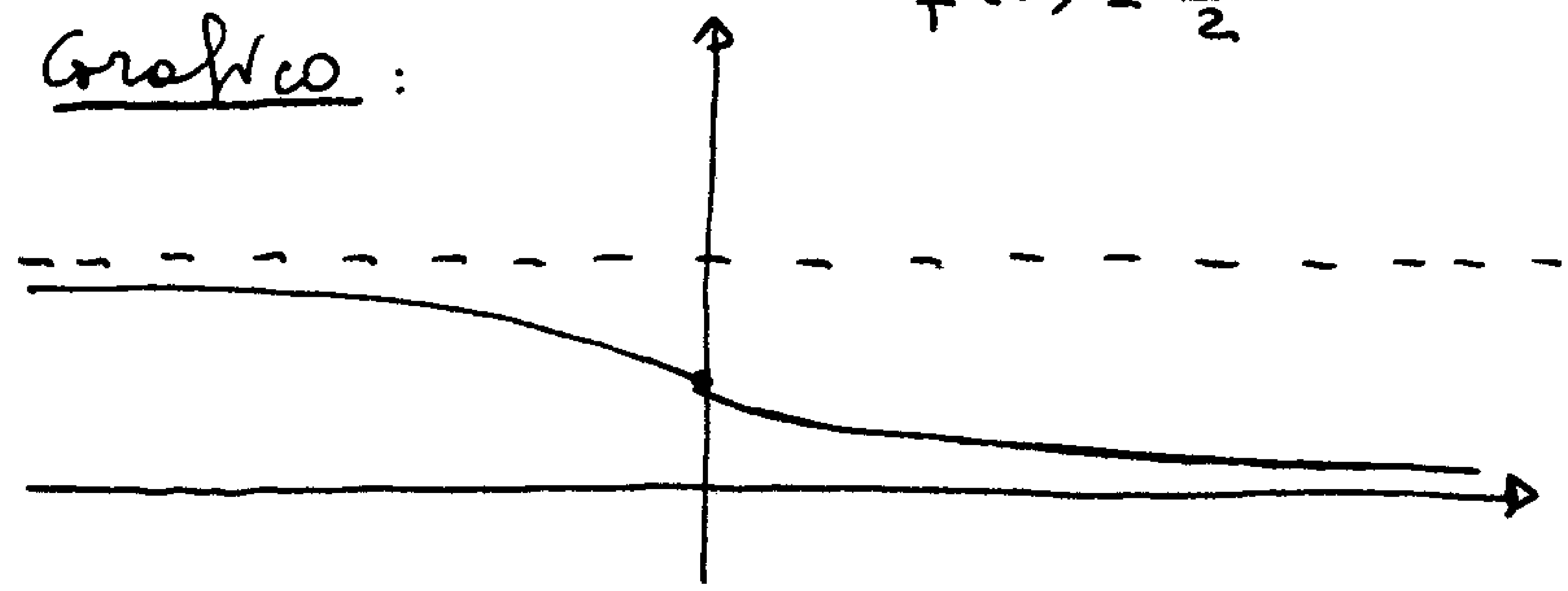
Compito di Matematica Generale del 8/10/2016 $\boxed{\text{CMG1}}$

1) $f(x)=\frac{1}{e^x+1}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=\frac{1}{0+1}=1$; $\lim_{x\to+\infty} f(x)=0^+$. $f(x)>0\ \forall x\in\mathbb{R}$.

$f'(x)=-\frac{e^x}{(e^x+1)^2}<0\ \forall x\in\mathbb{R}$ $\qquad f(0)=\frac{1}{2}$

Grafico :



$$f''(x)=-\left(\frac{e^x\cdot(e^x+1)^2-e^x\cdot 2(e^x+1)\cdot e^x}{(e^x+1)^4}\right)=$$

$$=\frac{e^x(e^x+1)(e^x-1)}{(e^x+1)^4}\geq 0 \text{ per } e^x\geq 1 \Rightarrow x\geq 0.$$

2) $\lim_{x\to 0}\frac{\operatorname{sen}5x-\operatorname{sen}3x}{x}=\lim_{x\to 0}\left(\frac{\operatorname{sen}5x}{5x}\cdot 5-\frac{\operatorname{sen}3x}{3x}\cdot 3\right)=1\cdot 5-1\cdot 3=2.$

$\lim_{x\to+\infty}\frac{3x^2-x^3+2\log x}{x^3+3x^2-x+10}=\lim_{x\to+\infty}\frac{-x^3}{x^3}=-1.\ \left(x^2=o(x^3);\ \log x=o(x^3);\ 3x^2-x=o(x^3)\right)$

3) $\lim_{x\to+\infty}\frac{1-3x+5x^2}{2+x+kx^2}=\lim_{x\to+\infty}\frac{5x^2}{kx^2}=\frac{5}{k}=-1\Rightarrow k=-5.$

4) $f(x)=\log x$; $g(x)=2x+3$. $F(x)=f(g(x))-g(f(x))=f(2x+3)-g(\log x)=\log(2x+3)-(2\log x+3)$.

