Compito di Matematica Generale del 11/7/2017 $\boxed{MG1}$

1) $f(x) = (x^2-1)\cdot e^x$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x) = 0^+$; $\lim_{x\to+\infty} f(x) = +\infty$.

$f(x) \geq 0$ per $x^2-1 \geq 0 \Rightarrow x \leq -1 \cup x \geq 1$. $f(0) = -1$.

$f'(x) = 2x\cdot e^x + (x^2-1)e^x = (x^2+2x-1)e^x \geq 0$ per

$x^2+2x-1 \geq 0$: $x = -1 \pm \sqrt{1+1} = -1 \pm \sqrt{2}$

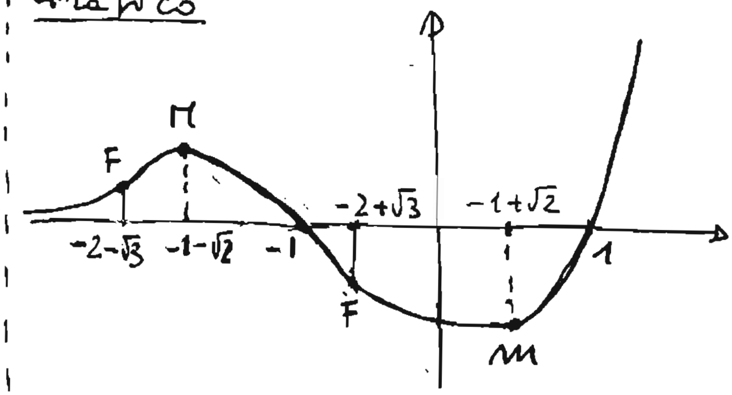
$f'(x) \geq 0$ per $x \leq -1-\sqrt{2} \cup x \geq -1+\sqrt{2}$

$f''(x) = (2x+2)e^x + (x^2+2x-1)e^x = (x^2+4x+1)e^x \geq 0$

per $x^2+4x+1 \geq 0$: $x = -2 \pm \sqrt{4-1} = -2 \pm \sqrt{3}$

$f''(x) \geq 0$ per $x \leq -2-\sqrt{3} \cup x \geq -2+\sqrt{3}$

Grafico



2) $\lim_{x\to0} \frac{e^{3x}-e^{2x}}{x-x^2} = \lim_{x\to0} \frac{x}{x-x^2}\cdot\left(\frac{e^{3x}-1}{3x}\cdot3 - \frac{e^{2x}-1}{2x}\cdot2\right) = 1\cdot(1\cdot3-1\cdot2) = 1.$

$\lim_{x\to+\infty}\left(\frac{\sqrt{x}-\log x+3x}{2x+\operatorname{sen}x}\right)^{1-x} = \lim_{x\to+\infty}\left(\frac{3x}{2x}\right)^{1-x} = \left(\to\frac{3}{2}\right)^{(\to-\infty)} = 0^+$ $\left(\begin{matrix}\sqrt{x}=o(3x);\ \log x = o(3x)\\ \operatorname{sen}x = o(2x)\end{matrix}\right)$

3) $f(x) = e^{1-x}$; $f(g(x)) = e^{1-g(x)} = \log x \Rightarrow 1-g(x) = \log(\log x) \Rightarrow g(x) = 1-\log(\log x)$.

C.E. $(g(x))$: $\begin{cases} x>0 \\ \log x>0\end{cases} \Rightarrow \begin{cases} x>0 \\ x>1\end{cases} \Rightarrow$ C.E.: $x>1$.

4) a) $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x) = 0^+$

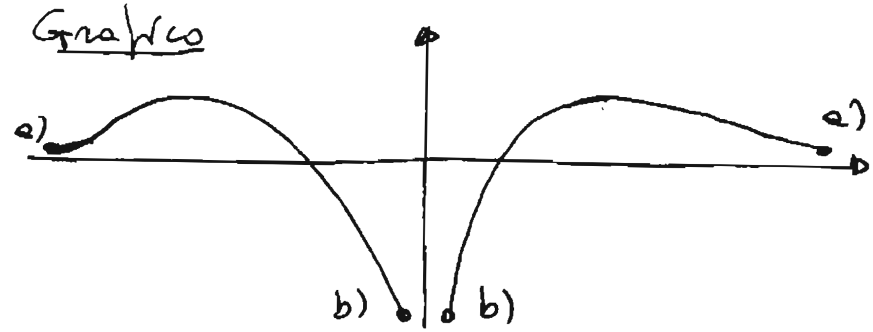
$\forall\varepsilon>0\ \exists\delta(\varepsilon)$: $x<\delta(\varepsilon) \Rightarrow 0<f(x)<\varepsilon$

$\forall\varepsilon>0\ \exists\delta(\varepsilon)$: $x>\delta(\varepsilon) \Rightarrow 0<f(x)<\varepsilon$

b) $\lim_{x\to0} f(x) = -\infty$

$\forall\varepsilon\ \exists\delta(\varepsilon)$: $0<|x-0|<\delta(\varepsilon) \Rightarrow f(x)<\varepsilon$

Grafico



5) $f(x) = 3x^2-2x+5$; $f'(x) = 6x-2$; $y = x+4 \Rightarrow m = 1 = f'(x) = 6x-2 \Rightarrow 6x = 3 \Rightarrow x = \frac{1}{2}$.

