Compito di Matematica Generale del 6/9/2017 $\boxed{MG1}$

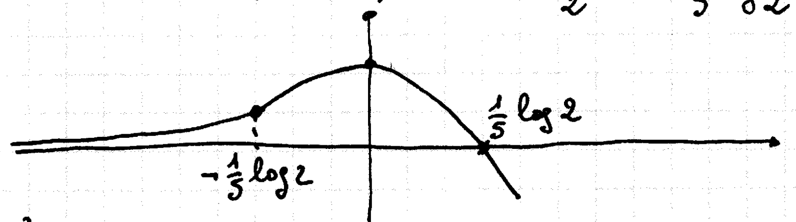
1) $f(x)=2e^{5x}-e^{10x}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=0^+$; $\lim\limits_{x\to+\infty} f(x)=-\infty$.

$f(x)\geq 0: e^{5x}\left(2-e^{5x}\right)\geq 0: \ e^{5x}\leq 2 \Rightarrow x\leq \frac{1}{5}\log 2$ $\quad + \ \frac{1}{5}\log 2 \ -$

$f'(x)=10e^{5x}-10e^{10x}=10e^{5x}\left(1-e^{5x}\right)\geq 0 \Rightarrow e^{5x}\leq 1 \Rightarrow x\leq 0.$ $\quad 0$

$f''(x)=50e^{5x}-100e^{10x}=50e^{5x}\left(1-2e^{5x}\right)\geq 0 \Rightarrow e^{5x}\leq\frac{1}{2}\Rightarrow x\leq\frac{1}{5}\log\frac{1}{2}=-\frac{1}{5}\log 2$ $\quad -\frac{1}{5}\log 2$

Grafico:



2) $\lim\limits_{x\to 0}\dfrac{(1+\operatorname{sen}x)^3-1}{3^x-1}=\lim\limits_{x\to 0}\dfrac{(1+\operatorname{sen}x)^3-1}{\operatorname{sen}x}\cdot\dfrac{\operatorname{sen}x}{x}\cdot\dfrac{x}{3^x-1}=3\cdot 1\cdot\dfrac{1}{\log 3}=3\cdot\log_3 e=\log_3 e^3.$

$\lim\limits_{x\to 0}\left(\dfrac{1+x+x^2+x^3}{x^2}\right)^{x-1}=\left(\dfrac{\to 1}{\to 0^+}\right)^{(\to -1)}=\left(\to +\infty\right)^{(\to -1)}=0^+.$

3)

A	B	C	$(A\Leftrightarrow B)$	$(C\Rightarrow A)$	$(B\Rightarrow C)$	$[(C\Rightarrow A)\ o\ (B\Rightarrow C)]$	$(A\Leftrightarrow B)$ e $[(C\Rightarrow A)\ o\ (B\Rightarrow C)]$
1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1
0	1	1	0	0	1	1	0
0	1	0	0	1	0	1	0

4) $f(x)=\log\left(1-e^x\right)$. C.E.: $1-e^x>0\Rightarrow e^x<1\Rightarrow x<0$.

$f'(x)=\dfrac{-e^x}{1-e^x}<0\ \forall x<0$; $f''(x)=-\dfrac{e^x(1-e^x)-e^x(-e^x)}{(1-e^x)^2}=\dfrac{-e^x}{(1-e^x)^2}<0\quad\forall x<0.$

Funzione invertibile e sempre concava $\forall x<0$.

$y=\log(1-e^x)\Rightarrow 1-e^x=e^y\Rightarrow e^x=1-e^y\Rightarrow x=\log(1-e^y)$. Inversa $f^{-1}(x)=\log\left(1-e^x\right)=f(x)$.

