# Compito di Matematica Generale del 16/3/2017 MG1

1) $f(x) = x^2 \cdot e^{x-1}$. C.E.: $\mathbb{R}$. $\lim_{x \to -\infty} f(x) = 0^+$; $\lim_{x \to +\infty} f(x) = +\infty$; $f(x) \geq 0 \, \forall x$; $f(0) = 0$.

$f'(x) = 2x e^{x-1} + x^2 e^{x-1} = x \cdot (2+x) \cdot e^{x-1} \geq 0$

per $x \leq -2 \cup x \geq 0$

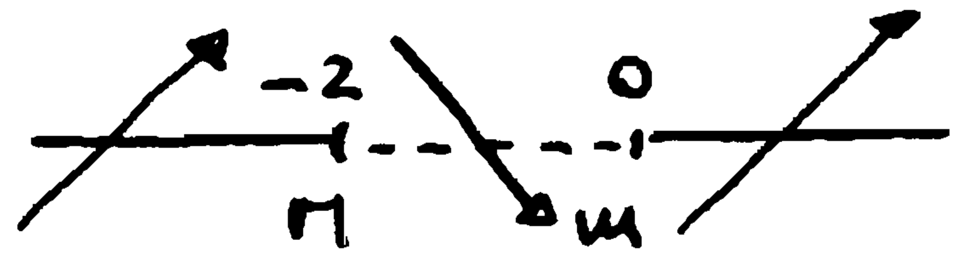
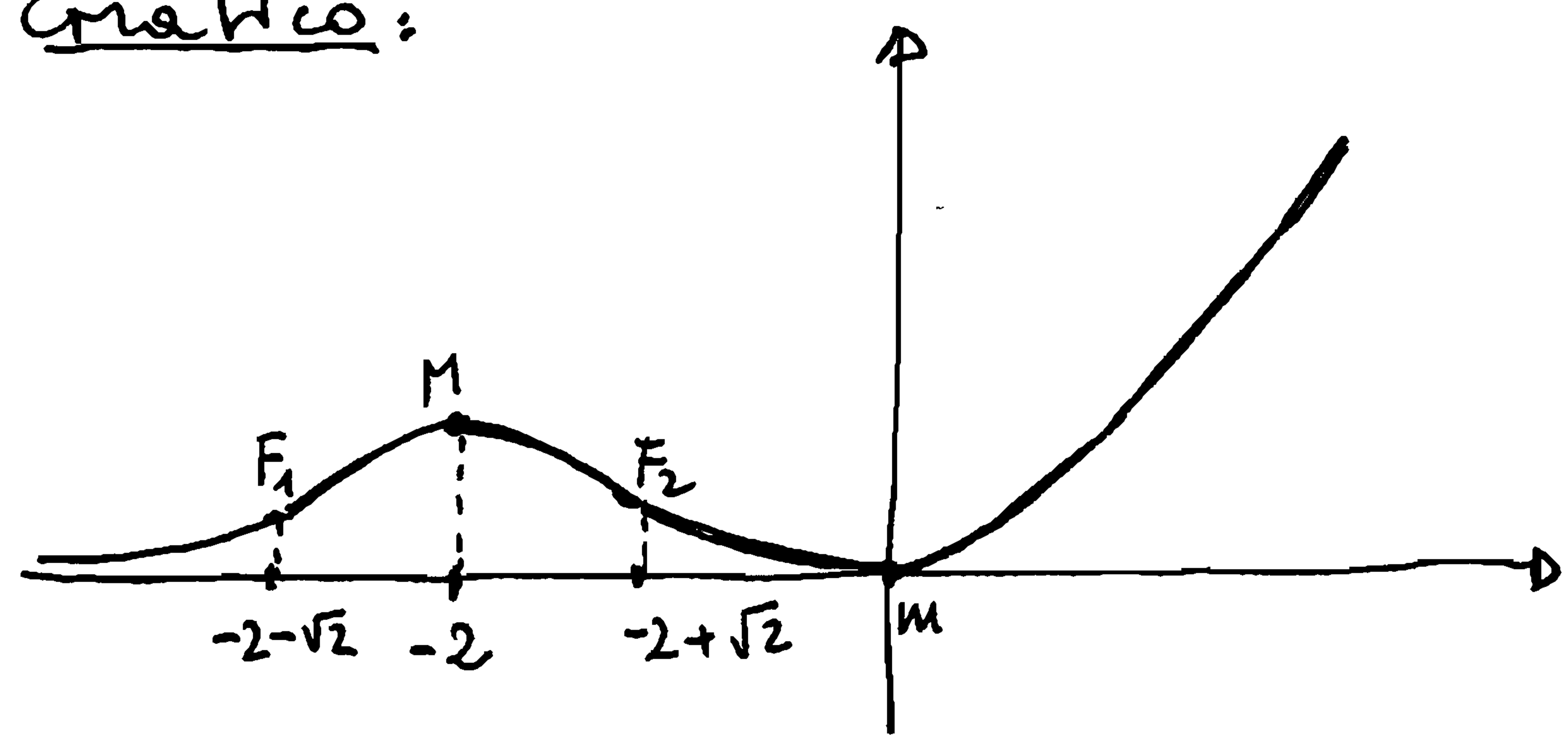



