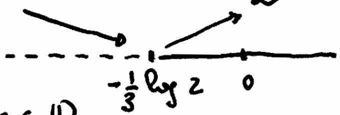
Compito di Matematica Generale del 22/9/2017 $\boxed{MG1}$

1) $f(x) = e^{2x} + e^{-x} = e^{2x} + \frac{1}{e^x} = \frac{e^{3x}+1}{e^x}$. C.E. $= \mathbb{R}$. $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x) = +\infty$.

$f(x) \geq 0 \ \forall x \in \mathbb{R}$. $f(0) = 2$.

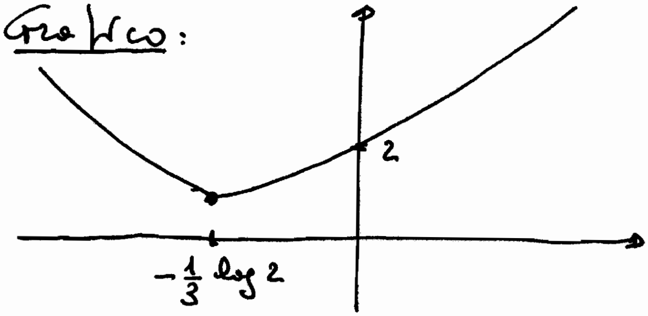
$f'(x) = 2e^{2x} - e^{-x} = \frac{2e^{3x}-1}{e^x} \geq 0 \Rightarrow e^{3x} \geq \frac{1}{2} \Rightarrow$

$\Rightarrow x \geq \frac{1}{3}\log\frac{1}{2} = -\frac{1}{3}\log 2$



$f''(x) = 4e^{2x} + e^{-x} > 0 \ \forall x \in \mathbb{R}$

funzione sempre convessa



2) $\lim_{x\to 0} \frac{\log(1+\operatorname{sen}x)}{3x+x^2} = \lim_{x\to 0} \frac{\log(1+\operatorname{sen}x)}{\operatorname{sen}x} \cdot \frac{\operatorname{sen}x}{x} \cdot \frac{x}{3x+x^2} = 1 \cdot 1 \cdot \frac{1}{3} = \frac{1}{3}$

$\lim_{x\to+\infty} \left(1 + \frac{1}{x^2+x}\right)^{x^2-1} = \lim_{x\to+\infty} \left[\left(1+\frac{1}{x^2+x}\right)^{x^2+x}\right]^{\frac{x^2-1}{x^2+x}} = \lim_{t\to+\infty}\left[\left(1+\frac{1}{t}\right)^t\right]^{\lim_{x\to+\infty}\frac{x^2-1}{x^2+x}} = e^1 = e.$

3)

A B C	non A	(non A ⇒ B)	C ⇒ B	A e (C ⇒ B)	(non A ⇒ B) o (A e (C ⇒ B))
~~1 1 1~~					
~~1 1 0~~					
1 0 1	0	1	0	0	1
~~1 0 0~~					
0 1 1	1	1	1	0	1
0 1 0	1	1	1	0	1
0 0 1	1	0	0	0	0 *
~~0 0 0~~					

La proposizione P non è una tautologia

4) $f(x) = \log(1+x)$; $f'(x) = \frac{1}{1+x}$; $f''(x) = -\frac{1}{(1+x)^2}$.

$f(2) = \log 3$; $f'(2) = \frac{1}{3}$; $f''(2) = -\frac{1}{9}$. $P_2(x;2) = \log 3 + \frac{1}{3}(x-2) + \frac{1}{2}\left(-\frac{1}{9}\right)\cdot(x-2)^2$.

$P_2(x;2) = \log 3 + \frac{1}{3}x - \frac{2}{3} - \frac{1}{18}x^2 + \frac{4}{18}x - \frac{4}{18} = \left(\log 3 - \frac{8}{9}\right) + \frac{5}{9}x - \frac{1}{18}x^2.$

5) $\int_0^\pi \cos x + \operatorname{sen}x \, dx = \left(\operatorname{sen}x - \cos x\right|_0^\pi = (0-(-1)) - (0-1) = 1+1 = 2.$

6) $f(x) = e^{1-x}$; $g(x) = x^2 - x$. $F(x) = f(g(x)) - g(f(x)) =$

$\boxed{MG2}$

$F(x) = f(x^2-x) - g(e^{1-x}) = e^{1-(x^2-x)} - (e^{1-x})^2 + e^{1-x} =$

$= F(x) = e^{1-x^2+x} - e^{2-2x} + e^{1-x}$. $F'(x) = (1-2x)e^{1-x^2+x} - (-2)e^{2-2x} - e^{1-x}$.

