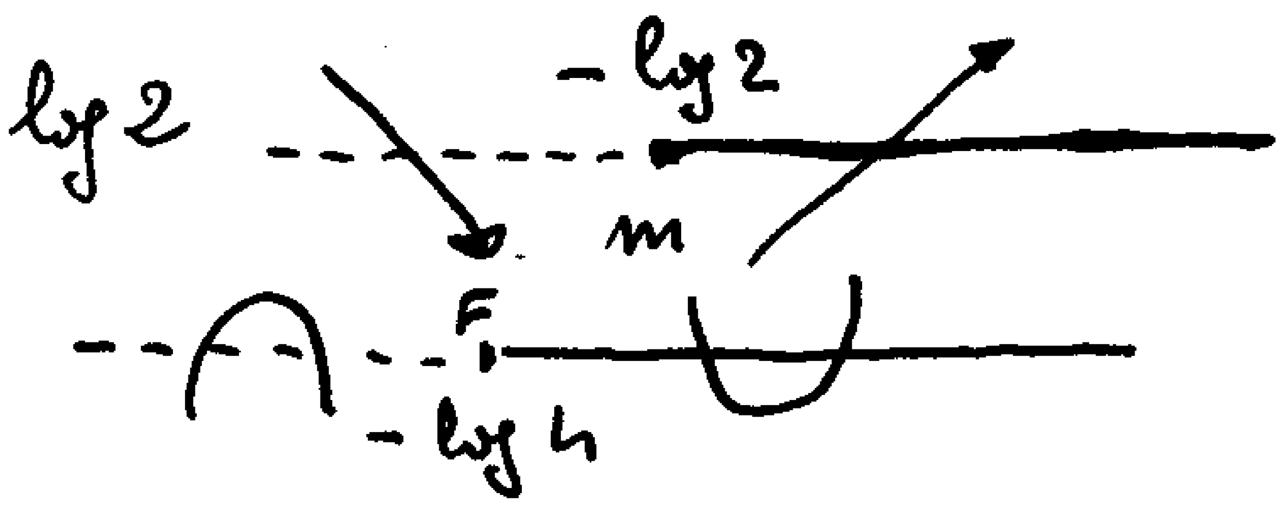
Compito di Matematica Generale del 12/6/2017 $\boxed{\text{CMG1}}$

1) $f(x)=e^{2x}-e^{x}-2$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=-2$; $\lim_{x\to+\infty} f(x)=+\infty$.

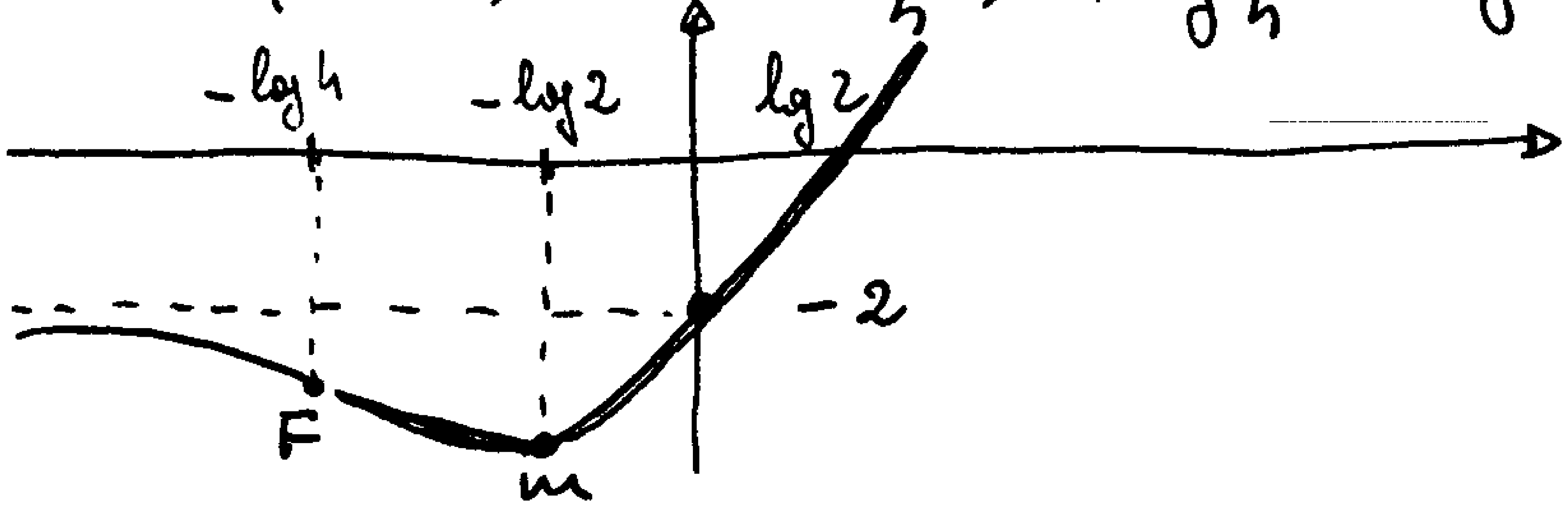
$f(x)=(e^{x}-2)(e^{x}+1)$; $f(x)\geq 0$ per $e^{x}\geq 2 \Rightarrow x\geq \log 2$. $f(\log 2)=0$; $f(0)=-2$.