Grafico:

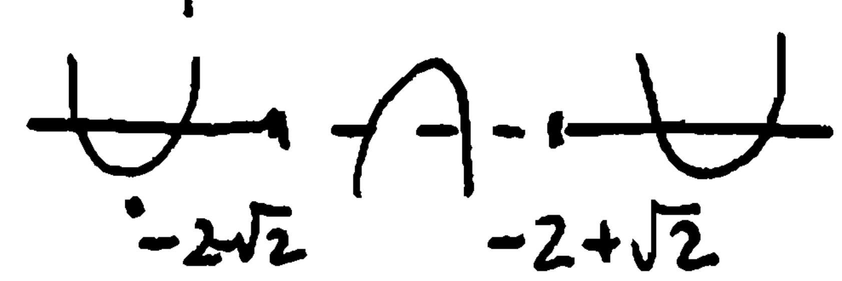



$f''(x) = (2+2x) e^{x-1} + (2x+x^2) e^{x-1} =$

$= (x^2+4x+2) e^{x-1} \geq 0$: $x^2+4x+2 \geq 0 \Rightarrow$

$x = -2 \pm \sqrt{4-2} = -2 \pm \sqrt{2} \Rightarrow f''(x) \geq 0$

per $x \leq -2-\sqrt{2} \cup x \geq -2+\sqrt{2}$:




2) $\lim_{x \to 0} \frac{2^{\operatorname{sen} x} - 1}{e^{\operatorname{sen} x} - 1} = \lim_{x \to 0} \frac{2^{\operatorname{sen} x} - 1}{\operatorname{sen} x} \cdot \frac{\operatorname{sen} x}{e^{\operatorname{sen} x} - 1} = \lim_{t \to 0} \frac{2^t - 1}{t} \cdot \frac{t}{e^t - 1} = \log 2 \cdot 1 = \log 2.$

$\lim_{x \to +\infty} \left(1 + \frac{1}{\log x}\right)^x = \lim_{x \to +\infty} \left[\left(1 + \frac{1}{\log x}\right)^{\log x}\right]^{\frac{x}{\log x}} = \left[\lim_{t \to +\infty} \left(1 + \frac{1}{t}\right)^t\right]^{\left(\lim_{x \to +\infty} \frac{x}{\log x}\right)} = (\to e)^{(\to +\infty)} = +\infty.$

3)

| A | B | C | (A o B) | (C ⇒ (A o B)) | (A o C) | (B ⇒ (A o C)) | (A e C) | ((A e C) ⇒ B) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| ~~0~~ | ~~1~~ | ~~0~~ | ~~1~~ | ~~1~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~1~~ |
| ~~0~~ | ~~0~~ | ~~1~~ | ~~0~~ | ~~0~~ | ~~1~~ | ~~1~~ | ~~0~~ | ~~1~~ |
| 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |

Dalla III riga si vede come la proposizione (A e C) ⇒ B possa risultare falsa anche se $P_1$ e $P_2$ risultano vere.

