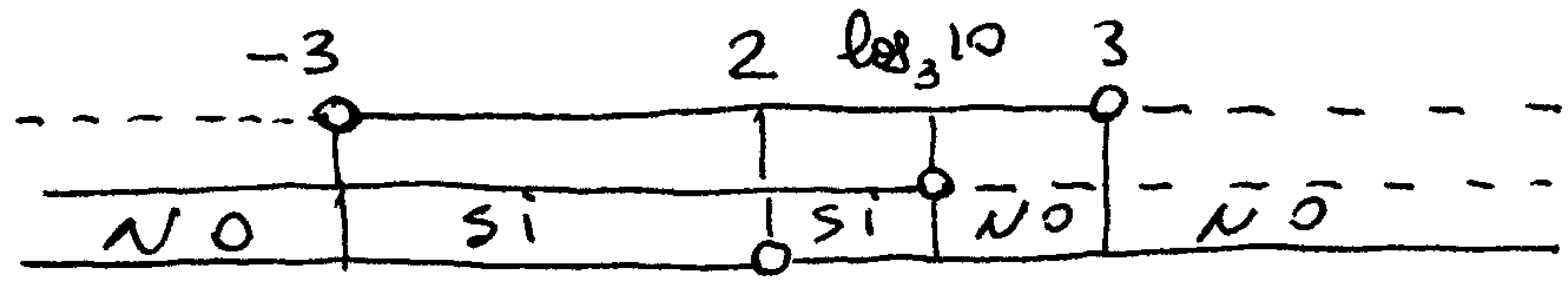
$\forall \varepsilon\ \exists \delta(\varepsilon): 0<|x+1|<\delta(\varepsilon) \Rightarrow f(x)>\varepsilon : \lim_{x\to -1} f(x) = +\infty$;

con asintoto obliquo a destra $y = 2x-3$.



5) $f(x) = \frac{\log(9-x^2)}{\log(10-3^x)}$. C.E.: $\begin{cases} 9-x^2>0 \\ 10-3^x>0 \\ 10-3^x \neq 1 \end{cases} \Rightarrow \begin{cases} x^2<9 \\ 3^x<10 \\ 3^x\neq 9 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} -3<x<3 \\ x<\log_3 10 \\ x\neq 2 \end{cases}$



C.E.: $]-3;2[\ \cup\ ]2;\log_3 10[$.