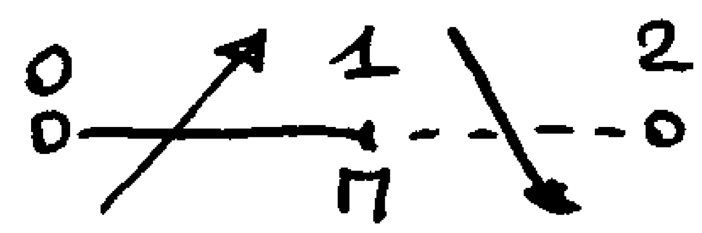
Compito di Matematica Generale del 19/3/2016 $\boxed{MG1}$

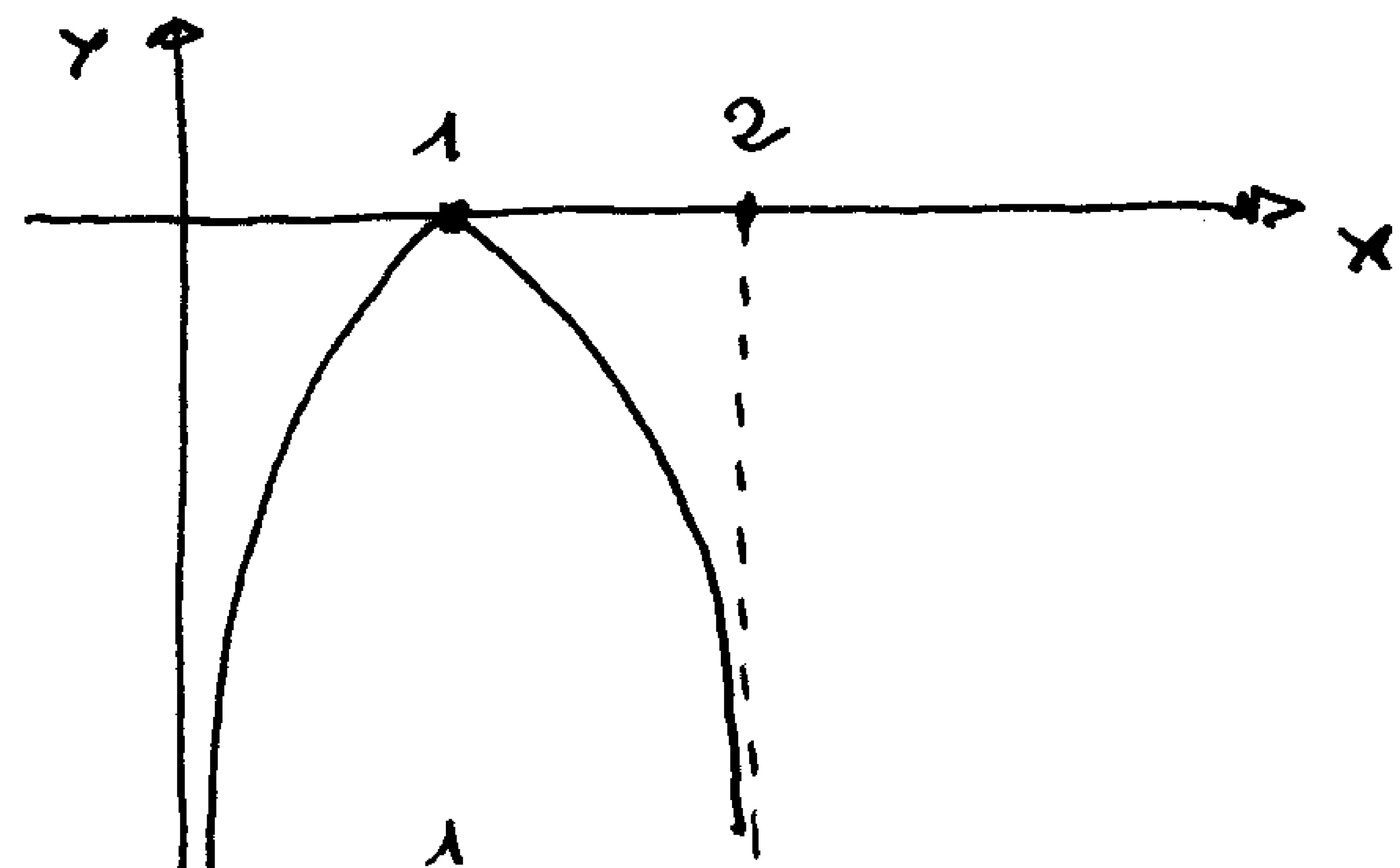
1) $f(x)=\log(2x-x^2)$. C.E.: $2x-x^2=x(2-x)>0 \Rightarrow 0<x<2$. $\lim_{x\to0^+} f(x)=\lim_{x\to2^-} f(x)=-\infty$.