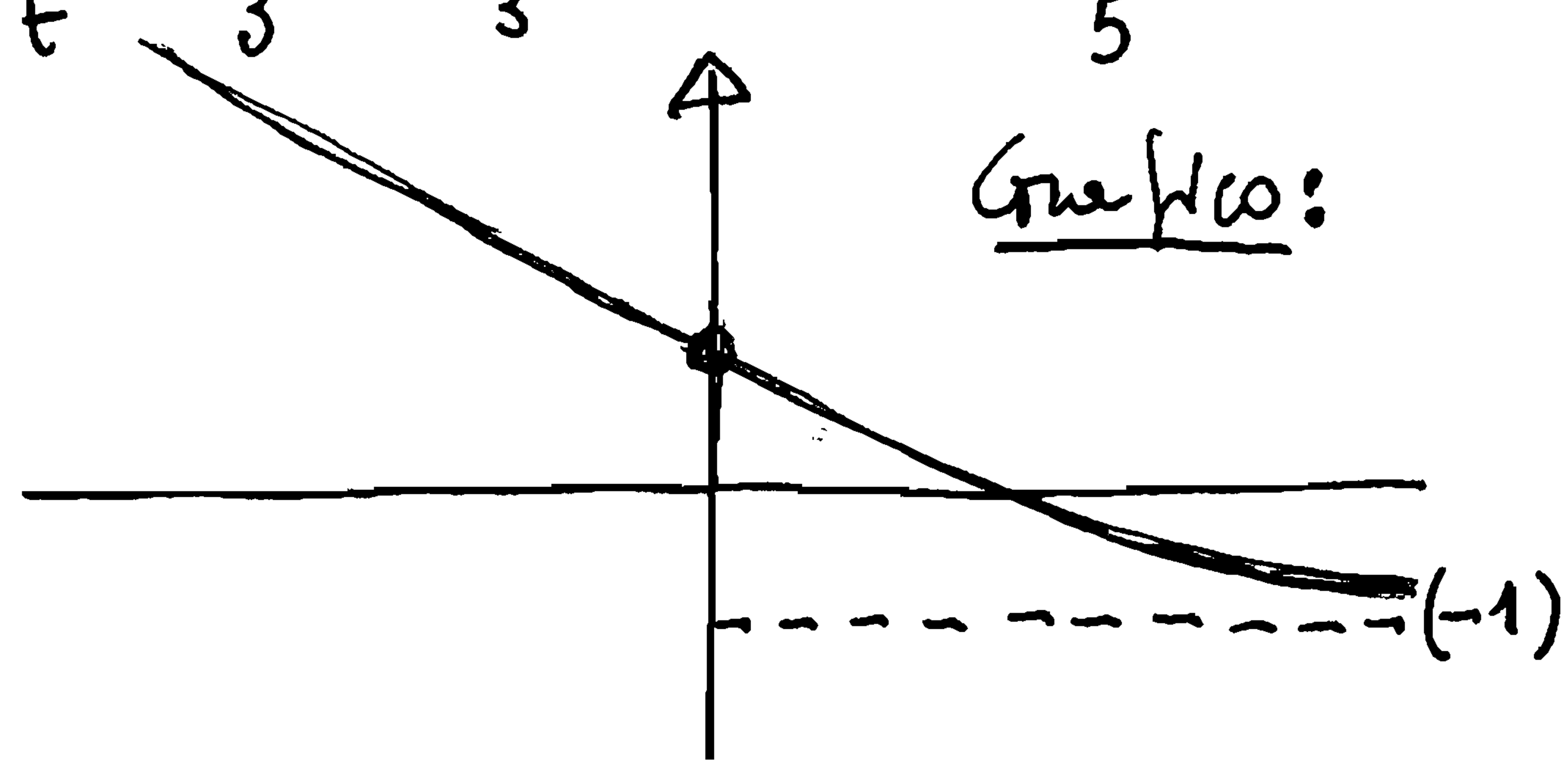
4) $\lim_{x \to 0} \frac{(1+Kx)^5 - 1}{3x} = \lim_{x \to 0} \frac{(1+Kx)^5 - 1}{Kx} \cdot \frac{K}{3} = \lim_{t \to 0} \frac{(1+t)^5 - 1}{t} \cdot \frac{K}{3} = 5 \cdot \frac{K}{3} = 2 \Rightarrow K = \frac{6}{5}.$

5) $\lim_{x \to -\infty} f(x) = +\infty$: $\forall \varepsilon \, \exists \delta(\varepsilon)$: $x < \delta(\varepsilon) \Rightarrow f(x) > \varepsilon$

