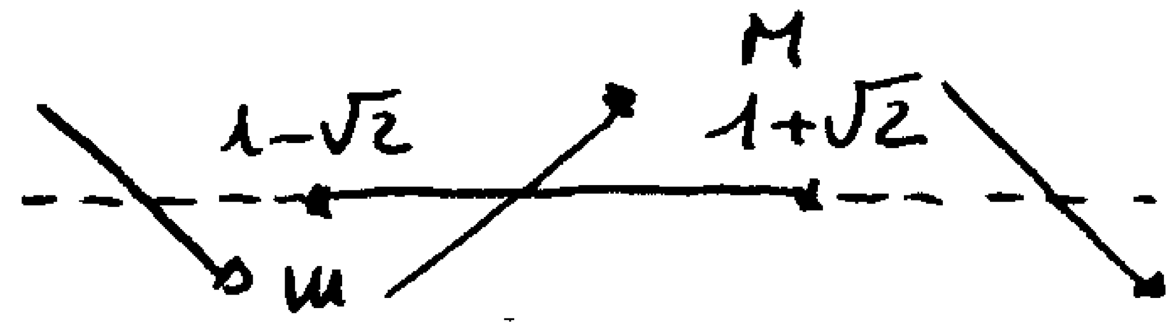
Compito di Matematica Generale del 27/1/2016 Compito A $\boxed{MGA1}$

1) $f(x)=(x^2-1)\cdot e^{2-x}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=0^+$.

$f(x)\geq 0$: $x^2-1\geq 0 \Rightarrow x\leq -1 \cup x\geq 1$ (+) -1 (−) 1 (+)

$f'(x)=2x\cdot e^{2-x}+(x^2-1)\cdot(-1)\cdot e^{2-x}=e^{2-x}(-x^2+2x+1)\geq 0$ per

$x^2-2x-1\leq 0$: $x=1\pm\sqrt{1+1}=1\pm\sqrt{2} \Rightarrow 1-\sqrt{2}\leq x\leq 1+\sqrt{2}$



$f''(x)=-e^{2-x}(-x^2+2x+1)+e^{2-x}(-2x+2)=e^{2-x}(+x^2-4x+1)\geq 0$ per

$x^2-4x+1\geq 0$: $x=2\pm\sqrt{4-1}=2\pm\sqrt{3} \Rightarrow x\leq 2-\sqrt{3} \cup x\geq 2+\sqrt{3}$

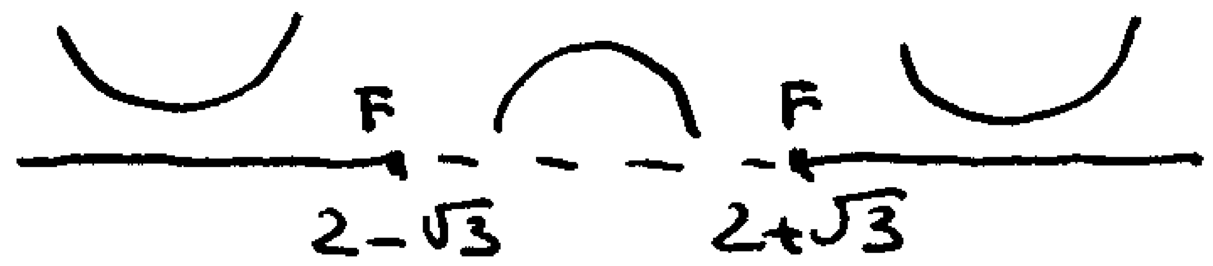
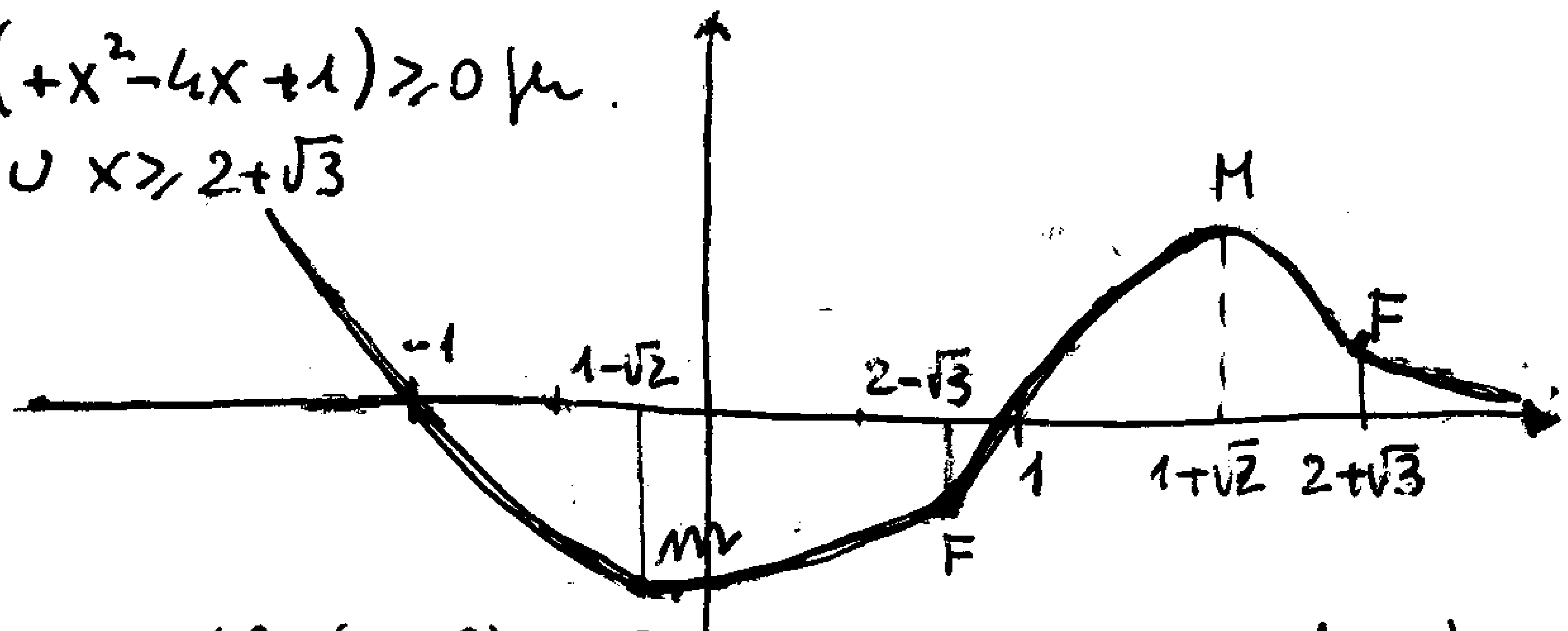


Grafico:



2) $\lim_{x\to 0^+}\dfrac{\log(1+x^2)-\sqrt[5]{x^3}+\operatorname{sen}x}{1-\cos x+\sqrt{x}}=\lim_{x\to 0^+}\dfrac{-x^{\frac{3}{5}}}{x^{\frac{1}{2}}}=0^-$ $\left(\log(1+x^2)\sim x^2;\ \operatorname{sen}x\sim x;\ 1-\cos x\sim\frac{1}{2}x^2\right.$ $\left.\log(1+x^2)=o(x^{\frac{3}{5}});\ 1-\cos x=o(x^{\frac{1}{2}});\ \operatorname{sen}x=o(x^{\frac{3}{5}})\right)$ $\left(\frac{3}{5}>\frac{1}{2}\right)$

