Prova Intermedia di Matematica Generale del 7/11/2016 **Compito A2**

1) $\lim_{x\to 0} \frac{3^{\sin x}-2^x}{\log(1+3x)} = \lim_{x\to 0} \frac{3^{\sin x}-1}{\sin x}\cdot\frac{\sin x}{x}\cdot\frac{3x}{\log(1+3x)}\cdot\frac{1}{3} - \frac{2^x-1}{x}\cdot\frac{3x}{\log(1+3x)}\cdot\frac{1}{3} = (\log 3-\log 2)\frac{1}{3} = \frac{1}{3}\log\frac{3}{2}$.

$\lim_{x\to+\infty}\left(\frac{1+x-3x^2}{5-x^2}\right)^{1-x} = \left(\to \frac{-3}{-1}\right)^{(\to-\infty)} = \left(\to 3\right)^{(\to-\infty)} = 0^+$.

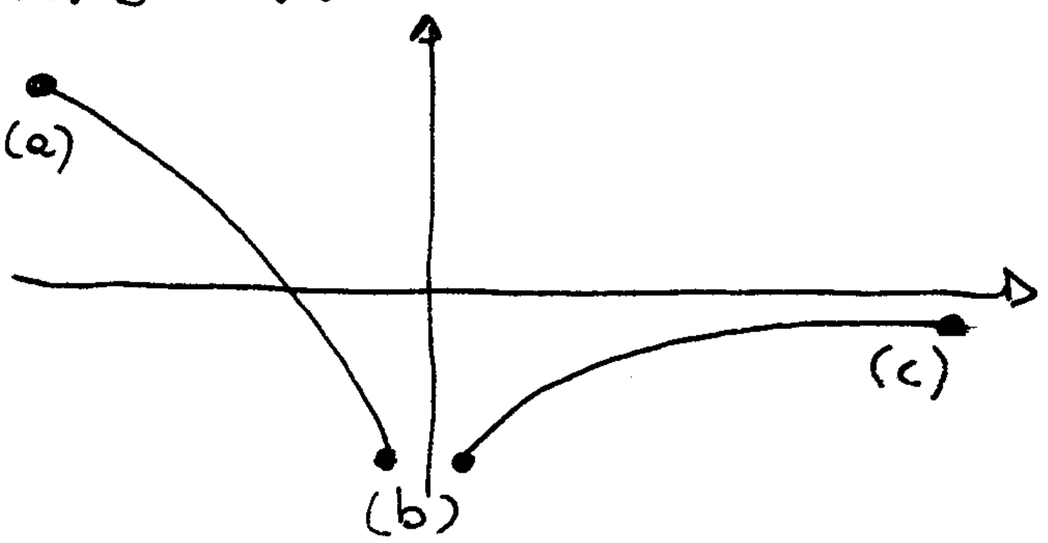
2) $\lim_{x\to 0} \frac{\sin kx^2}{1-\cos 2x} = \lim_{x\to 0} \frac{\sin kx^2}{kx^2}\cdot\frac{kx^2}{4x^2}\cdot\frac{4x^2}{1-\cos 2x} = 1\cdot\frac{k}{4}\cdot 2 = \frac{k}{2} = 4 \Rightarrow k = 8$.

3) $f^{-1}(x) = 3^{x+2} = y \Rightarrow x+2 = \log_3 y \Rightarrow x = \log_3 y - 2 \Rightarrow f(x) = \log_3 x - 2$; $g(x) = 2x-1$.

$f(g(f(x))) = f(g(\log_3 x-2)) = f(2\log_3 x-4-1) = f(2\log_3 x-5) = \log_3(2\log_3 x-5)-2$.

C.E.: $\begin{cases} x>0 \\ 2\log_3 x-5>0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \log_3 x > \frac{5}{2} \end{cases} \Rightarrow \begin{cases} x>0 \\ x > 3^{\frac{5}{2}} \end{cases} \Rightarrow x > 3^{\frac{5}{2}} = \sqrt{3^5} = 9\sqrt{3}$. C.E.: $x > 9\sqrt{3}$.

