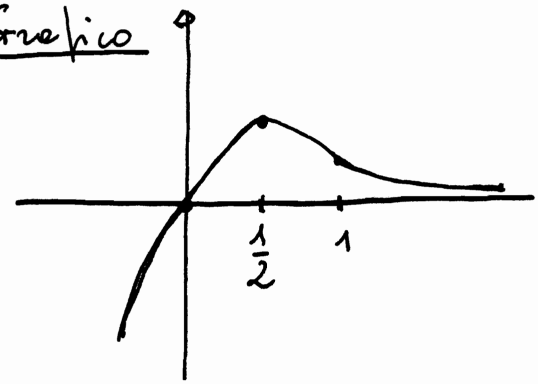
Compito di Matematica Generale del 14/10/2017 $\boxed{MG1}$

1) $f(x) = x \cdot e^{1-2x}$. C.E.: $\mathbb{R}$. $\lim\limits_{x \to -\infty} f(x) = (-\infty(+\infty)) = -\infty$; $\lim\limits_{x \to +\infty} f(x) = 0^+$.

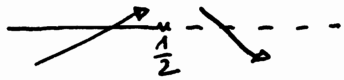
$f(x) \geq 0 \ \forall x \geq 0$. $f(0) = 0$.

Grafico



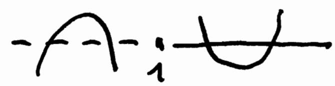
$f'(x) = 1 \cdot e^{1-2x} + x(-2)e^{1-2x} = (1-2x)e^{1-2x} \geq 0$

per $x \leq \frac{1}{2}$



$f''(x) = (-2)e^{1-2x} + (1-2x)(-2)e^{1-2x} = 4 \cdot e^{1-2x} \cdot (x-1) \geq 0$

per $x \geq 1$

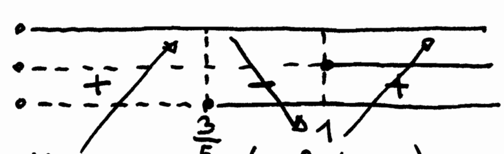


2) $\lim\limits_{x \to 0} \frac{1-\cos 2x}{1-\cos x} = \lim\limits_{x \to 0} \frac{1-\cos 2x}{4x^2} \cdot 4 \cdot \frac{x^2}{1-\cos x} = \frac{1}{2} \cdot 4 \cdot \frac{1}{\frac{1}{2}} = 4$. $\left(\lim\limits_{t \to 0} \frac{1-\cos t}{t^2} = \frac{1}{2}\right)$

$\lim\limits_{x \to +\infty} \left(1 - \frac{1}{2x}\right)^{3x-1} = \lim\limits_{x \to +\infty} \left[\left(1 + \frac{(-1)}{2x}\right)^{2x}\right]^{\frac{3x-1}{2x}} = \left(\to e^{-1}\right)^{\to \frac{3}{2}} = e^{-\frac{3}{2}} = \frac{1}{e\sqrt{e}}$.

3) $f(x) = x^3 \cdot (x-1)^2$. C.E. $= \mathbb{R}$. $f'(x) = 3x^2 \cdot (x-1)^2 + x^3 \cdot 2(x-1) = x^2(x-1) \cdot (3x-3+2x) \Rightarrow$

$\Rightarrow f'(x) = x^2 \cdot (x-1)(5x-3) \geq 0$ $\begin{cases} x^2 \geq 0 \ \forall x \\ x-1 \geq 0 \text{ per } x \geq 1 \\ 5x-3 \geq 0 \text{ per } x \geq \frac{3}{5} \end{cases}$

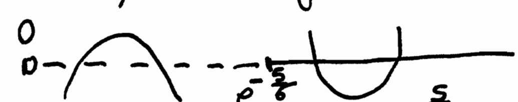


$x = \frac{3}{5}$ è punto di massimo (relativo); $x = 1$ è punto di minimo (relativo).

4) $f(x) = x^3 \cdot \log x$. C.E. $= \mathbb{R}^*_+ : x > 0$. $f'(x) = 3x^2 \log x + x^3 \cdot \frac{1}{x} = x^2(3\log x + 1)$.

$f''(x) = 2x(3\log x + 1) + x^{\cancel{2}} \cdot \frac{3}{\cancel{x}} = x(6\log x + 2 + 3) = x \cdot (6\log x + 5) \geq 0$ per

$\log x \geq -\frac{5}{6} \Rightarrow x \geq e^{-\frac{5}{6}} = \frac{1}{\sqrt[6]{e^5}}$.