$\lim_{x\to+\infty} x\cdot\log\left(1+\frac{1}{x}\right) \Rightarrow \left(x=\frac{1}{t}\right)=\lim_{t\to 0}\dfrac{\log(1+t)}{t}=1.$

3) per $x=2$: $x^2-x-k \Rightarrow 4-2-k=0 \Rightarrow k=2$. $\lim_{x\to 2^+}\dfrac{x^3-3x^2+5x-5}{x^2-x-2}=\lim_{x\to 2^+}\dfrac{x^3-3x^2+5x-5}{(x-2)(x+1)} \Rightarrow$

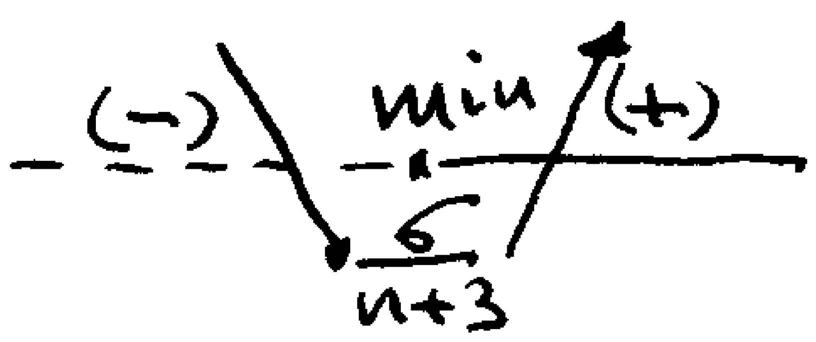
$\Rightarrow \dfrac{(\to 1)}{(\to 0^+)\cdot(\to 3)} \Rightarrow \lim_{x\to 2^+} f(x)=+\infty.$

4) $f(x)=(x-2)^n\cdot x^3$; $f(x)\in C(\mathbb{R})$. $f'(x)=n(x-2)^{n-1}\cdot x^3+3(x-2)^n\cdot x^2 \Rightarrow$

$\Rightarrow f'(x)=x^2\cdot(x-2)^{n-1}\cdot(nx+3x-6)\geq 0$.

Per n dispari ($n-1$ pari)

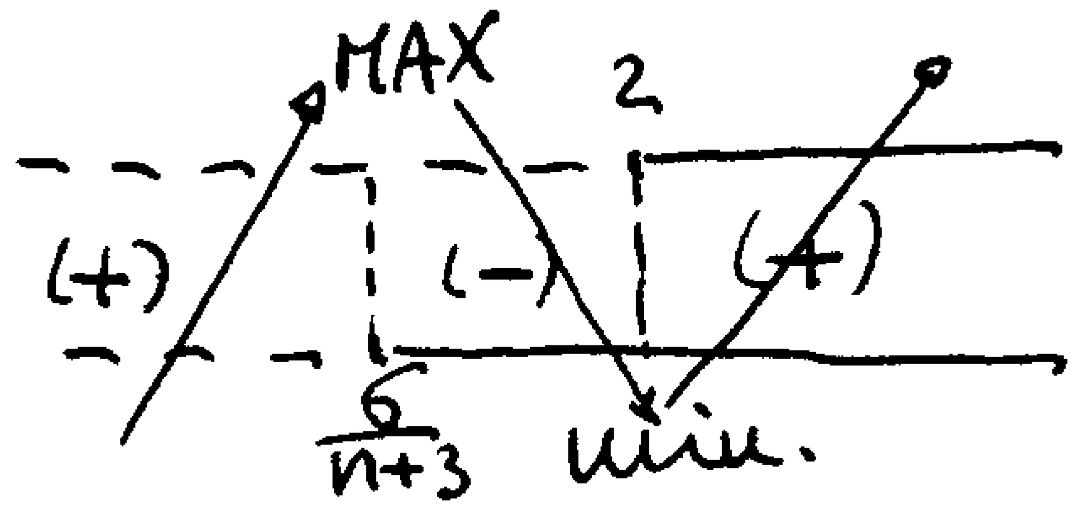
$nx+3x-6\geq 0$: $x\geq\dfrac{6}{n+3}$



Per n pari ($n-1$ dispari)

$x-2\geq 0$: $x\geq 2$

$nx+3x-6\geq 0$: $x\geq\dfrac{6}{n+3}$



Per n dispari punto di minimo in $x=\dfrac{6}{n+3}$; Per n pari punto di MAX in $x=\dfrac{6}{n+3}$; min in $x=2$.

5) $f(x;y)=e^{x^2+3y-y^3}$. $\nabla f(x;y)=\underline{0} \Rightarrow \begin{cases} f'_x=2x\cdot e^{x^2+3y-y^3}=0 \\ f'_y=3(1-y^2)e^{x^2+3y-y^3}=0\end{cases} \Rightarrow \begin{cases}x=0\\ y=1\end{cases} \cup \begin{cases}x=0\\ y=-1\end{cases}.$

MGA2

$$H(x;y)=\begin{Vmatrix} e^{x^2+3y-y^3}\cdot(4x^2+2) & e^{x^2+3y-y^3}\cdot(2x)(3-3y^2) \\ e^{x^2+3y-y^3}(2x)(3-3y^2) & e^{x^2+3y-y^3}((3-3y^2)^2-6y) \end{Vmatrix} \Rightarrow H(0;1)=\begin{Vmatrix} 2e^2 & 0 \\ 0 & -6e^2 \end{Vmatrix}: \text{Sella};\ H(0;-1)=\begin{Vmatrix} 2e^{-2} & 0 \\ 0 & 6e^{-2} \end{Vmatrix}: \text{Min.}$$

6) $\int_0^1 e^{2-3x}-\frac{1}{(x-2)^3}\,dx=\left(-\frac{1}{3}e^{2-3x}-\left(-\frac{1}{2}(x-2)^{-2}\right)\right|_0^1=-\frac{1}{3}e^{-1}+\frac{1}{2}-\left(-\frac{1}{3}e^2+\frac{1}{8}\right)=\frac{1}{3}e^2-\frac{1}{3e}+\frac{3}{8}.$

7) $f(x;y)=xe^{3y-x}+3y$. $\nabla f(x;y)=(e^{3y-x}-xe^{3y-x};3xe^{3y-x}+3)$; $\nabla f(3;1)=(-2;12)$.

$(x;y)\perp(-2;12)\Rightarrow -2x+12y=0\Rightarrow x=6y$. $\|6y\ \ y\|=\sqrt{36y^2+y^2}=\sqrt{37y^2}=\sqrt{37}\Rightarrow y^2=1\Rightarrow y=\pm 1.$

I vettori sono $V_1=(6;1)$ e $V_2=(-6;-1)$.

8) $f(x)=\frac{2^x+k}{2^k-1}$; $g(x)=1-x$. $f(g(x))=\frac{2^{1-x}+k}{2^{1-x}-1}$; $f(g(0))=\frac{2+k}{2-1}=\frac{2+k}{1}=5\Rightarrow k=3.$

$y=\frac{2^{1-x}+3}{2^{1-x}-1}\Rightarrow y2^{1-x}-y=2^{1-x}+3\Rightarrow 2^{1-x}=\frac{y+3}{y-1}\Rightarrow 1-x=\log_2\left(\frac{y+3}{y-1}\right)\Rightarrow x=1-\log_2\left(\frac{y+3}{y-1}\right).$

Inversa: $y=1-\log_2\left(\frac{x+3}{x-1}\right)$.

