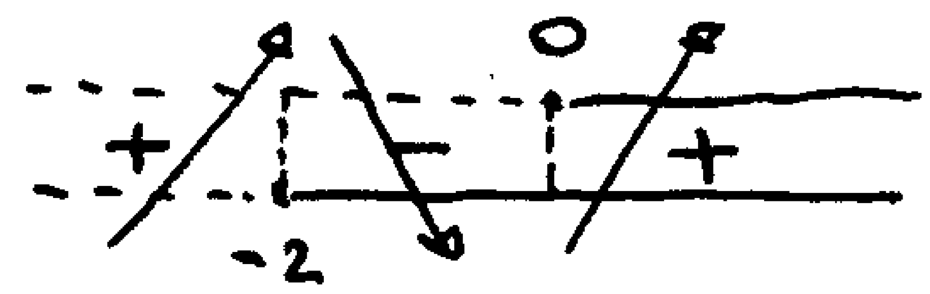
Compito di Matematica Generale del 23/9/16 $\boxed{MG1}$

1) $f(x) = x^2 \cdot e^x$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x) = 0^+$; $\lim\limits_{x\to+\infty} f(x) = +\infty$.

$f(x) \geq 0 : \forall x \in \mathbb{R}$; $f(0) = 0$.

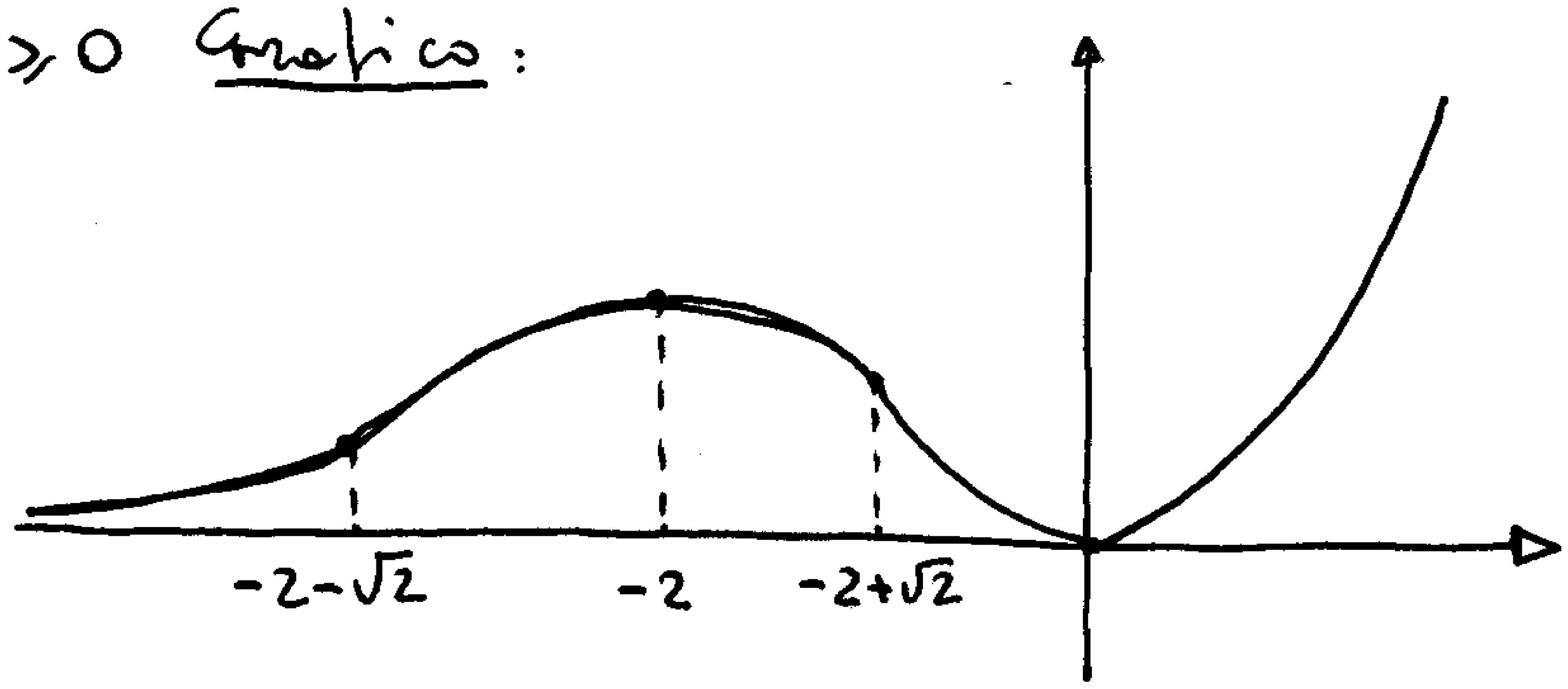
$f'(x) = 2x\,e^x + x^2 e^x = x(2+x)e^x \geq 0 \left\langle \begin{array}{l} x \geq 0 : x \geq 0 \\ 2+x \geq 0 : x \geq -2 \end{array} \right.$



$f''(x) = 2e^x + 2xe^x + 2x\,e^x + x^2 e^x = e^x \cdot (x^2 + 4x + 2) \geq 0$ Grafico:

$x = -2 \pm \sqrt{4-2} = -2 \pm \sqrt{2} \Rightarrow f''(x) \geq 0$

per $x \leq -2-\sqrt{2} \cup x \geq -2+\sqrt{2}$



2) $\lim\limits_{x\to 0} \dfrac{\cos 2x - \cos 3x}{x^2} = \lim\limits_{x\to 0} \dfrac{1-\cos 3x}{9x^2} \cdot 9 - \dfrac{1-\cos 2x}{4x^2} \cdot 4 = \dfrac{1}{2} \cdot 9 - \dfrac{1}{2} \cdot 4 = \dfrac{5}{2}$.