$f(x)\geq 0$: $2x-x^2\geq 1 \Rightarrow x^2-2x+1\leq 0 \Rightarrow (x-1)^2\leq 0$ vera solo per $x=1$. $f(x)\leq 0$ in C.E..

$f'(x)=\frac{2-2x}{2x-x^2}\geq 0 \Rightarrow 2(1-x)\geq 0$: $x\leq 1$



Grafico



$$f''(x)=2\cdot\frac{(-1)(2x-x^2)-(1-x)(2-2x)}{(2x-x^2)^2}=$$

$$=2\cdot\frac{-2x+x^2-2+2x+2x-2x^2}{(2x-x^2)^2}=2\cdot\frac{-x^2+2x-2}{(2x-x^2)^2}\geq 0$$

per $x^2-2x+2\leq 0$: $\Delta=1-2<0 \Rightarrow f''(x)<0\ \forall x\in$ C.E.

2) $\lim_{x\to0}\frac{\sqrt{1+\sin x}-1}{e^x-1}=\lim_{x\to0}\frac{(1+\sin x)^{\frac12}-1}{\sin x}\cdot\frac{\sin x}{x}\cdot\frac{x}{e^x-1}=\lim_{t\to0}\frac{(1+t)^{\frac12}-1}{t}\cdot1\cdot1=\frac12\cdot1=\frac12$.

$\lim_{x\to0}\frac{\sqrt[3]{x^2}-\sin x+x^2}{x-3\sqrt[3]{x^2}}=\lim_{x\to0}\frac{\sqrt[3]{x^2}}{-3\sqrt[3]{x^2}}=-\frac13\cdot\left(\sin x\sim x=o\left(x^{\frac23}\right);\ x^2=o\left(x^{\frac23}\right)\right)$

3) $f(x)=\frac{1}{e^{\frac1x}-1}$. C.E.: $\begin{cases}x\neq0\\ e^{\frac1x}-1\neq0\end{cases}\Rightarrow\begin{cases}x\neq0\\ \frac1x\neq0\ \forall x\end{cases}\Rightarrow$ C.E. $\mathbb{R}\setminus\{0\}$.

$\lim_{x\to0^-} f(x)=\left(\frac{1}{e^{(\to-\infty)}-1}\right)=\frac{1}{-1}=-1$; $\lim_{x\to0^+} f(x)=\left(\frac{1}{e^{(\to+\infty)}-1}\right)=\left(\frac{1}{\to+\infty}\right)=0^+$.

$x=0$ è un punto di discontinuità di I° specie.

4) $f(x)=x^2-3x+1$; $f'(x)=2x-3$; $f'(1)=-1$. $g(x)=kx-x^2$; $g'(x)=k-2x$; $g'(1)=k-2$.

Tangenti parallele se: $-1=k-2\Rightarrow k=1\Rightarrow g(x)=x-x^2$. $f(1)=-1$; $g(1)=0\Rightarrow$

Equazione tangente a $f(x)$: $y+1=-1(x-1)\Rightarrow y=-x$; Eq. tang. a $g(x)$: $y-0=-1(x-1)\Rightarrow y=-x+1$.