9) $f(x)=e^{1-kx}$. Bisettrice I e III quadrante: $y=x\Rightarrow m=1=f'(0)$. $f'(x)=-k\cdot e^{1-kx}\Rightarrow$

$\Rightarrow f'(0)=-k\cdot e=1$ per $k=-\frac{1}{e}\Rightarrow f(x)=e^{1+\frac{x}{e}}$. $f(0)=e$; $f'(0)=1\Rightarrow$

$\Rightarrow$ Equazione retta tangente: $y-e=1(x-0)\Rightarrow y=x+e.$

10) $P:(\text{non } A)\Rightarrow(B \text{ o } C)$

A	B	C	non A	B o C	(non A) => (B o C)
1	1	1	0	1	1
1	1	0	0	1	1
1	0	1	0	1	1
1	0	0	0	0	1
0	1	1	1	1	1
0	1	0	1	1	1
0	0	1	1	1	1
0	0	0	1	0	0

La proposizione è falsa solo quando non ho il giorno libero e non studio e non lavoro.

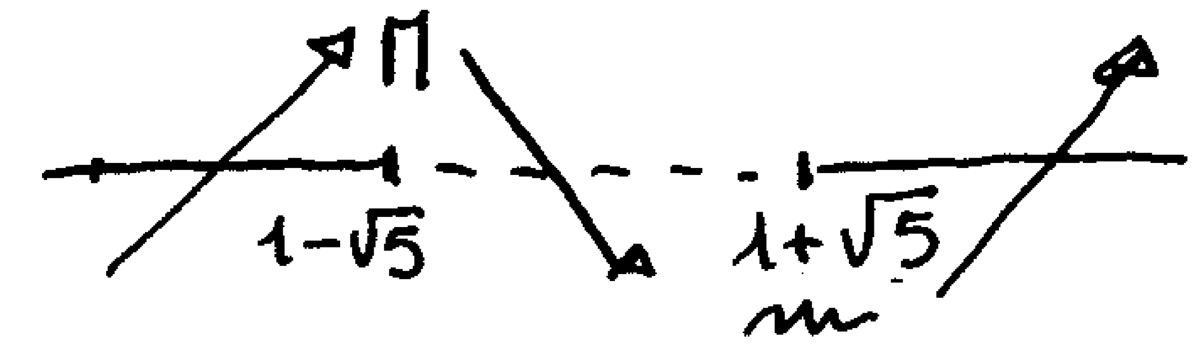
Compito di Matematica Generale del 27/1/2016 Compito B $\boxed{MGB1}$

1) $f(x)=(4-x^2)\cdot e^{1-x}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=-\infty$; $\lim_{x\to+\infty} f(x)=0^-$.

$f(x)\geq 0$: $4-x^2\geq 0 \Rightarrow -2\leq x\leq 2$ (−) −2 (+) 2 (−)

$f'(x)=-2x\cdot e^{1-x}+(4-x^2)\cdot(-e^{1-x})=e^{1-x}(x^2-2x-4)\geq 0$ per

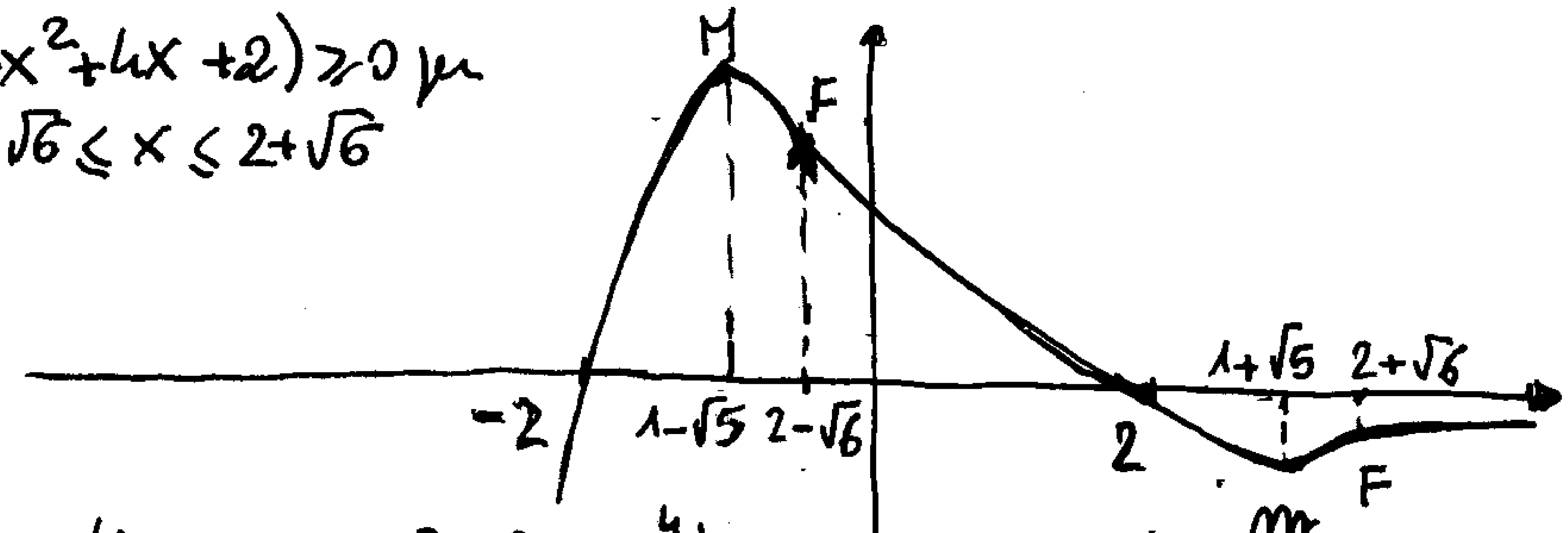
$x^2-2x-4\geq 0$: $x=1\pm\sqrt{1+4}=1\pm\sqrt{5}\Rightarrow x\leq 1-\sqrt{5}\cup x\geq 1+\sqrt{5}$



$f''(x)=-e^{1-x}(x^2-2x-4)+e^{1-x}\cdot(2x-2)=e^{1-x}\cdot(-x^2+4x+2)\geq 0$ per

$x^2-4x-2\leq 0$: $x=2\pm\sqrt{4+2}=2\pm\sqrt{6}\Rightarrow 2-\sqrt{6}\leq x\leq 2+\sqrt{6}$

Grafico:



2) $\lim_{x\to0}\frac{x^3-\log(1+x^3)-\sqrt[3]{x^4}}{\operatorname{sen}^2x+x^3}=\lim_{x\to0}\frac{-x^{\frac43}}{\operatorname{sen}^2x}=-\infty$ $\left(\begin{array}{l}\log(1+x^3)\sim x^3;\ x^3=o(x^{\frac43})\\ x^3=o(\operatorname{sen}^2x)\ ;\ \operatorname{sen}^2x=o(x^{\frac43})\end{array}\right)$.

$\lim_{x\to+\infty}(x+1)\cdot\operatorname{sen}\frac1x\Rightarrow\left|x=\frac1t\right|=\lim_{t\to0}(1+t)\cdot\frac{\operatorname{sen}t}{t}=1\cdot1=1$.