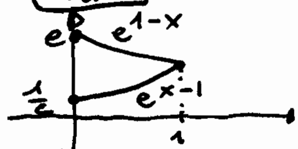
Equazione tangente in $x=\frac{1}{2}$: $y - f\left(\frac{1}{2}\right) = 1\cdot\left(x-\frac{1}{2}\right) \Rightarrow y = x-\frac{1}{2}+\frac{19}{4} \Rightarrow y = x+\frac{17}{4}$.

La retta $y = x+4$ non è la retta tangente; è solo parallela alla retta tangente.

MG2

6) $\int_0^1 e^{x-1} - e^{1-x}\,dx = \left(e^{x-1}+e^{1-x}\right|_0^1 = (1+1)-\left(\frac{1}{e}+e\right) = 2-\frac{1}{e}-e<0.$

Tale integrale non esprime un'area nel senso tradizionale



7) $A\cdot B\cdot X = \begin{Vmatrix}1 & x\\ 1 & 1\\ x & 1\end{Vmatrix}\cdot\begin{Vmatrix}1 & -1\\ -1 & 1\end{Vmatrix}\cdot\begin{Vmatrix}2\\ -2\end{Vmatrix} = \begin{Vmatrix}1 & x\\ 1 & 1\\ x & 1\end{Vmatrix}\cdot\begin{Vmatrix}4\\ -4\end{Vmatrix} = \begin{Vmatrix}4-4x\\ 0\\ 4x-4\end{Vmatrix}.$ $(4-4x;4x-4)\cdot(1;1;1)=4-4x+4x-4=0$ $\forall x\in\mathbb{R}.$

8) $f(x;y) = x^3-3x+xy+y^2-5y.$

$\begin{cases} f'_x = 3x^2-3+y=0\\ f'_y = x+2y-5=0\end{cases} \Rightarrow \begin{cases} y = 3-3x^2\\ x+6-6x^2-5=0\end{cases} \Rightarrow 6x^2-x-1=0 \Rightarrow x = \frac{1\pm\sqrt{1+24}}{12} = \frac{1\pm 5}{12} \left\langle \begin{matrix}\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\right.$

$\Rightarrow \begin{cases} x=\frac{1}{2}\\ y=\frac{9}{4}\end{cases}$ e $\begin{cases} x=-\frac{1}{3}\\ y=\frac{8}{3}\end{cases}$. $H(x;y) = \begin{Vmatrix}6x & 1\\ 1 & 2\end{Vmatrix}$

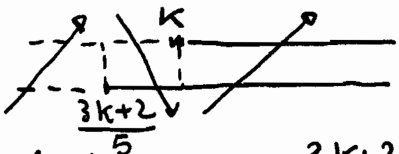
$H\left(\frac{1}{2};\frac{9}{4}\right) = \begin{Vmatrix}3 & 1\\ 1 & 2\end{Vmatrix} \Rightarrow \begin{cases}3>0;\ 2>0\\ 6-1>0\end{cases}$: P. Minimo; $H\left(-\frac{1}{3};\frac{8}{3}\right) = \begin{Vmatrix}-2 & 1\\ 1 & 2\end{Vmatrix} \Rightarrow -4-1<0$: P. Sella.

9) $f(x) = (x-1)^3(x-k)^2$. $f'(x) = 3(x-1)^2\cdot(x-k)^2+(x-1)^3\cdot 2(x-k) \Rightarrow$

$\Rightarrow f'(x) = (x-1)^2\cdot(x-k)\cdot\left(3(x-k)+2(x-1)\right) = (x-1)^2(x-k)\cdot(5x-3k-2) \geq 0$

$x-k\geq 0:\ x\geq k$

$5x-3k-2\geq 0:\ x\geq \frac{3k+2}{5}$



$\frac{3k+2}{5}<k \Rightarrow 3k+2<5k \Rightarrow 2k>2 \Rightarrow k>1.$

Per avere P. Max in $x=3$ dovrà essere: $\frac{3k+2}{5}=3 \Rightarrow 3k+2=15 \Rightarrow k=\frac{13}{3}.$

10)

A B C	A ⇒ B	B ⇒ C	A ⇒ C	C ⇒ A	
1 1 1	1	1	1	1	
1 1 0	~~1~~	~~0~~	~~0~~	~~1~~	
1 0 1	~~0~~	~~1~~	~~1~~	~~1~~	
1 0 0	~~0~~	~~1~~	~~0~~	~~1~~	
0 1 1	1	1	1	0	*
0 1 0	~~1~~	~~0~~	~~1~~	~~1~~	
0 0 1	1	1	1	0	*
0 0 0	1	1	1	1	

Vengono tolte le righe II; III; IV; VI dove A ⇒ B oppure B ⇒ C risultano false. Quindi si vede che solo la P_3: A ⇒ C risulta vera.