TASKS of MATHEMATICS
for Economic Applications AA. 2016/17

Intermediate Test December 2017

I M 1) Given the complex number $z = (2i)^{19} \cdot (1+i)^{16}$, find the square roots of $\overline{z}$, the conjugate number of z.

I M 2) Given the linear system $\begin{cases} mx_1 + nx_2 + x_3 + x_4 = 0 \\ mx_1 + nx_2 + kx_3 + x_4 = 0 \\ mx_1 + nx_2 + kx_3 + hx_4 = 0 \end{cases}$ depending from parameters m, n, k and h, find conditions on parameters such that the system has ∞^1 solutions.

I M 3) Consider the linear map $F : \mathbb{R}^4 \to \mathbb{R}^4, F(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} m & 1 & 0 & 0 \\ 1 & k & 0 & 0 \\ 0 & 0 & k & 1 \\ 0 & 0 & 1 & m \end{Vmatrix}$; check, depending on parameters m and k, the dimension of the image of F, and when such dimension is minimum find a basis for the kernel and a basis for the image of F.

I M 4) Find the matrix $\mathbb{A}_2$ which admits the eigenvector $(1,1)$ for the eigenvalue $\lambda = 1$ and the eigenvector $(1,-1)$ for the eigenvalue $\lambda = -1$.

I M 5) The matrix $\mathbb{A} = \begin{Vmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & k \end{Vmatrix}$ admits the multiple eigenvalue $\lambda = 1$. Find the value of k and determine an orthogonal matrix which diagonalizes $\mathbb{A}$.

I Winter Exam Session 2017

I M 1) Determine the three solutions of the equation $(x-i)^3 = i$.

I M 2) Given $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 1 \end{Vmatrix}$, determine its eigenvalues and the corresponding eigenvectors. Check if the matrix is or not diagonalizable.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 0 & 1 \\ 0 & -2 & k \end{Vmatrix}$ consider the linear map $\mathbb{R}^3 \to \mathbb{R}^4$: $\mathbb{A} \cdot \mathbb{X} = \mathbb{Y}$. Since $\text{Dim}(\text{Imm}) = 2$, determine the value for the real parameter k and then find all the vectors $\mathbb{X}$ for which $\mathbb{A} \cdot \mathbb{X} = (2,1,3,-1)$.

I M 4) - Given the linear system $\begin{cases} mx_1 + nx_2 + x_3 + x_4 + x_5 = 0 \\ mx_1 + nx_2 + kx_3 + x_4 + x_5 = 0 \\ mx_1 + nx_2 + kx_3 + hx_4 + x_5 = 0 \end{cases}$ depending from parameters m, n, k and h, find conditions on the parameters such that the system has ∞^2 solutions.

II M 1) With the equation $f(x,y,z) = xyz + x^2y + xz^2 - yz = 0$, satisfied at the point $P = (1,-1,1)$, we can define an implicit function $(x,y) \to z$. Calculate $\nabla z(1,-1)$.

II M 2) Given $f(x,y) = x^2 + y^2 - x^2y^2$, analyze its stationary points.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = y - x \\ \text{u.c.} \begin{cases} x^2 + y^2 \le 1 \\ x \le y \end{cases} \end{cases}$.

II M 4) Given the function $f(x,y) = xy\, e^{x-y}$ and the unit vectors $v = (\cos\alpha, \sin\alpha)$ and $w = (\sin\alpha, \cos\alpha)$, determine proper values for α such that $D^2_{v,w} f(1,1) = 0$.

II Winter Exam Session 2017

I M 1) Calculate the three roots $\sqrt[3]{(i^2-1)^5 (i+1)^2}$.

I M 2) The matrix $\mathbb{A} = \begin{Vmatrix} 3 & 2 & 1 \\ 1 & 4 & k \\ 1 & 2 & -3 \end{Vmatrix}$ admits the multiple eigenvalue $\lambda = 2$. Find the value of the parameter k and check if the matrix is a diagonalizable one.

I M 3) Given $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 2 & -2 \\ 1 & 2 & 1 & 1 \\ 3 & 2 & m & k \end{Vmatrix}$ consider the linear map $\mathbb{R}^4 \to \mathbb{R}^3$: $\mathbb{A} \cdot \mathbb{X} = \mathbb{Y}$. If the dimension of Ker($\mathbb{A}$) is the maximum possible, determine the values of the parameters m and k and then find a basis for Ker($\mathbb{A}$) and a basis for Imm($\mathbb{A}$).