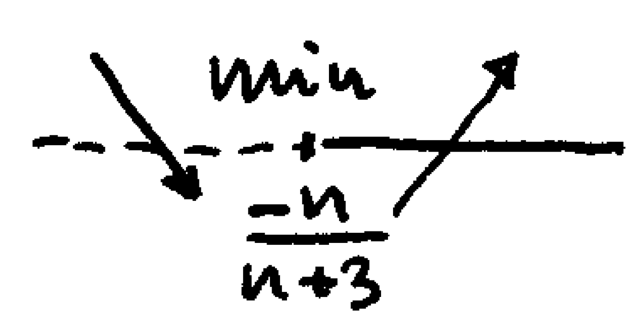
3) per $x=1$: $x^2-4x-k\Rightarrow 1-4-k=0\Rightarrow k=-3$. $\lim_{x\to1^-}\frac{x^5+3x^2-2x}{x^2-4x+3}=\lim_{x\to1^-}\frac{x^5+3x^2-2x}{(x-1)(x-3)}\Rightarrow$

$\Rightarrow\frac{(\to2)}{(\to0^-)(\to-2)}\Rightarrow\lim_{x\to1^-}f(x)=+\infty$.

4) $f(x)=x^n\cdot(x+1)^3$; $f(x)\in C(\mathbb{R})$. $f'(x)=n\cdot x^{n-1}\cdot(x+1)^3+3x^n\cdot(x+1)^2=x^{n-1}\cdot(x+1)^2\cdot(nx+n+3x)\geq 0$.

Per n dispari (n−1 pari)

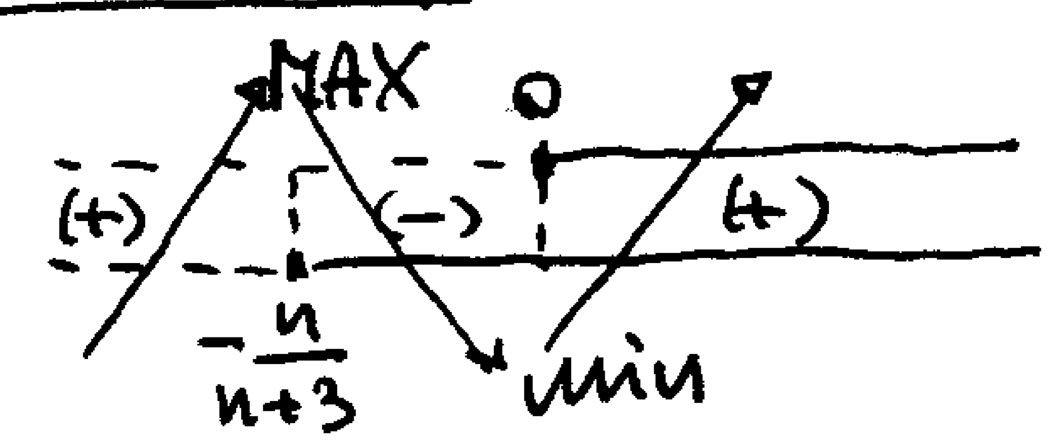
$nx+n+3x\geq 0$: $x\geq\frac{-n}{n+3}$



Per n pari (n−1 dispari)

$x\geq 0$: $x\geq 0$

$nx+n+3x\geq 0$: $x\geq\frac{-n}{n+3}$



Per n dispari punto di minimo in $x=\frac{-n}{n+3}$; per n pari punto di Max in $x=\frac{-n}{n+3}$ e punto di min in $x=0$.

5) $f(x;y)=e^{y^3-3y-x^2}$. $\nabla f(x;y)=\underline{0}\Rightarrow\begin{cases}f'_x=(-2x)e^{y^3-3y-x^2}=0\\ f'_y=3(y^2-1)\cdot e^{y^3-3y-x^2}=0\end{cases}\Rightarrow\begin{cases}x=0\\ y=1\end{cases}\cup\begin{cases}x=0\\ y=-1\end{cases}$.

$H(x;y)=\begin{Vmatrix}e^{y^3-3y-x^2}\cdot(4x^2-2) & e^{y^3-3y-x^2}\cdot(-2x)(3y^2-3)\\ e^{y^3-3y-x^2}\cdot(-2x)(3y^2-3) & e^{y^3-3y-x^2}\cdot\{(3y^2-3)^2+6y\}\end{Vmatrix}\Rightarrow H(0;1)=\begin{Vmatrix}-2e^{-2} & 0\\ 0 & 6e^{-2}\end{Vmatrix}$: Sella. $H(0;-1)=\begin{Vmatrix}-2e^{2} & 0\\ 0 & -6e^{2}\end{Vmatrix}$: MAX.

MGB2

6) $\int_0^1 \frac{1}{(x+1)^2} + e^{1-2x}\,dx = \left(-\frac{1}{x+1} - \frac{1}{2}e^{1-2x}\right|_0^1 = -\frac{1}{2} - \frac{1}{2}e^{-1} - \left(-1 - \frac{1}{2}e\right) = \frac{1}{2} - \frac{1}{2e} + \frac{1}{2}e.$

7) $f(x;y) = y e^{2y-x} - 3x$. $\nabla f(x;y) = \left(-y e^{2y-x} - 3 ; e^{2y-x} + 2y e^{2y-x}\right)$; $\nabla f(2;1) = (-4;3)$.

$(x;y) \perp (-4;3) \Rightarrow -4x + 3y = 0 \Rightarrow y = \frac{4}{3}x$. $\| x \ \frac{4}{3}x \| = \sqrt{x^2 + \frac{16}{9}x^2} = \sqrt{\frac{25}{9}x^2} = 5 \Rightarrow \frac{25}{9}x^2 = 25 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$.

I due vettori sono $V_1 = (3;4)$ e $V_2 = (-3;-4)$.

8) $f(x) = \frac{3^x + 2}{3^x - k}$; $g(x) = x + 2$. $f(g(x)) = \frac{3^{x+2} + 2}{3^{x+2} - k}$; $f(g(-1)) = \frac{3+2}{3-k} = \frac{5}{3-k} = 5 \Rightarrow 3 - k = 1 \Rightarrow k = 2$.

$y = \frac{3^{x+2} + 2}{3^{x+2} - 2} \Rightarrow y \cdot 3^{x+2} - 2y = 3^{x+2} + 2 \Rightarrow 3^{x+2} = \frac{2y+2}{y-1} \Rightarrow x + 2 = \log_3 \frac{2y+2}{y-1} \Rightarrow x = \log_3 \frac{2y+2}{y-1} - 2$.

Inversa: $y = \log_3 \frac{2y+2}{y-1} - 2$.

9) $f(x) = e^{kx+2}$. Bisettrice I e III quadrante: $y = x \Rightarrow m = 1 = f'(0)$. $f'(x) = k \cdot e^{kx+2} \Rightarrow$

$\Rightarrow f'(0) = k \cdot e^2 = 1$ per $k = \frac{1}{e^2} \Rightarrow f(x) = e^{\frac{1}{e^2} x + 2}$. $f(0) = e^2$; $f'(0) = 1 \Rightarrow$

$\Rightarrow$ Equazione retta tangente: $y - e^2 = 1 \cdot (x - 0) \Rightarrow y = x + e^2$.

10) P: B $\Rightarrow$ (A e non C)

A	B	C	non C	A e non C	B $\Rightarrow$ (A e non C)
1	1	1	0	0	0
1	1	0	1	1	1
1	0	1	0	0	1
1	0	0	1	1	1
0	1	1	0	0	0
0	1	0	1	0	0
0	0	1	0	0	1
0	0	0	1	0	1

La proposizione è falsa se ho il giorno libero, studio e lavoro; quando non ho il giorno libero ma studio e lavoro; quando non ho il giorno libero e studio e non lavoro.