$f'(x)=2e^{2x}-e^{x}=e^{x}(2e^{x}-1)\geq 0 \Rightarrow e^{x}\geq \frac{1}{2} \Rightarrow x\geq \log\frac{1}{2}=-\log 2$

$f''(x)=4e^{2x}-e^{x}=e^{x}(4e^{x}-1)\geq 0 \Rightarrow e^{x}\geq \frac{1}{4} \Rightarrow x\geq \log\frac{1}{4}=-\log 4$



Grafico



2) $\lim_{x\to 0}\frac{(1+x)^5-2^x}{x}=\lim_{x\to 0}\frac{(1+x)^5-1}{x}-\frac{2^x-1}{x}=5-\log 2.$

$\lim_{x\to+\infty}\left(x+\frac{\log x}{x}\right)^{1-\log x}=\left(\to+\infty+0\right)^{(\to-\infty)}=\left(\to+\infty\right)^{(\to-\infty)}=0^{+}.$

3)

A	B	(A e B)	non (A e B)	(non A)	(non B)	(non A e non B)	(non (A e B)) ⇒ (non A e non B)
1	1	1	0	0	0	0	1
1	0	0	1	0	1	0	0
0	1	0	1	1	0	0	0
0	0	0	1	1	1	1	1

La proposizione (non (A e B)) ⇒ (non A e non B) non è una tautologia.

4) Da $\operatorname{sen} x = x-\frac{x^3}{6}+o(x^3)$ segue $\operatorname{sen} x^2 = x^2-\frac{x^6}{6}+o(x^6) \Rightarrow$

$\Rightarrow \lim_{x\to 0}\frac{x^2-\operatorname{sen} x^2}{x^\alpha}=\lim_{x\to 0}\frac{\cancel{x^2}-\cancel{x^2}+\frac{x^6}{6}+o(x^6)}{x^\alpha}=\frac{1}{6}$ se $\alpha=6$.

5) $f(x)=\frac{1}{x}$; $g(x)=e^{x-1}$. $f(g(f(x)))=f(g(\frac{1}{x}))=f(e^{\frac{1}{x}-1})=\frac{1}{e^{\frac{1}{x}-1}}=F(x)=y \Rightarrow$

$\Rightarrow \frac{1}{y}=e^{\frac{1}{x}-1} \Rightarrow \frac{1}{x}-1=\log\frac{1}{y} \Rightarrow \frac{1}{x}=\log\frac{1}{y}+1 \Rightarrow x=\frac{1}{\log\frac{1}{y}+1}$. Inversa: $y=\frac{1}{\log\frac{1}{x}+1}$.

$y=\frac{1}{\log\frac{1}{x}+1}=\frac{1}{1-\log x} \Rightarrow$ C.E. $\begin{cases} x>0 \\ \log x\neq 1\end{cases} \Rightarrow \begin{cases} x>0 \\ x\neq e.\end{cases}$

6) $\int_0^1 x\,(1+x^2)^5\,dx \Rightarrow \frac{1}{2}\int 2x\,(1+x^2)^5\,dx=\frac{1}{2}\int(1+x^2)^5\,d(1+x^2)=\frac{1}{2}\cdot\frac{1}{6}\cdot(1+x^2)^6 \Rightarrow$

$\boxed{\text{CMG2}}$

$$\Rightarrow \int_0^1 x(1+x^2)^5\,dx = \frac{1}{12}\left(1+x^2\right)^6\Big|_0^1 = \frac{1}{12}(64-1) = \frac{63}{12} = \frac{21}{4}.$$

7) $f(x) = e^{x^2-x}$; $f(0) = e^0 = 1$;