$\lim\limits_{x\to+\infty} \left(\dfrac{1+3x+x^2}{3x^2}\right)^{1-x} = \left(\to \dfrac{1}{3}\right)^{(\to -\infty)} = +\infty$.

3) $\lim\limits_{x\to+\infty} \left(1 + \dfrac{k}{x}\right)^{x-1} = \lim\limits_{x\to+\infty} \left(1 + \dfrac{k}{x}\right)^{x} = e^k = 5 \Rightarrow k = \log 5$.

4) $f(x) = e^{2x}$; $g(x) = 3x-1$; $h(x)$. $f(g(h(x))) = f(3h(x)-1) = e^{6h(x)-2} = x^3 \Rightarrow$

$\Rightarrow 6h(x) - 2 = \log x^3 \Rightarrow 6h(x) = 3\log x + 2 \Rightarrow h(x) = \dfrac{1}{6}(3\log x + 2) = \dfrac{1}{2}\log x + \dfrac{1}{3}$.

5) $\displaystyle\int_1^e \frac{x^2-2x+1}{x}\,dx = \int_1^e x - 2 + \frac{1}{x}\,dx = \left(\frac{x^2}{2} - 2x + \log x\right|_1^e = \left(\frac{e^2}{2} - 2e + \log e\right) - \left(\frac{1}{2} - 2 + 0\right) =$

$= \dfrac{e^2}{2} - 2e + 1 - \dfrac{1}{2} + 2 = \dfrac{e^2}{2} - 2e + \dfrac{5}{2}$.

6) $\left\|\begin{matrix} 1 & k \\ k & 1 \end{matrix}\right\| \cdot \left\|\begin{matrix} 1 & -1 \\ -1 & 2 \end{matrix}\right\| \cdot \left\|\begin{matrix} 1 \\ 1 \end{matrix}\right\| = \left\|\begin{matrix} 1-k & -1+2k \\ k-1 & -k+2 \end{matrix}\right\| \cdot \left\|\begin{matrix} 1 \\ 1 \end{matrix}\right\| = \left\|\begin{matrix} 1-k & -1+2k \\ k-1 & -k+2 \end{matrix}\right\| = \left\|\begin{matrix} k \\ 1 \end{matrix}\right\| = \left\|\begin{matrix} 0 \\ 1 \end{matrix}\right\| \Rightarrow k = 0$.

7) $f(x) = e^{k-3x}$. Coefficiente Angolare retta tangente: $f'(-1) \Rightarrow$

$f'(x) = -3 \cdot e^{k-3x} \Rightarrow f'(-1) = -3\,e^{k+3}$. $f'(-1) = -1$ per il parallelismo $\Rightarrow$

$\Rightarrow -3\,e^{k+3} = -1 \Rightarrow e^{k+3} = \dfrac{1}{3} \Rightarrow k+3 = \log\dfrac{1}{3} \Rightarrow k = \log\dfrac{1}{3} - 3 = -\log 3 - 3$.

MG2

8) $f(x) = e^{x-1}$; $f(1) = e^0 = 1$. $f'(x) = e^{x-1}$; $f'(1) = 1$.
$f''(x) = e^{x-1}$; $f''(1) = e^0 = 1$; $f'''(x) = e^{x-1}$; $f'''(1) = 1$.

$P_3(x;1) = 1 + (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{6}\cdot(x-1)^3$.

9) $f(x;y) = xy - x^2 + xy^2$.

$\begin{cases} f'_x = y - 2x + y^2 = 0 \\ f'_y = x + 2xy = x(1+2y) = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y^2 + y = y(y+1) = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} x=0 \\ y=-1 \end{cases}$ oppure

$\begin{cases} 1+2y = 0 \\ y - 2x + y^2 = 0 \end{cases} \Rightarrow \begin{cases} y = -\frac{1}{2} \\ -\frac{1}{2} - 2x + \frac{1}{4} = 0 \end{cases} \Rightarrow \begin{cases} 2x = -\frac{1}{4} \\ y = -\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x = -\frac{1}{8} \\ y = -\frac{1}{2} \end{cases}$.

Ci sono tre punti stazionari: $P_0 = (0;0)$; $P_1 = (0;-1)$ e $P_2 = \left(-\frac{1}{8}; -\frac{1}{2}\right)$.

$H(x;y) = \begin{Vmatrix} -2 & 1+2y \\ 1+2y & 2x \end{Vmatrix}$. $H(0;0) = \begin{Vmatrix} -2 & 1 \\ 1 & 0 \end{Vmatrix} \Rightarrow -2\cdot 0 - 1 < 0$: Punto di Sella.

$H(0;-1) = \begin{Vmatrix} -2 & -1 \\ -1 & 0 \end{Vmatrix} \Rightarrow -2\cdot 0 - 1 < 0$: P. Sella; $H\left(-\frac{1}{8}; -\frac{1}{2}\right) = \begin{Vmatrix} -2 & 0 \\ 0 & -\frac{1}{4} \end{Vmatrix} \Rightarrow \begin{cases} -2 < 0; -\frac{1}{4} < 0 \\ \frac{1}{2} - 0 > 0 \end{cases}$: P. MAX.

10)

A	B	non B	A ⇒ non B	A e B	non (A e B)
1	1	0	0	1	0
1	0	1	1	0	1
0	1	0	1	0	1
0	0	1	1	0	1

Le due proposizioni sono logicamente equivalenti.

O anche (Da $(A \Rightarrow B) \Leftrightarrow (\text{non } A \text{ o } B)$)

$(A \Rightarrow \text{non } B) \Leftrightarrow (\text{non } A \text{ o non } B) \Leftrightarrow \text{non } (A \text{ e } B)$.