4) $\lim_{x\to-\infty} f(x) = +\infty$; $\lim_{x\to 0} f(x) = -\infty$; $\lim_{x\to+\infty} f(x) = 0^-$



a) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow f(x)>\varepsilon$
b) $\forall\varepsilon\ \exists\delta(\varepsilon): 0<|x-0|<\delta(\varepsilon) \Rightarrow f(x)<\varepsilon$
c) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow 0-\varepsilon<f(x)<0$.

5) P_1: Maria canta ; P_2: Lucia balla.

(Se Maria non canta allora Lucia balla) e (Maria canta oppure Lucia non balla)
(non $P_1 \Rightarrow P_2$) e (P_1 o non P_2)

P_1 P_2	non P_1	(non $P_1 \Rightarrow P_2$)	non P_2	(P_1 o non P_2)	(non $P_1 \Rightarrow P_2$) e (P_1 o non P_2)
1 1	0	1	0	1	1
1 0	0	1	1	1	1
0 1	1	1	0	0	0
0 0	1	0	1	1	0

La proposizione è vera quando Maria canta indipendentemente da quello che fa Lucia.

Prova intermedia di Matematica Generale del 7/11/2016 — Compito B2

1) $\lim_{x\to 0} \frac{(1+\sin x)^2-1}{3^{2x}-1} = \lim_{x\to 0} \frac{(1+\sin x)^2-1}{\sin x} \cdot \frac{\sin x}{x} \cdot \frac{2x}{3^{2x}-1} \cdot \frac{1}{2} = 2 \cdot 1 \cdot \frac{1}{\log 3} \cdot \frac{1}{2} = \log_3 e.$

$\lim_{x\to+\infty} \left(\frac{2-3x+x^2}{2x^2-1} \right)^{x-1} = \left(\to \frac{1}{2} \right)^{(\to+\infty)} = 0^+.$

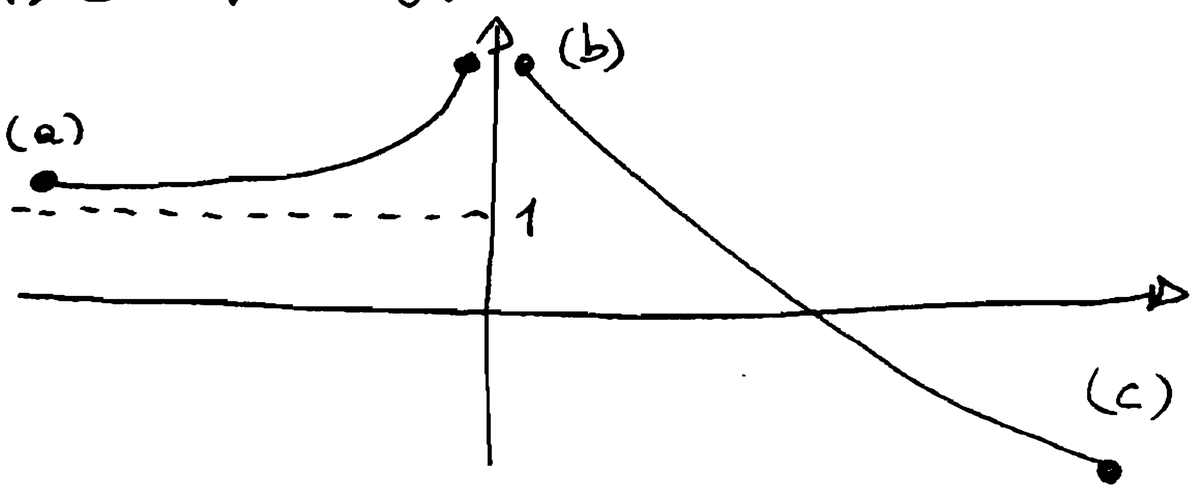
2) $\lim_{x\to 0} \frac{\log(1+Kx^2)}{1-\cos 3x} = \lim_{x\to 0} \frac{\log(1+Kx^2)}{Kx^2} \cdot \frac{Kx^2}{9x^2} \cdot \frac{9x^2}{1-\cos 3x} = 1 \cdot \frac{K}{9} \cdot 2 = 5 \Rightarrow K = \frac{45}{2}.$

3) $f^{-1}(x) = 2^{x+3} = y \Rightarrow x+3 = \log_2 y \Rightarrow x = \log_2 y - 3 \Rightarrow f(x) = \log_2 x - 3 ; g(x) = 3x-1.$