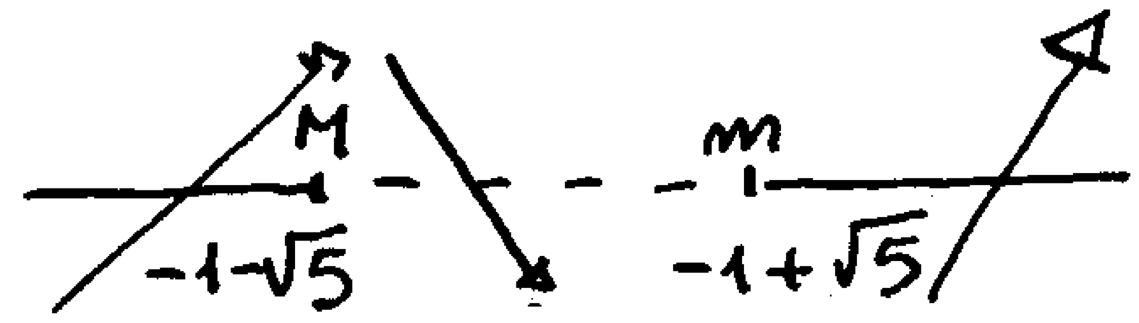
Compito di Matematica Generale del 27/1/2016 Compito C $\boxed{MGC1}$

1) $f(x)=(x^2-4)\cdot e^{x-1}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=0^+$; $\lim_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$: $x^2-4\geq 0 \Rightarrow x\leq -2 \cup x\geq 2$ (+) -2 (−) 2 (+)

$f'(x)=2x\cdot e^{x-1}+(x^2-4)\cdot e^{x-1}=e^{x-1}\cdot(x^2+2x-4)\geq 0$ per

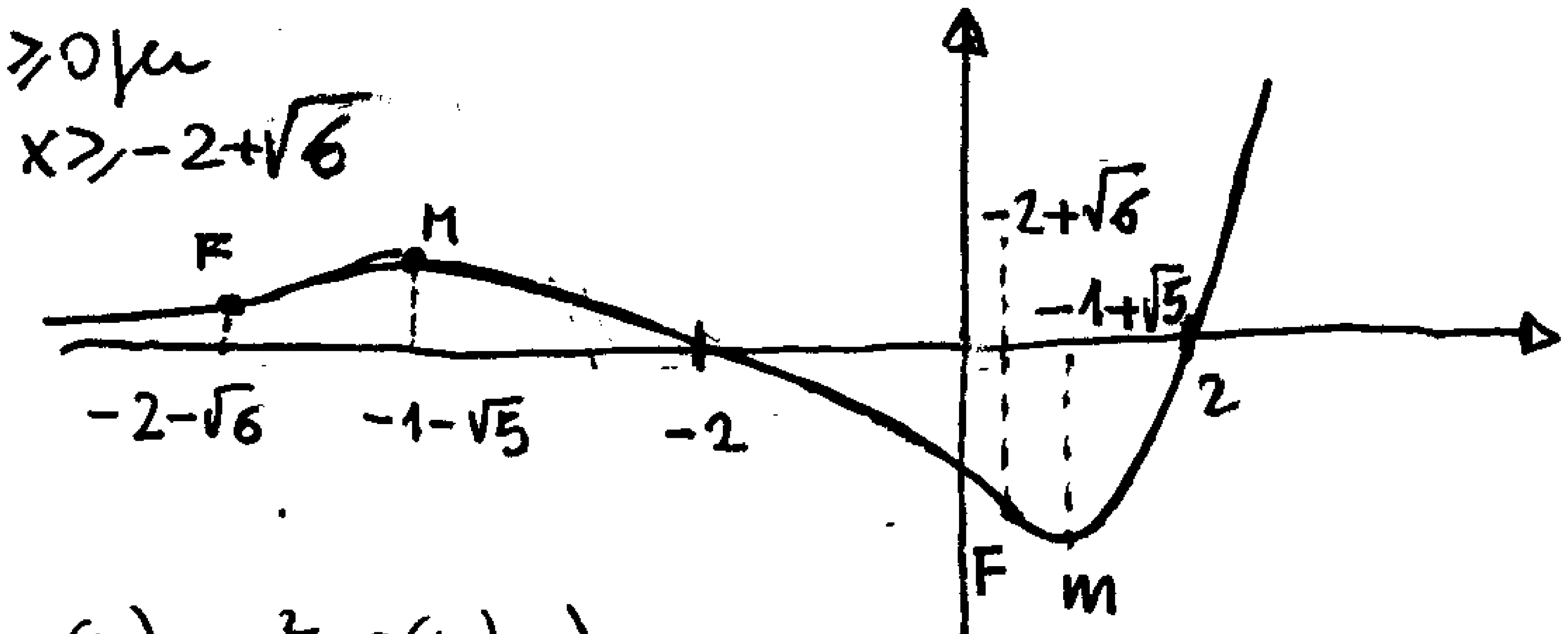
$x^2+2x-4\geq 0$: $x=-1\pm\sqrt{1+4}=-1\pm\sqrt5 \Rightarrow x\leq -1-\sqrt5 \cup x\geq -1+\sqrt5$



$f''(x)=e^{x-1}\cdot(x^2+2x-4)+e^{x-1}\cdot(2x+2)=e^{x-1}\cdot(x^2+4x-2)\geq 0$ per

$x^2+4x-2\geq 0$: $x=-2\pm\sqrt{4+2}=-2\pm\sqrt6 \Rightarrow x\leq -2-\sqrt6 \cup x\geq -2+\sqrt6$

Grafico:



2) $\lim_{x\to0}\dfrac{\sqrt[3]{x^4}-\log(1+x)+x^2}{\operatorname{sen}^2x+x}=\lim_{x\to0}\dfrac{-\log(1+x)}{x}=-1$ $\left(\begin{array}{l} x^{\frac43}=o(x);\ x^2=o(x)\\ \log(1+x)\sim x;\ \operatorname{sen}^2x=o(x)\end{array}\right)$

$\lim_{x\to+\infty} x\cdot\left(2^{\frac1x}-1\right)\Rightarrow\left(\frac1x=t\right)=\lim_{t\to0}\dfrac{2^t-1}{t}=\log 2.$

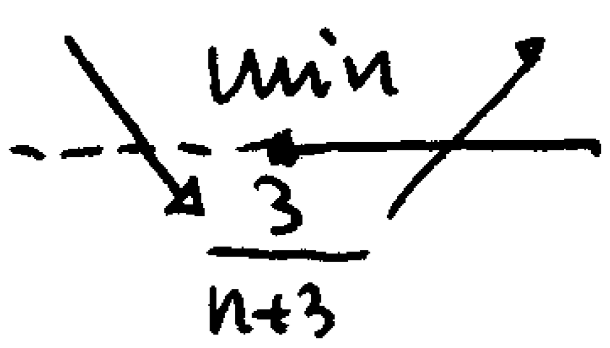
3) per $x=2$: $x^2-3x-k\Rightarrow 4-6-k=0\Rightarrow k=-2$. $\lim_{x\to2^-}\dfrac{x^2+3x-5}{x^2-3x+2}=\lim_{x\to2^-}\dfrac{x^2+3x-5}{(x-1)(x-2)}\Rightarrow$