5) A: $f(x)=\log(2-3x)\Rightarrow$ C.E. $2-3x>0\Rightarrow x<\frac23$: A è FALSA;
B: $g(x)=e^{x^2-2x}\Rightarrow g'(x)=(2x-2)e^{x^2-2x}\geq0\Rightarrow x\geq1$: B è FALSA;

A B C	(A ⇒ B)	non B	(non B ⇒ C)	(A ⇒ B) e (non B ⇒ C)
0 0 1	1	1	1	1
0 0 0	1	1	0	0

MG2

6) $A \cdot B \cdot X = \begin{Vmatrix} 1 & -1 \\ -1 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & K \\ K & -1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1-K & K+1 \\ -1+K & -K-1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1-K+K+1 \\ -1+K-K-1 \end{Vmatrix} = \begin{Vmatrix} 2 \\ -2 \end{Vmatrix} \quad \forall K \in \mathbb{R}.$

7) $\int_0^1 e^{2x} + e^{K-x} dx = \left(\frac{1}{2} e^{2x} - e^{K-x} \right|_0^1 = \cancel{\frac{1}{2} e^2} - e^{K-1} - \left(\frac{1}{2} - e^K \right) = \cancel{\frac{1}{2} e^2} \Rightarrow e^K - e^{K-1} = \frac{1}{2} \Rightarrow e^K \cdot \left(1 - \frac{1}{e}\right) = \frac{1}{2} \Rightarrow$

$\Rightarrow e^K \cdot \frac{e-1}{e} = \frac{1}{2} \Rightarrow e^K = \frac{e}{2(e-1)} \Rightarrow K = \log \frac{e}{2(e-1)}.$

8) $f(x;y) = x^2 y - xy^2 + 2xy$. $\nabla f = \underline{0} \Rightarrow \begin{cases} f'_x = 2xy - y^2 + 2y = y \cdot (2x - y + 2) = 0 \\ f'_y = x^2 - 2xy + 2x = x \cdot (x - 2y + 2) = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \cup \begin{cases} y = 0 \\ x = -2 \end{cases} \cup \begin{cases} y = 2 \\ x = 0 \end{cases} \cup \begin{cases} y = 2x+2 \\ x - 4x - 4 + 2 = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{2}{3} \\ y = \frac{2}{3} \end{cases}$. $H(x;y) = \begin{Vmatrix} 2y & 2x - 2y + 2 \\ 2x - 2y + 2 & -2x \end{Vmatrix}.$

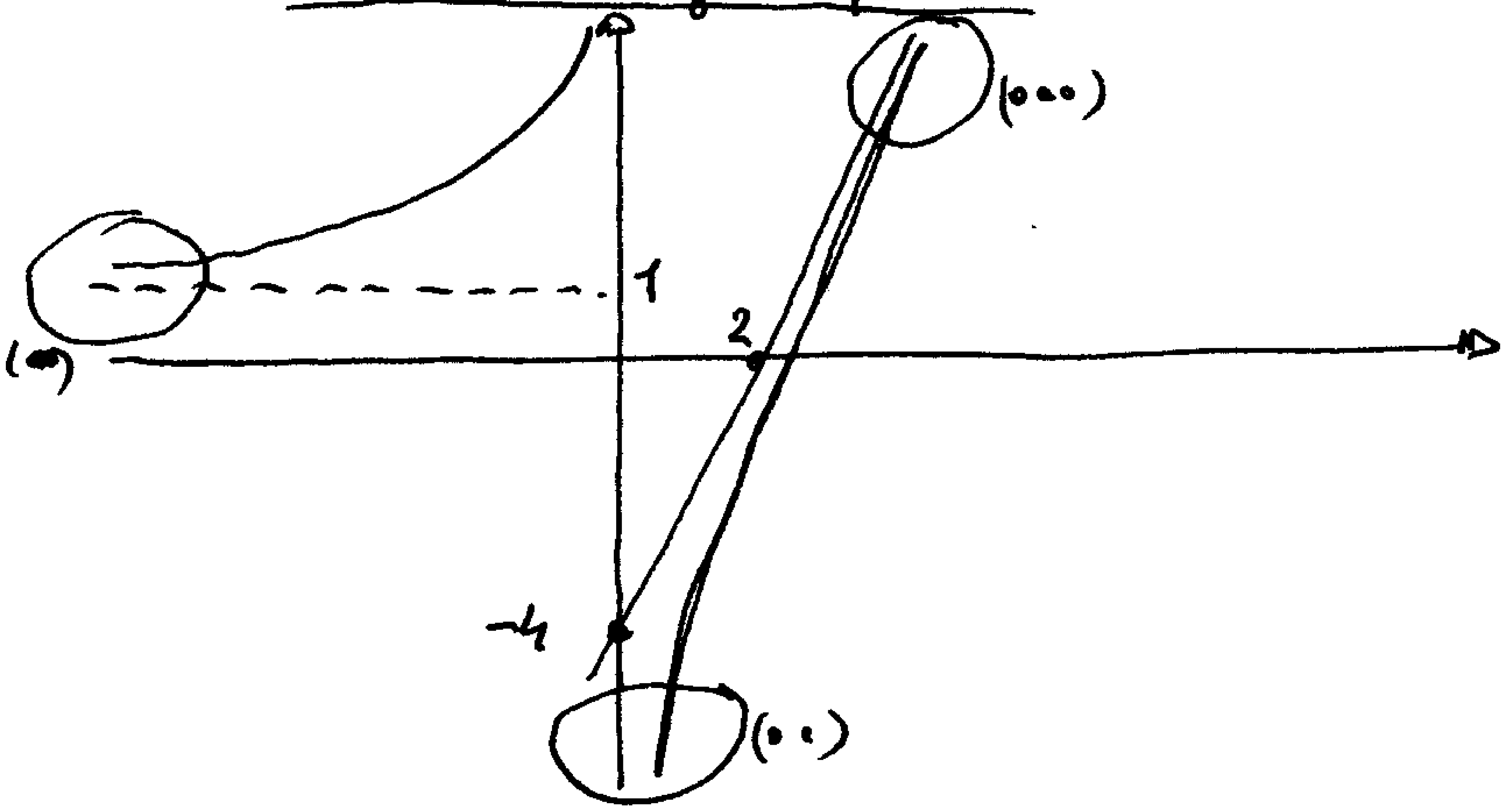
$H(0;0) = \begin{Vmatrix} 0 & 2 \\ 2 & 0 \end{Vmatrix}$: Sella; $H(-2;0) = \begin{Vmatrix} 0 & -2 \\ -2 & 4 \end{Vmatrix}$: Sella; $H(0;2) = \begin{Vmatrix} 4 & -2 \\ -2 & 0 \end{Vmatrix}$: Sella; $H\left(-\frac{2}{3}; \frac{2}{3}\right) = \begin{Vmatrix} \frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{Vmatrix}$: <u>MINIMO</u>