$f(g(f(x))) = f(g(\log_2 x - 3) = f(3\log_2 x - 9 - 1) = f(3\log_2 x - 10) = \log_2 (3\log_2 x - 10) - 3.$

$\text{C.E.}: \begin{cases} x>0 \\ 3\log_2 x - 10 > 0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \log_2 x > \frac{10}{3} \end{cases} \Rightarrow \begin{cases} x>0 \\ x > 2^{\frac{10}{3}} \end{cases} \Rightarrow x > 2^{\frac{10}{3}} = \sqrt[3]{2^{10}} = 8\sqrt[3]{2}.$ C.E.: $x > 8\sqrt[3]{2}$.

4) $\lim_{x\to-\infty} f(x) = 1^+ ; \lim_{x\to 0} f(x) = +\infty ; \lim_{x\to+\infty} f(x) = -\infty$



a) $\forall \varepsilon > 0 \ \exists \delta(\varepsilon) : x < \delta(\varepsilon) \Rightarrow 1 < f(x) < 1+\varepsilon$
b) $\forall \varepsilon \ \exists \delta(\varepsilon) : 0 < |x-0| < \delta(\varepsilon) \Rightarrow f(x) > \varepsilon$
c) $\forall \varepsilon \ \exists \delta(\varepsilon) : x > \delta(\varepsilon) \Rightarrow f(x) < \varepsilon$.

5) P_1: Maria canta ; P_2: Lucia balla.

(Lucia balla e Maria canta) oppure (Se Lucia non balla allora Maria non canta)

P_1	P_2	$(P_2 \text{ e } P_1)$	non P_2	non P_1	$(\text{non } P_2 \Rightarrow \text{non } P_1)$	$(P_1 \text{ e } P_2)$ o $(\text{non } P_2 \Rightarrow \text{non } P_1)$
1	1	1	0	0	1	1
1	0	0	1	0	0	0
0	1	0	0	1	1	1
0	0	0	1	1	1	1

La proposizione è falsa solo quando Maria canta e Lucia non balla.

Prova Intermedia di Matematica Generale del 7/11/2016 **Compito C2**

1) $\lim_{x\to 0} \frac{2^x - 3^{tgx}}{\log(1-x)} = \lim_{x\to 0} \left(\frac{2^x-1}{x} - \frac{3^{tgx}-1}{tgx} \cdot \frac{tgx}{x} \right) \cdot \frac{(-x)}{\log(1+(-x))} \cdot (-1) = (\log 2 - \log 3) \cdot 1 \cdot (-1) = \log \frac{3}{2}$.

$\lim_{x\to+\infty} \left(\frac{1+3x+5x^2}{2x^2+1} \right)^{2x-1} = \left(\to \frac{5}{2} \right)^{(\to+\infty)} = +\infty$.

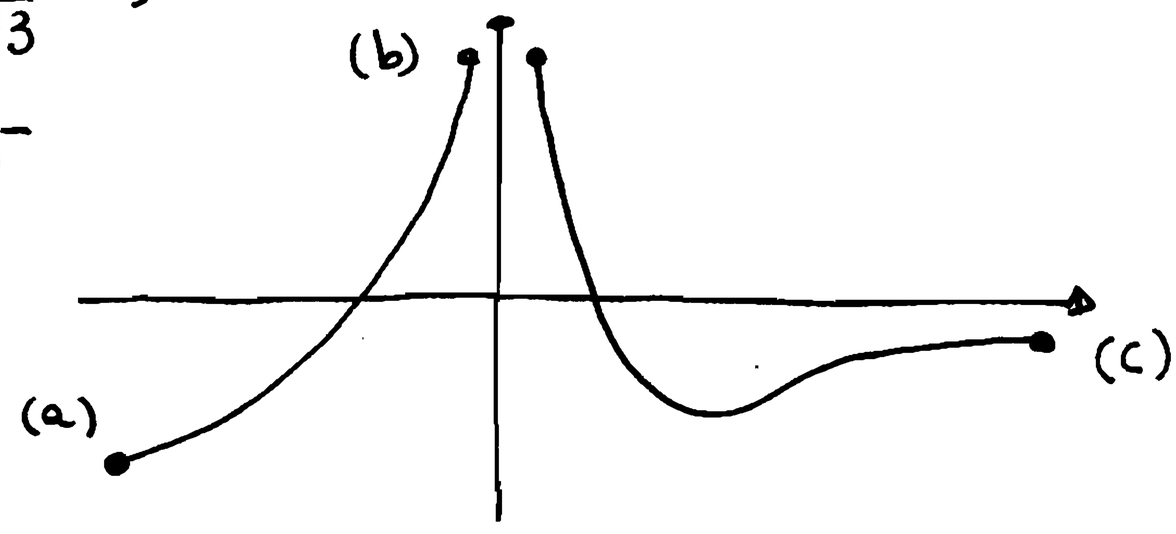
2) $\lim_{x\to 0} \frac{(1+kx^2)^5 - 1}{sen^2 x} = \lim_{x\to 0} \frac{(1+kx^2)^5-1}{kx^2} \cdot \frac{kx^2}{sen^2 x} = 5 \cdot k = 3 \Rightarrow k = \frac{3}{5}$.