$\Rightarrow\dfrac{(\to5)}{(\to1)(\to0^-)}=-\infty.$

4) $f(x)=x^3\cdot(x-1)^n$; $f(x)\in C(\mathbb{R})$. $f'(x)=3x^2\cdot(x-1)^n+n\cdot x^3(x-1)^{n-1}=x^2\cdot(x-1)^{n-1}\cdot(3x-3+nx)\geq 0.$

Per n dispari (n−1 pari)

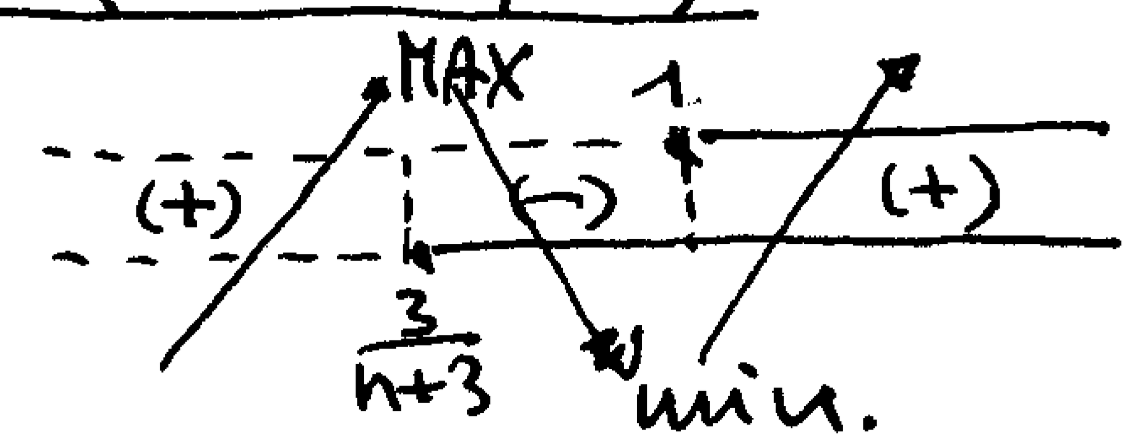
$3x-3+nx\geq 0$: $x\geq\dfrac{3}{n+3}$



Per n pari (n−1 dispari)

$x-1\geq 0$: $x\geq 1$

$3x-3+nx\geq 0$: $x\geq\dfrac{3}{n+3}$



Per n dispari punto di minimo per $x=\frac{3}{n+3}$; per n pari punto di Max in $x=\frac{3}{n+3}$ e punto di min in $x=1$.

5) $f(x;y)=e^{x^3-3x+y^2}$. $\nabla f=0\Rightarrow\begin{cases}f'_x=3(x^2-1)\cdot e^{x^3-3x+y^2}=0\\ f'_y=2y\cdot e^{x^3-3x+y^2}=0\end{cases}\Rightarrow\begin{cases}x=1\\y=0\end{cases}\cup\begin{cases}x=-1\\y=0\end{cases}.$

$H(x;y)=\left\|\begin{matrix}e^{x^3-3x+y^2}\cdot((3x^2-3)^2+6x) & e^{x^3-3x+y^2}\cdot(3x^2-3)(2y)\\ e^{x^3-3x+y^2}\cdot(3x^2-3)(2y) & e^{3x^3-3x+y^2}\cdot(4y^2+2)\end{matrix}\right\|\Rightarrow H(1;0)=\left\|\begin{matrix}6\cdot e^{-2} & 0\\ 0 & 2e^{-2}\end{matrix}\right\|$: Min. $H(-1;0)=\left\|\begin{matrix}-6e^2 & 0\\ 0 & 2e^2\end{matrix}\right\|$: Sella.

MGC2

6) $\int_0^1 e^{3x-1} - \frac{1}{(x+2)^3}\,dx = \left(\frac{1}{3} e^{3x-1} + \frac{1}{2}\frac{1}{(x+2)^2}\right|_0^1 = \frac{1}{3}e^2 + \frac{1}{18} - \left(\frac{1}{3}e^{-1} + \frac{1}{8}\right) = \frac{1}{3}e^2 - \frac{1}{3e} - \frac{5}{72}$.

7) $f(x;y) = 2y + xe^{y-2x}$. $\nabla f(x;y) = \left(e^{y-2x} - 2xe^{y-2x}; 2 + xe^{y-2x}\right)$; $\nabla f(1;2) = (-1;3)$.

$(x;y) \perp (-1;3) \Rightarrow -x+3y=0 \Rightarrow x=3y$. $\|3y\ y\| = \sqrt{9y^2+y^2} = \sqrt{10y^2} = \sqrt{20} \Rightarrow 10y^2 = 20 \Rightarrow y^2 = 2 \Rightarrow y = \pm\sqrt{2}$.

I due vettori sono $V_1 = (3\sqrt{2}; \sqrt{2})$ e $V_2 = (-3\sqrt{2}; -\sqrt{2})$.

8) $f(x) = \frac{2^x - k}{2^x + 4}$; $g(x) = 2-x$. $f(g(x)) = \frac{2^{2-x} - k}{2^{2-x}+4}$; $f(g(1)) = \frac{2-k}{2+4} = \frac{2-k}{6} = -1 \Rightarrow 2-k = -6 \Rightarrow k = 8$.

$y = \frac{2^{2-x}-8}{2^{2-x}+4} \Rightarrow y \cdot 2^{2-x} + 4y = 2^{2-x} - 8 \Rightarrow 2^{2-x} = \frac{4y+8}{1-y} \Rightarrow 2-x = \log_2 \frac{4y+8}{1-y} \Rightarrow x = 2 - \log_2 \frac{4y+8}{1-y}$.

Inversa: $y = 2 - \log_2 \frac{4x+8}{1-x}$.

9) $f(x) = e^{kx-1}$. Bisettrice I e III quadrante: $y = x \Rightarrow m = 1 = f'(0)$. $f'(x) = k \cdot e^{kx-1} \Rightarrow$

$\Rightarrow f'(0) = k \cdot e^{-1} = 1$ per $k = e \Rightarrow f(x) = e^{ex-1}$. $f(0) = \frac{1}{e}$; $f'(0) = 1 \Rightarrow$

$\Rightarrow$ Equazione retta tangente: $y - \frac{1}{e} = 1 \cdot (x-0) \Rightarrow y = x + \frac{1}{e}$.

10) P: (B e non C) $\Rightarrow$ A

A B C	non C	(B e non C)	(B e non C) $\Rightarrow$ A
1 1 1	0	0	1
1 1 0	1	1	1
1 0 1	0	0	1
1 0 0	1	0	1
0 1 1	0	0	1
0 1 0	1	1	0
0 0 1	0	0	1
0 0 0	1	0	1

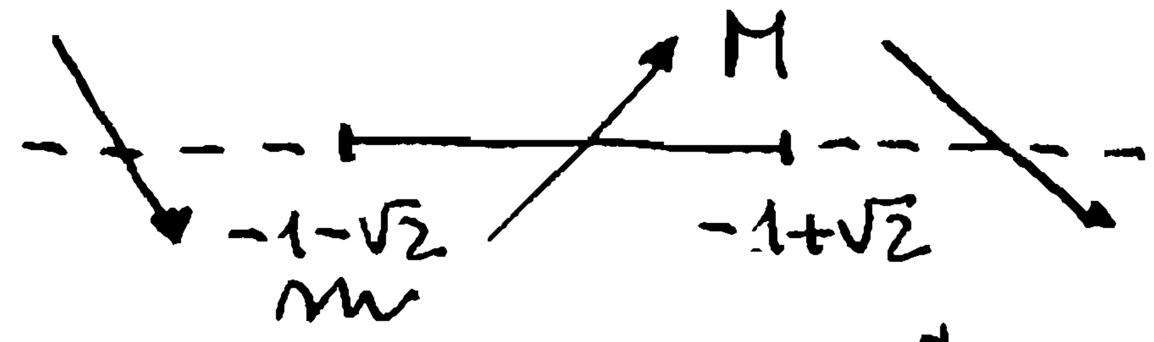
La proposizione è falsa se non ho il giorno libero, studio e non lavoro.