9) $\cdot \forall \varepsilon > 0 \ \exists \delta(\varepsilon) : x < \delta(\varepsilon) \Rightarrow 1 < f(x) < 1 + \varepsilon$:
$\lim_{x \to -\infty} f(x) = 1^+$

$\cdot\cdot \forall \varepsilon \ \exists \delta(\varepsilon) : 0 < x < \delta(\varepsilon) \Rightarrow f(x) < \varepsilon$:
$\lim_{x \to 0^+} f(x) = -\infty$

$\cdot\cdot\cdot$ Asintoto obliquo sulla destra di equazione $y = 2x - 4$

<u>Possibile grafico</u>



10) $f(x) = \frac{x-1}{x+2}$; $g(x) = \frac{1}{x}$.

$K(x) = f(g(f(x))) = f\left(g\left(\frac{x-1}{x+2}\right)\right) = f\left(\frac{x+2}{x-1}\right) = \frac{\frac{x+2}{x-1} - 1}{\frac{x+2}{x-1} + 2} = \frac{\frac{x+2-x+1}{x-1}}{\frac{x+2+2x-2}{x-1}} = \frac{\frac{3}{x-1}}{\frac{3x}{x-1}} = \frac{3}{3x} = \frac{1}{x} = g(x).$

$H(x) = g(f(g(x))) = g\left(f\left(\frac{1}{x}\right)\right) = g\left(\frac{\frac{1}{x} - 1}{\frac{1}{x} + 2}\right) = g\left(\frac{\frac{1-x}{x}}{\frac{1+2x}{x}}\right) = g\left(\frac{1-x}{1+2x}\right) = \frac{1+2x}{1-x} = y \Rightarrow$

$\Rightarrow 1 + 2x = y - xy \Rightarrow 2x + xy = y - 1 \Rightarrow x(2+y) = y - 1 \Rightarrow x = \frac{y-1}{y+2}.$

Quindi $H^{-1}(x) : y = \frac{x-1}{x+2} = f(x).$