I M 4) Verify that the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 2 \\ 1 & 3 \end{Vmatrix}$ is similar to the matrix $\mathbb{B} = \begin{Vmatrix} 0 & -1 \\ 1 & 4 \end{Vmatrix}$ and find a matrix $\mathbb{P}$ that realizes the similarity between $\mathbb{A}$ and $\mathbb{B}$.

II M 1) With the equation $f(x,y) = e^{y^2-x^2} - e^{x-y} = 0$, satisfied at the point $P = (1,1)$, we can define an implicit function $x \to y(x)$. Calculate $y'(1)$ and $y''(1)$.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y,z) = x^2 + y^2 + z^2 \\ \text{u.c. } x + y + z = -1 \end{cases}$.

II M 3) Given $f(x,y) = x^2 y + xy^2$, $P_0 = (1, -1)$, v and w the unit vectors of $(1,1)$ and $(-1,1)$, calculate $D_v f(P_0)$ and $D^2_{v,w} f(P_0)$.

II M 4) Calculate the Gradient vector of the function $f(x,y,z) = x^{2y-z^3} + (xz)^{(y+2)^3}$.

I Additional Exam Session 2017

I M 1) Determine the algebraic form of the complex number $z = e^{1-6i}$ and then draw it in the complex plane.

I M 2) Find a matrix which diagonalizes $\mathbb{A} = \begin{Vmatrix} 2 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{Vmatrix}$.

I M 3) Given the matrix $\mathbb{A}$ of exercise I M 2) above, find its inverse matrix.

I M 4) Given the linear map $f : \mathbb{R}^3 \to \mathbb{R}^3$: $f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, if the vectors $\mathbb{X}_1 = (0,1,1)$ and $\mathbb{X}_2 = (1,1,0)$ belong to the Kernel of the map and if $f(1,1,1) = (2,1,-1)$ find the matrix $\mathbb{A}$ and a basis for the Image of the map.

II M 1) The equation $f(x,y,z) = xe^{y+z} - ye^{x+z} + ze^{x+y} = 0$ defines an implicit function $z = z(x,y)$ at point $P = (1,1,0)$. Determine $\nabla z(1,1)$.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = x^2 + xy^2 \\ \text{u.c. } 4x^2 + y^2 \le 4 \end{cases}$.

II M 3) For $f(x,y,z) = \log(x-y) - e^{z-x} + x^2 y - yz^3$ and $P_0 = (1,0,1)$, calculate $D_v f(P_0)$, where v is the unit vector of $(1,1,1)$.

II M 4) Given the function $f(x,y)=(x+y)e^{(x-y)}$, find the expression of its Mac Laurin polynomial of degree 2 at point $(0,0)$.

I Summer Exam Session 2017

I M 1) After finding the solutions of the equation $z^4-3i\,z^3-z^2-3i\,z-2=0$, determine the square roots of the solution having the maximum modulus.

I M 2) Given the matrix $\mathbb{A}=\begin{Vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & k & 0 \\ 0 & k & 0 & 1 \\ 0 & 0 & 1 & 0 \end{Vmatrix}$, since it admits the eigenvalue $\lambda=1$, find an orthogonal matrix which diagonalizes $\mathbb{A}$.

I M 3) In the basis $\mathbb{W}=\{(1,2,1);(2,-1,0);(k,k,k)\}$ the vector $\mathbb{Y}=(-1,4,1)$ has coordinates $(2,-1,1)$. Determine the value of the parameter k.

I M 4) Given the linear system $\begin{cases} x_1+2x_2-x_4=2 \\ 2x_1+x_2-x_3+m\,x_4=k \\ -x_1+4x_2+2k\cdot x_3+m\,x_4=4 \end{cases}$, verify that it always admits solutions for every value of the parameters m and k. Finally, determine the number of such solutions by varying m and k.

II M 1) With the system $\begin{cases} f(x,y,z)=x^2+y^2+z^2=3 \\ g(x,y,z)=xy-yz+xz=1 \end{cases}$ and the point $P=(-1,1,-1)$ which type of implicit function can we define ? Then calculate its first order derivatives.

II M 2) Find $\text{Max/min}\ f(x,y)=x^2+y^3$ in the region $\{(x,y): x\geq 0, y\geq 0, y\leq 1-x\}$.

