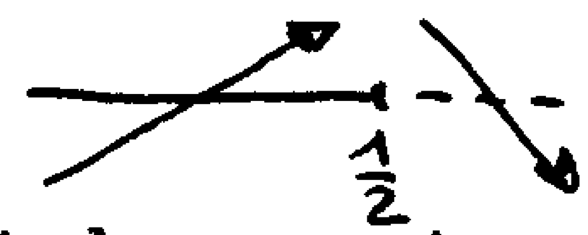
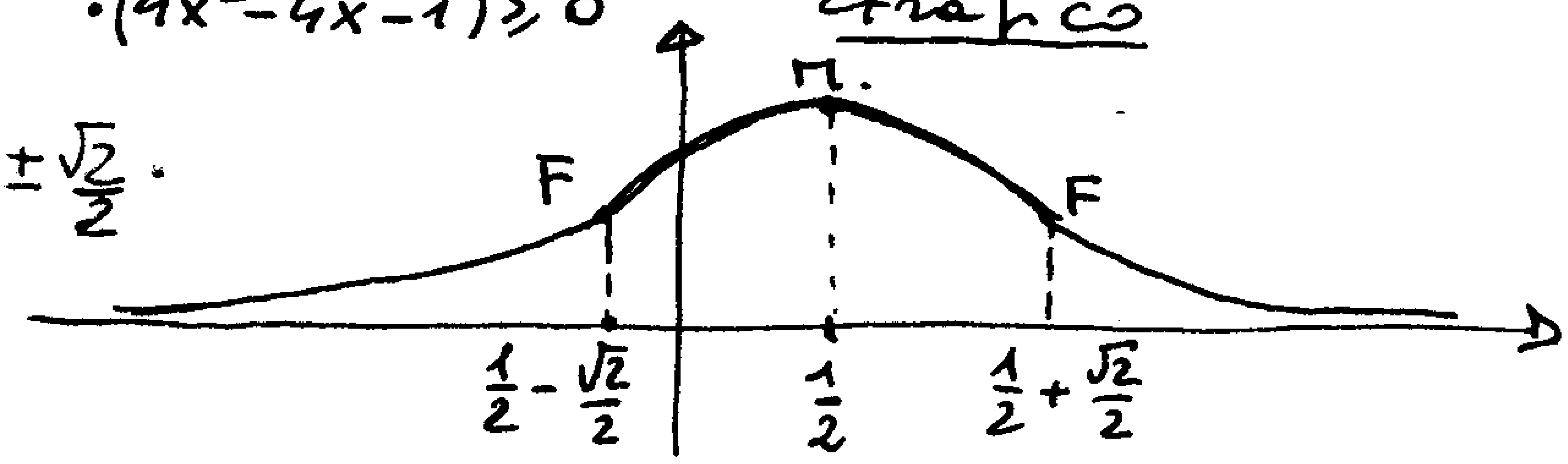
Compito di Matematica Generale del 27/6/2016 $\boxed{\text{CMG1}}$

1) $f(x)=e^{1+x-x^2}$. C.E.: $\mathbb{R}$. $\lim_{x\to+\infty} f(x)=\lim_{x\to-\infty} f(x)=0^+$. $f(x)>0\ \forall x\in\mathbb{R}$.

$f'(x)=e^{1+x-x^2}\cdot(1-2x)\geq 0$ per $x\leq\frac{1}{2}$:  . $f(\frac{1}{2})=e^{\frac{5}{4}}=\sqrt[4]{e^5}$

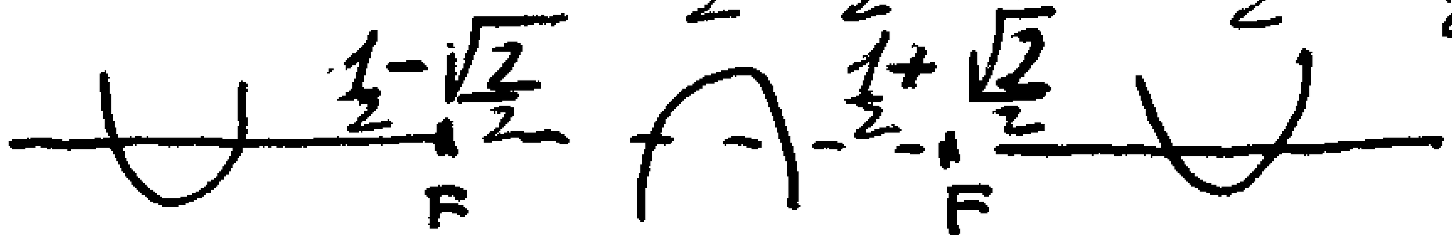
$f''(x)=e^{1+x-x^2}\cdot((1-2x)^2+(-2)) = e^{1+x-x^2}\cdot(4x^2-4x-1)\geq 0$

Grafico



$x=\frac{2\pm\sqrt{4+4}}{4}=\frac{2\pm\sqrt{8}}{4}=\frac{2\pm2\sqrt{2}}{4}=\frac{1}{2}\pm\frac{\sqrt{2}}{2}$.