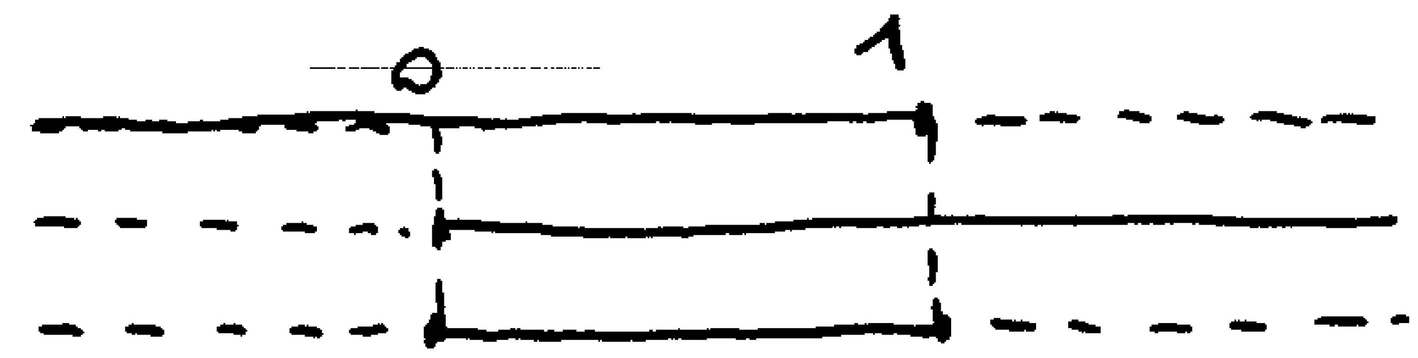
$f'(x) = (2x-1)\,e^{x^2-x}$; $f'(0) = (-1)\cdot 1 = -1$;

$f''(x) = 2\,e^{x^2-x} + (2x-1)^2 \cdot e^{x^2-x} = \left((2x-1)^2+2\right)e^{x^2-x}$; $f''(0) = 3\cdot 1 = 3$;

$f'''(x) = \left(2(2x-1)\cdot 2\right)e^{x^2-x} + \left((2x-1)^2+2\right)(2x-1)\,e^{x^2-x}$; $f'''(0) = -4-3 = -7$.

$P_3(x;0) = 1 - x + \frac{3}{2}x^2 - \frac{7}{6}x^3$.

8) $H = \begin{Vmatrix} 1-k & 0 \\ 0 & k \end{Vmatrix}$. $\begin{cases} 1-k>0 : k<1 \\ k>0 \quad : k>0 \\ |H_2| = k(1-k)>0 \text{ per } 0<k<1 \end{cases}$

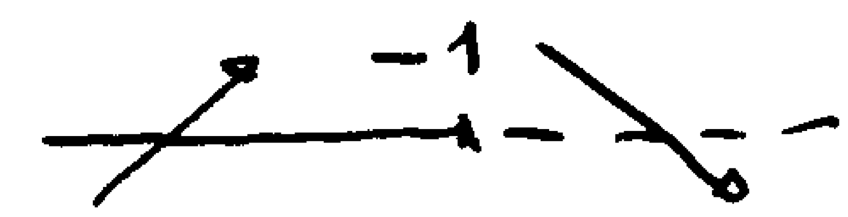


Se $0<k<1$: $|H_1|>0$ e $|H_2|>0$ $\Rightarrow$ Punto di Minimo;
Se $k<0$ o se $k>1$: $|H_2|<0$ $\Rightarrow$ Punto di Sella;
Se $k=0$ o se $k=1$: $|H_2|=0$ e nulla si può dire.

9) $f(x) = e^{kx-x^2}$: funzione continua e derivabile $\forall x \in \mathbb{R}$.

$f'(x) = (k-2x)\,e^{kx-x^2} \geq 0$ per $k-2x \geq 0 \Rightarrow x \leq \frac{k}{2} = -1$ se $k=-2$.

$f'(x) = 0$ per $x=-1$ se $f(x) = e^{-2x-x^2}$. $f'(x) \geq 0$ per $x \leq -1$:



Quindi $x=-1$ è un punto di Massimo.

10) $A = \begin{Vmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \end{Vmatrix}$; $X = \begin{Vmatrix} x \\ 1 \\ x \end{Vmatrix}$. $A\cdot X = \begin{Vmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ 1 \\ x \end{Vmatrix} = \begin{Vmatrix} 1\cdot x + 1\cdot 1 - 1\cdot x \\ 0\cdot x + 1\cdot 1 + 2\cdot x \end{Vmatrix} = \begin{Vmatrix} 1 \\ 1+2x \end{Vmatrix} = Y$.

$$\|Y\| = \sqrt{1^2 + (1+2x)^2} = \sqrt{1+1+4x^2+4x} = \sqrt{4x^2+4x+2} = 1 \Rightarrow$$

$$\Rightarrow 4x^2+4x+2 = 1 \Rightarrow 4x^2+4x+1 = 0 \Rightarrow (2x+1)^2 = 0 \Rightarrow x = -\frac{1}{2}.$$