7) $f(x) = \dfrac{e^x-1}{e^x+1}$. C.E. $= \mathbb{R}$. $\lim\limits_{x\to-\infty} f(x) = -1$; $\lim\limits_{x\to+\infty} f(x) = 1$.

$f'(x) = \dfrac{e^x(e^x+1) - e^x(e^x-1)}{(e^x+1)^2} = \dfrac{2e^x}{(e^x+1)^2} > 0 \ \forall x$. Funzione strettamente crescente e quindi invertibile su tutto $\mathbb{R}$. $f: \mathbb{R} \to]-1;1[$. $f^{-1}(x):]-1;1[\to \mathbb{R}$.

$y = \dfrac{e^x-1}{e^x+1} \Rightarrow ye^x + y = e^x - 1 \Rightarrow e^x(1-y) = y+1 \Rightarrow e^x = \dfrac{1+y}{1-y} \Rightarrow x = \log\left(\dfrac{1+y}{1-y}\right)$.

Inversa: $f^{-1}(x) = \log\left(\dfrac{1+x}{1-x}\right)$.

8) $f(x;y) = x^2+y^2-x^3y^2 \Rightarrow \begin{cases} f'_x = 2x-3x^2y^2 = x(2-3xy^2) = 0 \\ f'_y = 2y-2x^3y = 2y(1-x^3) = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \cancel{\begin{cases} x=0 \\ x=1 \end{cases}} \cup \cancel{\begin{cases} x=\frac{2}{3y^2} \\ y=0 \end{cases}} \cup \begin{cases} x=1 \\ 2-3y^2=0 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=\pm\sqrt{\frac{2}{3}} \end{cases}$. $H(x;y) = \begin{Vmatrix} 2-6xy^2 & -6x^2y \\ -6x^2y & 2-2x^3 \end{Vmatrix}$.

$H(0;0) = \begin{Vmatrix} 2 & 0 \\ 0 & 2 \end{Vmatrix} \Rightarrow \begin{cases} |H_1|>0 \\ |H_2|>0 \end{cases}$: Minimo. $H\left(1;\sqrt{\frac{2}{3}}\right) = \begin{Vmatrix} -2 & -6\sqrt{\frac{2}{3}} \\ -6\sqrt{\frac{2}{3}} & 0 \end{Vmatrix}$: Sella; $H\left(1;-\sqrt{\frac{2}{3}}\right) = \begin{Vmatrix} -2 & 6\sqrt{\frac{2}{3}} \\ 6\sqrt{\frac{2}{3}} & 0 \end{Vmatrix}$: Sella.

9) $f(x;y) = xe^y - ye^x$. $\nabla f(x;y) = (e^y - ye^x; xe^y - e^x)$. $\nabla f(0;0) = (1;-1)$.

$H(f(x;y)) = \begin{Vmatrix} -ye^x & e^y-e^x \\ e^y-e^x & xe^y \end{Vmatrix}$. $H(1;1) = \begin{Vmatrix} -e & 0 \\ 0 & e \end{Vmatrix}$

$\nabla f(0;0) \cdot H(1;1) \cdot (\nabla f(0;0))^T = \begin{Vmatrix} 1 & -1 \end{Vmatrix} \cdot \begin{Vmatrix} -e & 0 \\ 0 & e \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -1 \end{Vmatrix} = \begin{Vmatrix} 1 & -1 \end{Vmatrix} \cdot \begin{Vmatrix} -e \\ -e \end{Vmatrix} = -e + e = 0$.

10) $f(x) = \log^2 x$. C.E. $=]0;+\infty[$. $\lim\limits_{x\to0^+} f(x) = \lim\limits_{x\to+\infty} f(x) = +\infty$. $f(x) \geq 0$.

$f'(x) = 2\log x \cdot \dfrac{1}{x}$. $f''(x) = 2\left(\dfrac{1}{x}\cdot\dfrac{1}{x} + \log x\left(-\dfrac{1}{x^2}\right)\right) = \dfrac{2}{x^2}\cdot(1-\log x) \geq 0$

per $\log x \leq 1 \Rightarrow x \leq e$. $f'(x) \geq 0$ per $x \geq 1$

Dato che la funzione è convessa in $]0;e[$ e dato che $x = \frac{1}{e}$ è interno a tale intervallo, la retta tangente sta al di sotto del grafico della funzione.