Compito di Matematica Generale del 27/1/2016 Compito D $\boxed{MGD1}$

1) $f(x)=(1-x^2)\cdot e^{x-2}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=0^-$; $\lim\limits_{x\to+\infty} f(x)=-\infty$.

$f(x)\geq 0$: $1-x^2\geq 0 \Rightarrow -1\leq x\leq 1$ $\quad (-)\ -1\ (+)\ 1\ (-)$

$f'(x)=-2x\cdot e^{x-2}+(1-x^2)\cdot e^{x-2}=e^{x-2}\cdot(-x^2-2x+1)\geq 0$ per
$x^2+2x-1\leq 0$: $x=-1\pm\sqrt{1+1}=-1\pm\sqrt{2} \Rightarrow -1-\sqrt{2}\leq x\leq -1+\sqrt{2}$



$f''(x)=e^{x-2}\cdot(-x^2-2x+1)+e^{x-2}\cdot(-2x-2)=e^{x-2}\cdot(-x^2-4x-1)\geq 0$ per
$x^2+4x+1\leq 0$: $x=-2\pm\sqrt{4-1}=-2\pm\sqrt{3} \Rightarrow -2-\sqrt{3}\leq x\leq -2+\sqrt{3}$

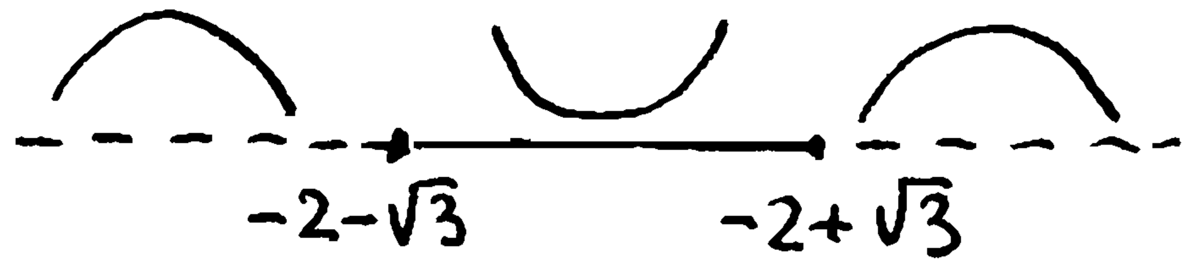
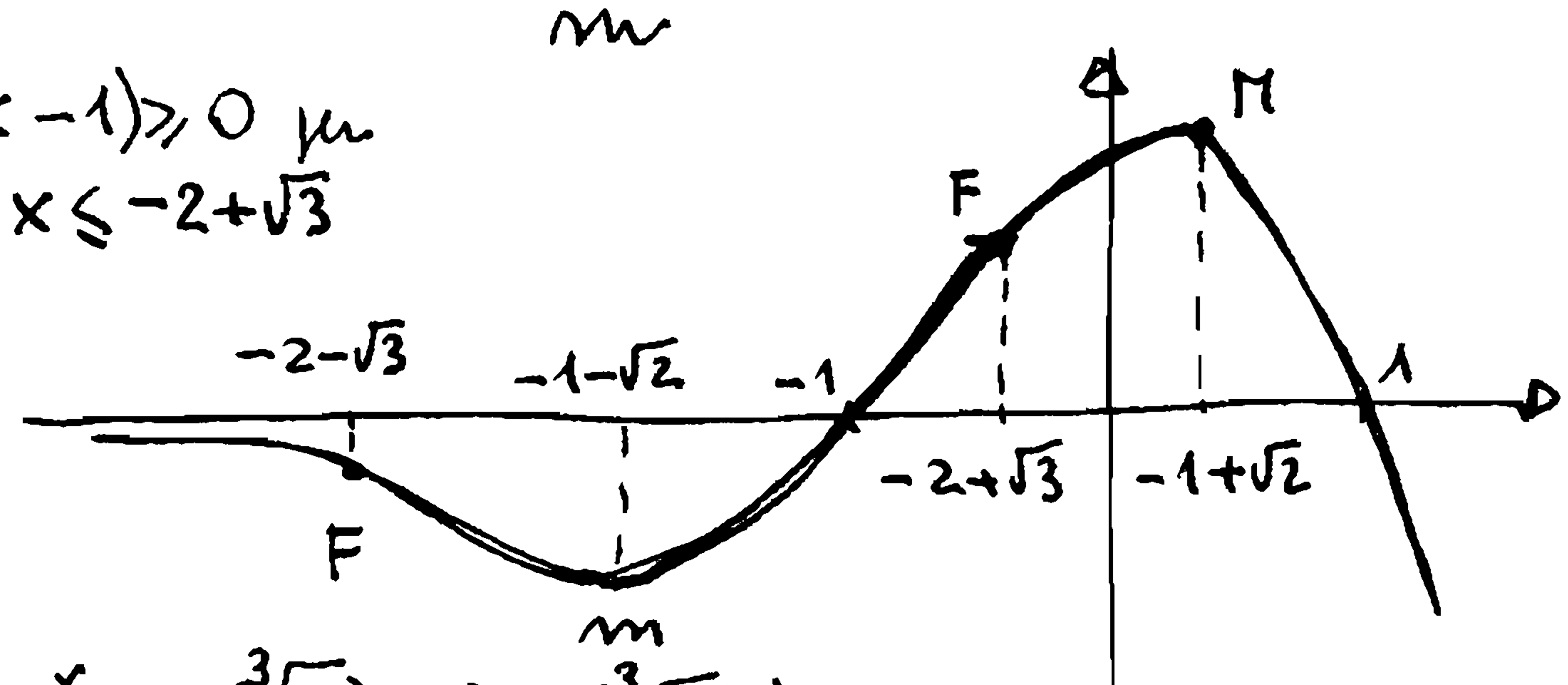


Grafico:



2) $\lim\limits_{x\to 0}\dfrac{3^x-1+\sqrt[3]{x}-x^2}{\operatorname{sen}x+x^3}=\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x}}{\operatorname{sen}x}=+\infty$ $\left(\begin{array}{l}3^x-1\simeq x;\ 3^x-1=o(\sqrt[3]{x});\ x^2=o(\sqrt[3]{x})\\ x^3=o(\operatorname{sen}x);\ \operatorname{sen}x=o(\sqrt[3]{x})\end{array}\right)$

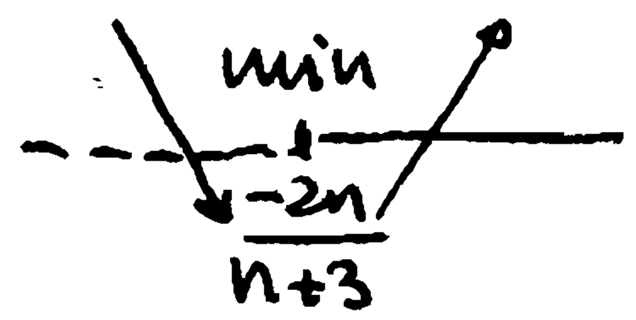
$\lim\limits_{x\to+\infty} x^2\cdot\left(1-\cos\frac{1}{x}\right)\Rightarrow\left(\frac{1}{x}=t\right)=\lim\limits_{t\to 0}\dfrac{1-\cos t}{t^2}=\dfrac{1}{2}$.