La funzione è concava in $]0; e^{-\frac{5}{6}}[$, convessa in $]e^{-\frac{5}{6}}; +\infty[$.

Ha un unico punto di flesso in $x = e^{-\frac{5}{6}}$.

5) $\int_0^1 \frac{x}{x+2}\,dx = \int_0^1 \frac{x+2-2}{x+2}\,dx = \int_0^1 \frac{x+2}{x+2} - \frac{2}{x+2}\,dx = \int_0^1 1 - \frac{2}{x+2}\,dx = \Big(x - 2\log(x+2)\Big|_0^1 =$

$= (1 - 2\log 3) - (0 - 2\log 2) = 1 + 2(\log 2 - \log 3) = \log e + 2\log\frac{2}{3} = \log e + \log\frac{4}{9} = \log\left(\frac{4}{9}e\right)$.

$\boxed{MG2}$

6) $\begin{Vmatrix}1 & 2 & -1\\ 2 & 0 & 1\end{Vmatrix}\cdot\begin{Vmatrix}1 & k\\ m & m\\ k & 1\end{Vmatrix}\cdot\begin{Vmatrix}1\\ 1\end{Vmatrix} = \begin{Vmatrix}1 & 2 & -1\\ 2 & 0 & 1\end{Vmatrix}\cdot\begin{Vmatrix}1+k\\ m+m\\ k+1\end{Vmatrix} = \begin{Vmatrix}1+k+4m-k-1\\ 2+2k+0+k+1\end{Vmatrix} =$

$= \begin{Vmatrix}4m\\ 3k+3\end{Vmatrix} = \begin{Vmatrix}4\\ 6\end{Vmatrix}$ se $m = 1 = k$.

7) $f(x;y) = x^3+y^2-3x+2y$. $\nabla f(x;y) = 0 \Rightarrow \begin{cases} f'_x = 3x^2-3 = 3(x^2-1) = 0\\ f'_y = 2y+2 = 2(y+1) = 0\end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = 1\\ y = -1\end{cases} \cup \begin{cases} x = -1\\ y = -1\end{cases}$. $H(x;y) = \begin{Vmatrix}6x & 0\\ 0 & 2\end{Vmatrix}$.

$H(1;-1) = \begin{Vmatrix}6 & 0\\ 0 & 2\end{Vmatrix} : \begin{cases} 6>0;\ 2>0\\ 12>0\end{cases}$: P. di Minimo. $H(-1;-1) = \begin{Vmatrix}-6 & 0\\ 0 & 2\end{Vmatrix} : \begin{cases} -6<0\\ 2>0\end{cases}$: P. di Sella.

8) P: Marco è buono SE Marta è gentile Significa: Marta è gentile $\Rightarrow$ Marco è buono.

Quindi basta fare la tavola di verità di $P_2 \Rightarrow P_1$.

P_2	P_1	$P_2 \Rightarrow P_1$
1	1	1
1	0	0
0	1	1
0	0	1

9) $f(x) = x^2-x$. Eq. Retta tangente: $y-f(x_0) = f'(x_0)(x-x_0)$. In $x=1$:

$f(1) = 0$; $f'(x) = 2x-1$; $f'(1) = 2-1 = 1 \Rightarrow$ Eq. r. tg: $y-0 = 1(x-1) \Rightarrow y = x-1$. $(m=1)$.

Equazione retta perpendicolare in $x=1$: $y-0 = (-1)(x-1) \Rightarrow y = 1-x$. $\left(-1 = -\frac{1}{m}\right)$

10) $f(x) = 2x-3$; $g(x) = 3^{x-1}$.

$f(g(x)) = f(3^{x-1}) = 2\cdot 3^{x-1}-3 = y \Rightarrow 2\cdot 3^{x-1} = y+3 \Rightarrow 3^{x-1} = \frac{1}{2}(y+3) \Rightarrow$

$\Rightarrow x-1 = \log_3\left(\frac{1}{2}(y+3)\right) \Rightarrow x = \log_3\left(\frac{1}{2}(y+3)\right)+1$. Inversa: $y = \log_3\left(\frac{1}{2}(x+3)\right)+1$.

$g(f(x)) = g(2x-3) = 3^{2x-3-1} = 3^{2x-4} = y \Rightarrow 2x-4 = \log_3 y \Rightarrow$

$\Rightarrow 2x = \log_3 y + 4 \Rightarrow x = \frac{1}{2}\log_3 y + 2$. Inversa: $y = \frac{1}{2}\log_3 x + 2$.