II M 3) For $f(x,y)=x^2y-xy^2$ and $P_0=(1,1)$, given $v=(\cos\alpha,\sin\alpha)$, verify that if $D_v f(P_0)=0$, then surely also $D^2_{v,v}f(P_0)=0$.

II M 4) For $f(x,y,z)=\log\left(x^2-y\right)+ze^{zx}+\cos(y-z)$ calculate $\nabla f(1,0,1)$.

II Summer Exam Session 2017

I M 1) Find the solutions z_1, z_2 and z_3 of the equation $z^3-z^2+4z-4=0$ and then verify that $z_1\cdot z_2\cdot z_3=e^{\log 4+2\pi i}$.

I M 2) Given the linear map $F:\mathbb{R}^4\to\mathbb{R}^3$, defined as:
$F(x_1,x_2,x_3,x_4)=(x_1+x_2+x_3+x_4, x_1-x_2+x_3-x_4, mx_1+mx_2+x_3+x_4)$,
study, by varying the parameter m, the dimensions of the Kernel and of the Image of F and find a basis for the Kernel and a basis for the Image when the dimension of the Kernel is maximum.

I M 3) Given the matrices $\mathbb{A}=\begin{Vmatrix} 2 & 1 \\ 1 & 2 \end{Vmatrix}$ and $\mathbb{B}=\begin{Vmatrix} a & 0 \\ 0 & b \end{Vmatrix}$, since $\mathbb{A}$ and $\mathbb{B}$ have the same caractheristic polynomial, find the values of a and b, and determine the dimensions of the eigenspaces associated to the eigenvalues of the matrix $\mathbb{B}$.

I M 4) Find, if it exists, a modal matrix which diagonalizes $\mathbb{A}=\begin{Vmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 2 & 2 & -2 \end{Vmatrix}$.

II M 1) Given the function $f(x,y)=x^3-y^3$ and the vectors $v=(1,0)$ and $w=(0,1)$, find the point P where $D_v f(P)=D_w f(P)$.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y,z) = x - 2y + 4z \\ \text{u.c. } x^2 + y^2 + z^2 = \dfrac{21}{4} \end{cases}$.

II M 3) Given the function $f(x,y,z) = xe^y - ye^z + xz$, find the expression of its Mac Laurin polynomial of degree 2 at point $(0,0,0)$.

II M 4) With the system $\begin{cases} f(x,y,z) = x \operatorname{sen} y - z\cos x + e^{z(x-y)} = 0 \\ g(x,y,z) = x^3 y - yz + xyz = 0 \end{cases}$, satisfied at point $P = (0,0,1)$, we define an implicit function $x \to (y,z)$. Calculate $\dfrac{\partial(y,z)}{\partial(x)}$.

I Autumn Exam Session 2017

I M 1) Calculate the cubic roots of the complex number $z = -27i^3$.

I M 2) The matrix $\mathbb{A} = \begin{Vmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & -9 & -3 \end{Vmatrix}$ admits the eigenvalue $\lambda = 2$. Check if the matrix is a diagonalizable one.

I M 3) In the basis $\mathbb{W} = \{(1,-1,2);(2,1,1);(1,2,k)\}$ the vector $\mathbb{X} = (1,-1,2)$ has co-ordinates $(2,-1,1)$. Determine the value of the parameter k and calculate the modulus of the vector $\mathbb{X}$.

I M 4) The two non-singular matrices $\mathbb{A}$ and $\mathbb{B}$ have the eigenvector $\mathbb{X}$ corresponding to the eigenvalue λ. Find, for the matrix $\mathbb{M} = \mathbb{A}^2 \cdot \mathbb{B}^{-1} + 3\mathbb{B} \cdot \mathbb{A}^{-1} \cdot \mathbb{B}$, the eigenvalue to which corresponds the eigenvector $\mathbb{X}$.

II M 1) Determine if there are values (x,y) that minimize or maximise the value of the determinant of the matrix $\mathbb{A}(x,y) = \begin{Vmatrix} x & y \\ x^2 & e^y \end{Vmatrix}$.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = x^2 y \\ \text{u.c. } x^2 + y^2 \le 1 \end{cases}$.

II M 3) Given the function $f(x,y) = x^2 + y - 2xy$, let u and v be the unit vectors respectively of $\mathbb{X}_1 = (1,1)$ and $\mathbb{X}_2 = (1,-1)$. Find the point P at which it results: $\begin{cases} \mathcal{D}_u f(P) = 0 \\ \mathcal{D}_v f(P) = \sqrt{2} \end{cases}$ and then calculate $\mathcal{D}^2_{u,v} f(P)$.