3) per $x=1$: $x^2-5x-k\Rightarrow 1-5-k=0\Rightarrow k=-4$. $\lim\limits_{x\to 1^+}\dfrac{x^3-3x+3}{x^2-5x+4}=\lim\limits_{x\to 1^+}\dfrac{x^3-3x+3}{(x-1)(x-4)}\Rightarrow$
$\Rightarrow\dfrac{(\to 1)}{(\to 0^+)(\to -3)}=-\infty$.

4) $f(x)=x^n\cdot(x+2)^3$; $f(x)\in C(\mathbb{R})$. $f'(x)=n\cdot x^{n-1}\cdot(x+2)^3+3\cdot x^n\cdot(x+2)^2=x^{n-1}\cdot(x+2)^2\cdot(nx+2n+3x)\geq 0$.

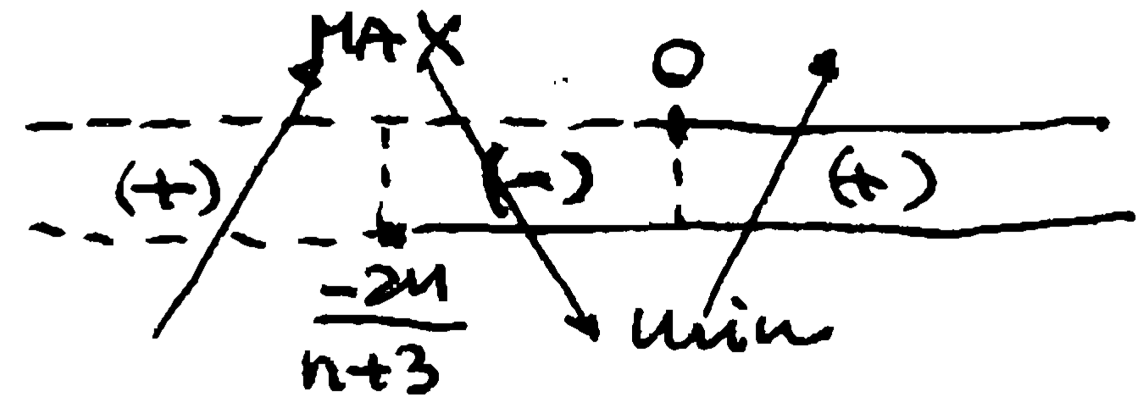
Per n dispari ($n-1$ pari)

$nx+2n+3x\geq 0$: $x\geq\dfrac{-2n}{n+3}$



Per n pari ($n-1$ dispari)

$x\geq 0$
$nx+2n+3x\geq 0$: $x\geq\dfrac{-2n}{n+3}$



Per n dispari punto di minimo per $x=\dfrac{-2n}{n+3}$; per n pari punto di Max per $x=\dfrac{-2n}{n+3}$ e punto di min in $x=0$.

5) $f(x;y)=e^{3x-x^3-y^2}$. $\nabla f(x;y)=\mathbb{0}\Rightarrow\begin{cases}f'_x=3(1-x^2)e^{3x-x^3-y^2}=0\\ f'_y=(-2y)\cdot e^{3x-x^3-y^2}=0\end{cases}\Rightarrow\begin{cases}x=1\\y=0\end{cases}\cup\begin{cases}x=-1\\y=0\end{cases}$.

$H(x;y)=\begin{Vmatrix}e^{3x-x^3-y^2}\cdot((3-3x^2)^2-6x) & e^{3x-x^3-y^2}\cdot(-2y)(3-3x^2)\\ e^{3x-x^3-y^2}\cdot(-2y)(3-3x^2) & e^{3x-x^3-y^2}\cdot(4y^2-2)\end{Vmatrix}\Rightarrow H(1;0)=\begin{Vmatrix}-6e^2 & 0\\ 0 & -2e^2\end{Vmatrix}$: MAX; $H(-1;0)=\begin{Vmatrix}6e^{-2} & 0\\ 0 & -2e^{-2}\end{Vmatrix}$: Sella.

MGD2

6) $\int_0^1 \frac{1}{(x-3)^2} + e^{2x-3}\,dx = \left(-\frac{1}{x-3} + \frac{1}{2}e^{2x-3}\right|_0^1 = +\frac{1}{2} + \frac{1}{2}e^{-1} - \left(\frac{1}{3} + \frac{1}{2}e^{-3}\right) = \frac{1}{6} + \frac{1}{2e} - \frac{1}{2e^3}$.

7) $f(x;y) = 3x + ye^{y-3x}$. $\nabla f(x;y) = \left(3 - 3ye^{y-3x}; e^{y-3x} + y \cdot e^{y-3x}\right)$; $\nabla f(1;3) = (-6;4)$.

$(x;y) \perp (-6;4) \Rightarrow -6x + 4y = 0 \Rightarrow y = \frac{3}{2}x$. $\left\|x\ \frac{3}{2}x\right\| = \sqrt{x^2 + \frac{9}{4}x^2} = \sqrt{\frac{13}{4}x^2} = \sqrt{13} \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$.

I due vettori sono $V_1 = (2;3)$ e $V_2 = (-2;-3)$.

8) $f(x) = \frac{3^x - 1}{3^x + k}$; $g(x) = x - 1$. $f(g(x)) = \frac{3^{x-1} - 1}{3^{x-1} + k}$; $f(g(2)) = \frac{3-1}{3+k} = \frac{2}{3+k} = 2 \Rightarrow 3 + k = 1 \Rightarrow k = -2$.

$y = \frac{3^{x-1} - 1}{3^{x-1} - 2} \Rightarrow y \cdot 3^{x-1} - 2y = 3^{x-1} - 1 \Rightarrow 3^{x-1} = \frac{2y-1}{y-1} \Rightarrow x - 1 = \log_3 \frac{2y-1}{y-1} \Rightarrow x = 1 + \log_3 \frac{2y-1}{y-1}$.

Inversa: $y = 1 + \log_3 \frac{2x-1}{x-1}$.

9) $f(x) = e^{2-kx}$. Bisettrice I e III quadrante: $y = x \Rightarrow m = 1 = f'(0)$. $f'(x) = -k \cdot e^{2-kx} \Rightarrow$

$\Rightarrow f'(0) = -k \cdot e^2 = 1$ per $k = -\frac{1}{e^2} \Rightarrow f(x) = e^{2 + \frac{1}{e^2} \cdot x}$. $f(0) = e^2$; $f'(0) = 1 \Rightarrow$

$\Rightarrow$ Equazione retta tangente: $y - e^2 = 1 \cdot (x - 0) \Rightarrow y = x + e^2$.

10) P: C $\Rightarrow$ (non A e non B)

A	B	C	non A	non B	(non A e non B)	C $\Rightarrow$ (non A e non B)
1	1	1	0	0	0	0
1	1	0	0	0	0	1
1	0	1	0	1	0	0
1	0	0	0	1	0	1
0	1	1	1	0	0	0
0	1	0	1	0	0	1
0	0	1	1	1	1	1
0	0	0	1	1	1	1

La proposizione è falsa quando ho il giorno libero, studio e lavoro; quando ho il giorno libero, non studio ma lavoro; quando non ho il giorno libero ma studio e lavoro.