$f''(x)\geq 0$ per $x\leq\frac{1}{2}-\frac{\sqrt{2}}{2}\cup x\geq\frac{1}{2}+\frac{\sqrt{2}}{2}$



2) $\lim_{x\to0}\frac{e^{2x}-\cos x}{x}=\lim_{x\to0}\frac{e^{2x}-1}{2x}\cdot 2+\frac{1-\cos x}{x}=1\cdot2+0=2$.

$\lim_{x\to0}\frac{x^2-\operatorname{sen}^3x+x}{x-\sqrt[3]{x^4}}=\lim_{x\to0}\frac{x}{x}=\lim_{x\to0}1=1$. $\left(\begin{matrix}x^2=o(x);\ \operatorname{sen}^3x\sim x^3=o(x)\\ \sqrt[3]{x^4}=x^{\frac{4}{3}}=o(x)\end{matrix}\right)$.

3) $f(x)=\log(e^{2x}+e^x)$. $e^{2x}+e^x>0\ \forall x\in\mathbb{R}\Rightarrow$ C.E.$=\mathbb{R}$. $f'(x)=\frac{2e^{2x}+e^x}{e^{2x}+e^x}>0\ \forall x\in\mathbb{R}$.

$\lim_{x\to-\infty}f(x)=-\infty$; $\lim_{x\to+\infty}f(x)=+\infty$. $f:\mathbb{R}\to\mathbb{R}$ invertibile su tutto $\mathbb{R}$. $f^{-1}:\mathbb{R}\to\mathbb{R}$.

$\log(e^{2x}+e^x)=y\Rightarrow e^{2x}+e^x=e^y\Rightarrow e^{2x}+e^x-e^y=0\Rightarrow e^x=\frac{-1\pm\sqrt{1+4e^y}}{2}\Rightarrow$

$\Rightarrow e^x=\frac{\sqrt{1+4e^y}-1}{2}$ $\left(\frac{-1-\sqrt{1+4e^y}}{2}<0 \text{ non accettabile}\right)\Rightarrow x=\log\left(\frac{\sqrt{1+4e^y}-1}{2}\right)\Rightarrow$

$\Rightarrow$ Inversa : $y=\log\left(\frac{\sqrt{1+4e^x}-1}{2}\right)$.

4) $f(x)=e^x$; $g(x)=x^2$. $h(x)=\frac{f(g(x))}{g(f(x))}=\frac{f(x^2)}{(f(x))^2}=\frac{e^{x^2}}{(e^x)^2}=\frac{e^{x^2}}{e^{2x}}$.

$h'(x)=\frac{e^{x^2}\cdot2x\cdot e^{2x}-e^{x^2}\cdot2e^{2x}}{(e^{2x})^2}=\frac{e^{x^2}\cdot e^{2x}\cdot(2x-2)}{e^{4x}}=0\Rightarrow 2x-2=0\Rightarrow x=1$.

5) $f(x;y)=x^2y-xy+xy^2$. $\nabla f(x;y)=(0;0)\Rightarrow\begin{cases}f'_x=2xy-y+y^2=y(2x-1+y)=0\\ f'_y=x^2-x+2xy=x\cdot(x-1+2y)=0\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}y=0\\x=0\end{cases}\cup\begin{cases}y=0\\x-1=0\Rightarrow x=1\end{cases}\cup\begin{cases}-1+y=0\\x=0\end{cases}\Rightarrow\begin{cases}x=0\\y=1\end{cases}\cup\begin{cases}y=1-2x\\x-1+2-4x=0\end{cases}\Rightarrow\begin{cases}y=1-2x\\3x=1\end{cases}\Rightarrow\begin{cases}x=\frac{1}{3}\\y=\frac{1}{3}\end{cases}$.

$P_0=(0;0)$; $P_1=(1;0)$; $P_2=(0;1)$; $P_3=\left(\frac{1}{3};\frac{1}{3}\right)$. $H(x;y)=\left\|\begin{matrix}2y & 2x-1+2y\\ 2x-1+2y & 2x\end{matrix}\right\|$.