II M 4) The equation $f(x,y) = (x+y) \cdot \log\left(1 + x^2 + y^2\right) = 0$ at point $P = (1,-1)$ defines an implicit function $y = y(x)$; calculate $y'(1)$.

II Autumn Exam Session 2017

I M 1) Given the complex number $z = 2 + i$, find the square roots of $w = \dfrac{i - \overline{z}}{i + z}$, where $\overline{z}$ is the conjugate of z.

I M 2) For the linear map $F\colon \mathbb{R}^4 \to \mathbb{R}^2$, with
$F(x_1,x_2,x_3,x_4) = (x_1 + x_2 + x_3 + x_4, mx_1 + x_2 + x_3 + mx_4)$,
since $F(1,1,1,1) = (4,4)$, find the dimension and a basis for the Kernel and for the Image of F.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 2 & 2 & 1 \\ 0 & 1 & -3 \\ 1 & 2 & k \end{Vmatrix}$, verify that it admits the same eigenvalue for every value of the parameter k and after determining the value of k for which this eigenvalue is a multiple one, examine in this case the diagonalizability of the given matrix.

I M 4) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 1 \end{Vmatrix}$ and the vector $\mathbb{X} = \begin{Vmatrix} 1 \\ k \\ 0 \end{Vmatrix}$, determine the values of the parameter k for which the vector $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$ has its modulus equal to 3.

II M 1) Analyze the nature of the stationary points of the function $f(x,y) = x^2 - y^2 + xy^2$.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = xy \\ \text{u.c. } 0 \le y \le 2x - x^2 \end{cases}$.

II M 3) Given $f(x,y) = e^{x-y} - e^{y-x}$ and the unit vector $v = (\cos\alpha, \operatorname{sen}\alpha)$, determinate α if $D_v f(1,1) = 0$ and then calculate $D^2_{v,w} f(1,1)$ where w is any unit vector.

II M 4) Given the system $\begin{cases} f(x,y,z) = x^2 y - e^{x-z} + e^{z-y} = 1 \\ g(x,y,z) = xyz + e^{x-z} - e^{z-y} = 1 \end{cases}$, verify that with it it is possible to define in a neighborhood of the point $(1,1,1)$ an implicit function $z \to (x(z), y(z))$ and then calculate the equation of the tangent line to that function at $z = 1$.

II Additional Exam Session 2017

I M 1) Calculate the cubic roots of the number $z = \dfrac{9}{i-1} + \dfrac{9}{1+i}$.

I M 2) For the linear map $F\colon \mathbb{R}^4 \to \mathbb{R}^3, F(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} 1 & 2 & -1 & 1 \\ 2 & 2 & 1 & k \\ 1 & 2 & m & 1 \end{Vmatrix}$, since the vector $(1,1,2,-1)$ belongs to the Kernel, find $F(1,1,2,2)$ and the dimension of the Kernel and of the Image of F.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & m & -3 \end{Vmatrix}$, it admits the same real eigenvalue for every value of the parameter m and also admits the complex eigenvalue $1+i$. Find the value of the parameter m and all the eigenvalues of the matrix.

I M 4) In the basis $\{\mathbb{W}_1 : (1,1,-1); \mathbb{W}_2 : (1,0,1); \mathbb{W}_3 : (x,y,z)\}$ the vector $\mathbb{X} = (1,1,1)$ has coordinates $(2,1,2)$. Determine the vector $\mathbb{W}_3$.

II M 1) Given the function $f(x,y) = x^2 + y^2 - xy^2 - x$, analyze the nature of its stationary points.

II M 2) Given the equation $f(x,y) = xy - e^{y-x} = 0$, verify that, with it, it is possible to define, in a neighborhood of the point $(1,1)$, an implicit function and calculate the first and second derivatives of such function at the proper point.

II M 3) Given $f(x,y) = e^x - e^y + x - y$ and the unit vector $v = (\cos\alpha, \operatorname{sen}\alpha)$, determinate the value of α if $D_v f(0,0) = 0$ and then calculate $D^2_{v,v} f(0,0)$.

II M 4) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 \\ \text{u.c. } 0 \le y \le 1 - x^2 \end{cases}$.