$\lim_{x \to 0} f(x) = 1$ : $\forall \varepsilon > 0 \, \exists \delta(\varepsilon)$: $0 < |x| < \delta(\varepsilon) \Rightarrow |f(x) - 1| < \varepsilon$

$\lim_{x \to +\infty} f(x) = -1$: $\forall \varepsilon > 0 \, \exists \delta(\varepsilon)$: $x > \delta(\varepsilon) \Rightarrow |f(x) + 1| < \varepsilon$

Grafico:



$\boxed{MG2}$

6) $\int_0^1 e^{3x} - kx\,dx = \left(\frac{1}{3}e^{3x} - k\cdot\frac{x^2}{2}\right|_0^1 = \frac{1}{3}e^3 - \frac{k}{2} - \left(\frac{1}{3} - 0\right) = \frac{1}{3}e^3 - \frac{k}{2} - \frac{1}{3} = \frac{1}{3} \Rightarrow \frac{k}{2} = \frac{1}{3}e^3 - \frac{2}{3} \Rightarrow k = \frac{2}{3}(e^3 - 2).$

7) Da $e^x = 1 + x + o(x) \Rightarrow \sqrt[10]{e} = e^{\frac{1}{10}} = 1 + \frac{1}{10} + o\left(\frac{1}{10}\right) \cong \frac{11}{10}$.

8) $f(x;y) = 3xy - x^2 + y^3$. $\nabla f(x;y) = (0;0) \Rightarrow \begin{cases} f'_x = 3y - 2x = 0 \\ f'_y = 3x + 3y^2 = 0 \end{cases} \Rightarrow \begin{cases} 3y + 2y^2 = y(3+2y) = 0 \\ x = -y^2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \cup \begin{cases} y = -\frac{3}{2} \\ x = -\frac{9}{4} \end{cases}$: $H(x;y) = \begin{Vmatrix} -2 & 3 \\ 3 & 6y \end{Vmatrix}$.

$H(0;0) = \begin{Vmatrix} -2 & 3 \\ 3 & 0 \end{Vmatrix}$ : $|H_2| = 0 - 9 < 0$ : Sella; $H\left(-\frac{9}{4}; -\frac{3}{2}\right) = \begin{Vmatrix} -2 & 3 \\ 3 & -9 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| : -2 < 0; -9 < 0 \\ |H_2| = 18 - 9 > 0 \end{cases}$ : P. di MAX.

9) $y = e^{x-1} \Rightarrow x - 1 = \log y \Rightarrow x = \log y + 1 \Rightarrow f(x) = 1 + \log x$;

$y = \log(x+1) \Rightarrow x + 1 = e^y \Rightarrow x = e^y - 1 \Rightarrow g(x) = e^x - 1$.

$f(g(x)) = f(e^x - 1) = 1 + \log(e^x - 1) = y \Rightarrow \log(e^x - 1) = y - 1 \Rightarrow e^x - 1 = e^{y-1} \Rightarrow$

$\Rightarrow e^x = e^{y-1} + 1 \Rightarrow x = \log\left(e^{y-1} + 1\right)$. Inversa di $f(g(x))$: $y = \log\left(e^{x-1} + 1\right)$.

10) $A \cdot B \cdot X = \begin{Vmatrix} 1 & -1 \\ 2 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & k \\ k & 1 \end{Vmatrix} \cdot \begin{Vmatrix} -1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1 & -1 \\ 2 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} -1+k \\ -k+1 \end{Vmatrix} =$

$= \begin{Vmatrix} 1(-1+k) - 1(-k+1) \\ 2(-1+k) + 2(-k+1) \end{Vmatrix} = \begin{Vmatrix} -1 + k + k - 1 \\ -2 + 2k - 2k + 2 \end{Vmatrix} = \begin{Vmatrix} 2k - 2 \\ 0 \end{Vmatrix} = Y$.

$\|Y\| = \sqrt{(2k-2)^2 + 0^2} = \sqrt{(2k-2)^2} = |2k - 2| = 2 \Rightarrow$
- $2k - 2 = 2 \Rightarrow 2k = 4 \Rightarrow k = 2.$
- $2k - 2 = -2 \Rightarrow 2k = 0 \Rightarrow k = 0.$