5) $\displaystyle\int_0^1\frac{e^{3x}-e^x}{e^{2x}}dx=\int_0^1 e^x-e^{-x}\,dx=\left(e^x+e^{-x}\right|_0^1=\left(e+\frac{1}{e}\right)-\left(1+1\right)=e+\frac{1}{e}-2.$

6) $Y=A\cdot X=\begin{Vmatrix}1 & -1 & 2\\ 2 & 1 & 0\end{Vmatrix}\cdot\begin{Vmatrix}1\\ m\\ k\end{Vmatrix}=\begin{Vmatrix}1-m+2k\\ 2+m+0\end{Vmatrix}$. $Y\perp(1;1):\left(1-m+2k;2+m\right)\cdot\left(1;1\right)=0\Rightarrow$

$\Rightarrow 1-m+2k+2+m=3+2k=0\Rightarrow k=-\frac{3}{2}\Rightarrow Y=\left(-m-2;m+2\right).$

$\|v\| = \sqrt{2} \Rightarrow \sqrt{(-m-2)^2 + (m+2)^2} = \sqrt{2} \Rightarrow 2 \cdot |m+2| = 2 \Rightarrow m+2 = \pm 1 \Rightarrow m = -1$ ó $m = -3$.

7) $f(x) = x^2 - 1$ e $g(x) = x^2 + 1$. $f(x) \sim g(x) \Leftrightarrow \lim_{x \to x_0} \frac{x^2-1}{x^2+1} = 1$ vere per $x \to -\infty$ e per $x \to +\infty$.

$f(x) = o(g(x)) \Leftrightarrow \lim_{x \to x_0} \frac{x^2-1}{x^2+1} = 0$ vere per $x \to -1$ e per $x \to +1$.

$g(x) = o(f(x)) \Leftrightarrow \lim_{x \to x_0} \frac{x^2+1}{x^2-1} = 0$: Impossibile.

8) $f(x;y) = x + y - x^2y^2$. $\begin{cases} f'_x = 1 - 2xy^2 = 0 \\ f'_y = 1 - 2x^2y = 0 \end{cases} \Rightarrow \begin{cases} 1 - 2 \cdot x \cdot \frac{1}{4x^4} = 1 - \frac{1}{2x^3} = 0 \\ y = \frac{1}{2x^2} \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x^3 = \frac{1}{2} \Rightarrow x = \frac{1}{\sqrt[3]{2}} \\ y = \frac{1}{2x^2} = \frac{1}{2} \cdot \sqrt[3]{4} = \frac{1}{\sqrt[3]{2}} \end{cases}$. $H(x;y) = \begin{Vmatrix} -2y^2 & -4xy \\ -4xy & -2x^2 \end{Vmatrix}$; $H\left(2^{-\frac{1}{3}}; 2^{-\frac{1}{3}}\right) = \begin{Vmatrix} -2 \cdot 2^{-\frac{2}{3}} & -4 \cdot 2^{-\frac{2}{3}} \\ -4 \cdot 2^{-\frac{2}{3}} & -2 \cdot 2^{-\frac{2}{3}} \end{Vmatrix}$

$H\left(2^{-\frac{1}{3}}; 2^{-\frac{1}{3}}\right) = 2^{-\frac{2}{3}} \begin{Vmatrix} -2 & -4 \\ -4 & -2 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| = -2 < 0 \\ |H_2| = 4 - 16 < 0 \end{cases}$: Punto di Sella.

9) $f(x;y;z) = x^3y - e^{x-y} + \cos(y-z)$

$\nabla f(x;y;z) = \left(3x^2y - e^{x-y} ; x^3 + e^{x-y} - \operatorname{sen}(y-z) ; \operatorname{sen}(y-z)\right)$.

$\nabla f(1;1;1) = (3 - e^0 ; 1 + e^0 - \operatorname{sen} 0 ; \operatorname{sen} 0) = (2 ; 2 ; 0)$

10) $f(x) = e^{1-kx}$; $f(0) = e$. $f'(x) = -k e^{1-kx}$; $f'(0) = -k \cdot e$. $f''(x) = k^2 e^{1-kx}$; $f''(0) = k^2 \cdot e$.

$P_2(x; 0) = e - k \cdot e \cdot x + \frac{1}{2} \cdot k^2 e x^2 = e - 2ex + 2ex^2$.

Vera per $k = 2$.