CMG2

$H(0;0)=\begin{Vmatrix}0 & -1\\ -1 & 0\end{Vmatrix}: 0-1<0$: P. Sella. ; $H(1;0)=\begin{Vmatrix}0 & 1\\ 1 & 2\end{Vmatrix}: 0\cdot 2-1<0$: P. Sella;

$H(0;1)=\begin{Vmatrix}2 & 1\\ 1 & 0\end{Vmatrix}: 2\cdot 0-1<0$: P. Sella ; $H(\frac{1}{3};\frac{1}{3})=\begin{Vmatrix}\frac{2}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{2}{3}\end{Vmatrix}: \begin{cases}\frac{2}{3}>0\\ \frac{4}{9}-\frac{1}{9}>0\end{cases}$: Punto di Minimo.

6) $A\cdot X=Y \Rightarrow \begin{Vmatrix}1 & 0 & 1\\ 0 & -1 & 1\\ 1 & -1 & 0\end{Vmatrix}\cdot\begin{Vmatrix}1\\ K\\ -1\end{Vmatrix}=\begin{Vmatrix}1\cdot 1+0\cdot K+1\cdot(-1)\\ 0\cdot 1+(-1)\cdot K+1\cdot(-1)\\ 1\cdot 1+(-1)\cdot K+0\cdot(-1)\end{Vmatrix}=\begin{Vmatrix}1-1\\ -K-1\\ 1-K\end{Vmatrix}=\begin{Vmatrix}0\\ -1-K\\ 1-K\end{Vmatrix}=Y.$

$\|Y\|=\sqrt{0^2+(-1-K)^2+(1-K)^2}=\sqrt{1+K^2+\cancel{2K}+1+K^2-\cancel{2K}}=\sqrt{2(1+K^2)}=\sqrt{10}\Rightarrow 1+K^2=5\Rightarrow K^2=4\Rightarrow K=\pm 2.$

7) $\int_0^1 e^{2x-1}-e^{1-x}\,dx=\left(\frac{1}{2}e^{2x-1}+e^{1-x}\right|_0^1=\frac{1}{2}e+1-\left(\frac{1}{2}e^{-1}+e\right)=1-\frac{1}{2e}-\frac{1}{2}e=\frac{2e-1-e^2}{2e}<0.$

8)

A	B	C	$A\cap C$	$A\cup C$	$(A\cup C)\setminus B$
1	1	1	1	1	0 *
1	1	0	0	1	0
1	0	1	1	1	1
1	0	0	0	1	1
0	1	1	0	1	0
0	1	0	0	0	0
0	0	1	0	1	1
0	0	0	0	0	0

* La I riga contraddice il fatto che $A\cap C$ sia un sottoinsieme di $(A\cup C)\setminus B$. Se allora vogliamo che un elemento stia sia in A che in B che in C, ovvero se $A\cap B\cap C=\phi$, allora $A\cap C$ è un sottoinsieme di $(A\cup C)\setminus B$.

9) $f(x)=\frac{x^3}{x^2-1}$. C.E.: $x^2-1\neq 0\Rightarrow x^2\neq 1\Rightarrow x\neq\pm 1$. $\lim_{x\to-\infty}f(x)=-\infty$; $\lim_{x\to+\infty}f(x)=+\infty$.

$\lim_{x\to-1^-}f(x)=-\infty$; $\lim_{x\to-1^+}f(x)=+\infty$: $x=-1$ Asintoto verticale.

$\lim_{x\to 1^-}f(x)=-\infty$; $\lim_{x\to 1^+}f(x)=+\infty$: $x=1$ Asintoto verticale.

$\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{x^3}{x^3-x}=1=(m?)\Rightarrow\lim_{x\to\pm\infty}f(x)-1\cdot x=\lim_{x\to\pm\infty}\frac{x^3}{x^2-1}-x=\lim_{x\to\pm\infty}\frac{x^3-x^3+x}{x^2-1}=$

$=\lim_{x\to\pm\infty}\frac{x}{x^2-1}=0=q\Rightarrow y=1\cdot x+0=x$ è asintoto obliquo a destra e a sinistra.

10) $e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+o(x^3)\Rightarrow e^{2x}=1+2x+\frac{4x^2}{2}+\frac{8x^3}{6}+o(x^3).$

$x\cdot e^x=x+x^2+\frac{x^3}{2}+\frac{x^4}{3!}+o(x^4)\Rightarrow f(x)=e^{2x}-xe^x=$

$=1+2x+2x^2+\frac{4}{3}x^3+o(x^3)-x-x^2-\frac{x^3}{2}-\frac{x^4}{6}-o(x^4)\Rightarrow$

$\Rightarrow f(x)=1+x+x^2+\frac{5}{6}x^3+o(x^3).$