3) $f^{-1}(x) = 3^{2-x} = y \Rightarrow 2-x = \log_3 y \Rightarrow x = 2 - \log_3 y \Rightarrow f(x) = 2 - \log_3 x$; $g(x) = 2x+1$.

$f(g(f(x))) = f(g(2-\log_3 x)) = f(4 - 2\log_3 x + 1) = f(5 - 2\log_3 x) = 2 - \log_3 (5 - 2\log_3 x)$.

$e.\varepsilon.: \begin{cases} x>0 \\ 5-2\log_3 x > 0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \log_3 x < \frac{5}{2} \end{cases} \Rightarrow \begin{cases} x>0 \\ x < 3^{\frac{5}{2}} = 9\sqrt{3} \end{cases} \Rightarrow e.\varepsilon.: 0 < x < 9\sqrt{3}$.

4) $\lim_{x\to-\infty} f(x) = -\infty$; $\lim_{x\to 0} f(x) = +\infty$; $\lim_{x\to+\infty} f(x) = 0^-$



a) $\forall \varepsilon \; \exists \delta(\varepsilon) : x < \delta(\varepsilon) \Rightarrow f(x) < \varepsilon$
b) $\forall \varepsilon \; \exists \delta(\varepsilon) : 0 < |x-0| < \delta(\varepsilon) \Rightarrow f(x) > \varepsilon$
c) $\forall \varepsilon > 0 \; \exists \delta(\varepsilon) : x > \delta(\varepsilon) \Rightarrow 0 - \varepsilon < f(x) < 0$.

5) P_1 : Maria canta ; P_2 : Lucia balla.

(Maria canta oppure Lucia non balla) e (Se Lucia balla allora Maria non canta)
$(P_1 \; o \; non\, P_2)$ e $(P_2 \Rightarrow non\, P_1)$

P_1	P_2	$non\,P_2$	$(P_1 \; o \; non\,P_2)$	$non\,P_1$	$(P_2 \Rightarrow non\,P_1)$	$(P_1 \; o \; non\,P_2)$ e $(P_2 \Rightarrow non\,P_1)$
1	1	0	1	0	0	0
1	0	1	1	0	1	1
0	1	0	0	1	1	0
0	0	1	1	1	1	1

La proposizione è vera quando Lucia non balla, indipendentemente da quello che fa Maria.

Prova intermedia di Matematica Generale del 7/11/2016 **Compito D2**

1) $\lim_{x\to 0} \frac{(1+tgx)^3-1}{sen\,2x} = \lim_{x\to 0} \frac{(1+tgx)^3-1}{tgx} \cdot \frac{tgx}{x} \cdot \frac{2x}{sen\,2x} \cdot \frac{1}{2} = 3\cdot 1\cdot 1\cdot \frac{1}{2} = \frac{3}{2}$.

$\lim_{x\to+\infty} \left(\frac{x^2-x+1}{x+3x^2}\right)^{2-x} = \left(\to \frac{1}{3}\right)^{(\to -\infty)} = +\infty$.