$F'(x)=\frac{2}{2x+3}-\frac{2}{x}.$

5) $\int_0^k 3e^x-2e^{2x}\,dx=\left[3e^x-e^{2x}\right]_0^k=(3e^k-e^{2k})-(3-1)=3e^k-e^{2k}-2=0\Rightarrow e^{2k}-3e^k+2=0\Rightarrow$

$\Rightarrow e^k=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm 1}{2}=\begin{cases}2\\1\end{cases}\Rightarrow e^k=2$ per $k=\log 2$; $e^k=1$ per $k=0$ non accettabile.

6) $X=\begin{Vmatrix}0&0&1\\0&1&0\\1&0&0\end{Vmatrix}\cdot\begin{Vmatrix}1\\-1\\x\end{Vmatrix}=\begin{Vmatrix}x\\-1\\1\end{Vmatrix}$. $\|X\|=\sqrt{x^2+1^2+1^2}=\sqrt{x^2+2}=\sqrt{6}\Rightarrow x^2=4\Rightarrow x=\pm 2.$

7) $f(x)=15x^2-36x+10$; $g(x)=2x^3+9x^2-30x+12$. Per avere rette tangenti parallele deve risultare: $f'(x_0)=g'(x_0)\Rightarrow$

$f'(x)=30x-36=g'(x)=6x^2+18x-30\Rightarrow 6x^2-12x+6=6(x^2-2x+1)=0\Rightarrow$

$\Rightarrow 6(x-1)^2=0\Rightarrow x_0=1$. Essendo $f(1)=-11$ e $g(1)=-7$ le due rette

CMG2

tangenti non sono coincidenti.

8) $f(x;y) = x^2 - xy^2 + y^2$.

$$\begin{cases} f'_x = 2x - y^2 = 0 \\ f'_y = 2y - 2xy = 2y(1-x) = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} y^2 = 2 \\ x = 1 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=\sqrt{2} \end{cases} \cup \begin{cases} x=1 \\ y=-\sqrt{2} \end{cases}.$$

$$H(x;y) = \begin{Vmatrix} 2 & -2y \\ -2y & 2-2x \end{Vmatrix}. \quad H(0;0) = \begin{Vmatrix} 2 & 0 \\ 0 & 2 \end{Vmatrix} \Rightarrow \begin{cases} 2>0 \\ 4>0 \end{cases} : \text{Punto di minimo.}$$

$$H(1;\sqrt{2}) = \begin{Vmatrix} 2 & -2\sqrt{2} \\ -2\sqrt{2} & 0 \end{Vmatrix} : |H_2| = -8 < 0 \text{ P. di Sella; } H(1;-\sqrt{2}) = \begin{Vmatrix} 2 & 2\sqrt{2} \\ 2\sqrt{2} & 0 \end{Vmatrix} : |H_2| = -8 < 0 \text{ P. di Sella.}$$

9) $f(x) = \log_3(1-2^{x-1})$. C.E.: $1-2^{x-1} > 0 \Rightarrow 2^{x-1} < 1 \Rightarrow x-1 < 0 \Rightarrow x < 1$.

$\lim\limits_{x\to-\infty} f(x) = 0^-$; $\lim\limits_{x\to 1^-} f(x) = -\infty$. $f(x): \,]-\infty;1[\, \to \,]-\infty;0[$.

$f'(x) = \dfrac{1}{1-2^{x-1}} \cdot \log_3 e \cdot (-2^{x-1}) \cdot \log 2 < 0 \quad \forall x \in$ C.E. Funzione strettamente decrescente e quindi invertibile. $f^{-1}(x): \,]-\infty;0[\, \to \,]-\infty;1[$.

$y = \log_3(1-2^{x-1}) \Rightarrow 1-2^{x-1} = 3^y \Rightarrow 2^{x-1} = 1-3^y \Rightarrow x-1 = \log_2(1-3^y) \Rightarrow$

$\Rightarrow x = \log_2(1-3^y)+1$. Inversa: $f^{-1}(x) = y = \log_2(1-3^x)+1$.

10)

A B C	A∘B	(A∘B) ⇒ C	non C	A ⇔ non C
~~1 1 1~~	~~1~~	~~1~~	~~0~~	~~0~~
1 1 0	1	0	1	1
~~1 0 1~~	~~1~~	~~1~~	~~0~~	~~0~~
1 0 0	1	0	1	1
~~0 1 1~~	~~1~~	~~1~~	~~0~~	~~1~~
0 1 0	1	0	1	0
~~0 0 1~~	~~0~~	~~1~~	~~0~~	~~1~~
~~0 0 0~~	~~0~~	~~1~~	~~1~~	~~0~~

Essendo per ipotesi la proposizione (A∘B) ⇒ C falsa si considerano solo le righe $\overline{\text{II}}$; $\overline{\text{IV}}$; $\overline{\text{VI}}$.