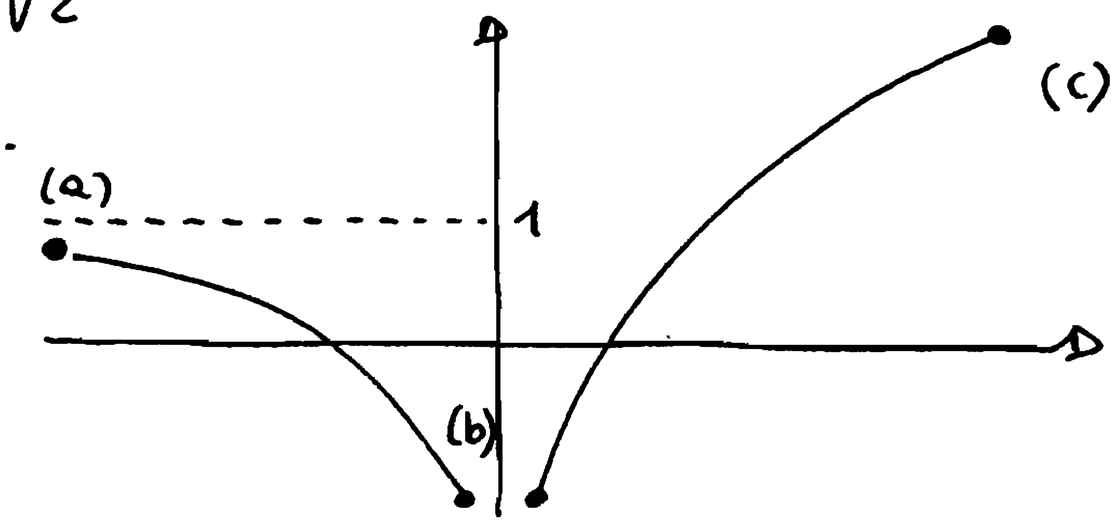
2) $\lim_{x\to 0} \frac{1-3^{kx}}{\log(1+2x)} = \lim_{x\to 0} \left(-\frac{3^{kx}-1}{kx}\right) \cdot \frac{kx}{2x} \cdot \frac{2x}{\log(1+2x)} = \frac{k}{2}\cdot(-\log 3) = 6 \Rightarrow k = -12\log_3 e$.

3) $f^{-1}(x) = 2^{3-x} = y \Rightarrow 3-x = \log_2 y \Rightarrow x = 3-\log_2 y \Rightarrow f(x) = 3-\log_2 x$; $g(x) = 3x+1$.

$f(g(f(x))) = f(g(3-\log_2 x)) = f(9-3\log_2 x+1) = f(10-3\log_2 x) = 3-\log_2(10-3\log_2 x)$.

c.e.: $\begin{cases} x>0 \\ 10-3\log_2 x>0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \log_2 x < \frac{10}{3} \end{cases} \Rightarrow \begin{cases} x>0 \\ x<2^{\frac{10}{3}} = 8\cdot\sqrt[3]{2} \end{cases} \Rightarrow$ c.e.: $0<x<8\cdot\sqrt[3]{2}$.

4) $\lim_{x\to-\infty} f(x) = 1^-$; $\lim_{x\to 0} f(x) = -\infty$; $\lim_{x\to+\infty} f(x) = +\infty$.



a) $\forall \varepsilon>0\ \exists \delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow 1-\varepsilon < f(x) < 1$;
b) $\forall \varepsilon\ \exists \delta(\varepsilon): 0<|x-0|<\delta(\varepsilon) \Rightarrow f(x)<\varepsilon$;
c) $\forall \varepsilon\ \exists \delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow f(x)>\varepsilon$.

5) P_1: Maria canta ; P_2: Lucia balla.

(Se Maria canta allora Lucia non balla) oppure (Maria non canta e Lucia balla)
$(P_1 \Rightarrow \text{non } P_2)$ o $(\text{non } P_1 \text{ e } P_2)$

P_1	P_2	non P_2	$(P_1 \Rightarrow \text{non } P_2)$	non P_1	$(\text{non } P_1 \text{ e } P_2)$	$(P_1 \Rightarrow \text{non } P_2)$ o $(\text{non } P_1 \text{ e } P_2)$
1	1	0	0	0	0	0
1	0	1	1	0	0	1
0	1	0	1	1	1	1
0	0	1	1	1	0	1

La proposizione è falsa solo quando Maria canta e Lucia balla.