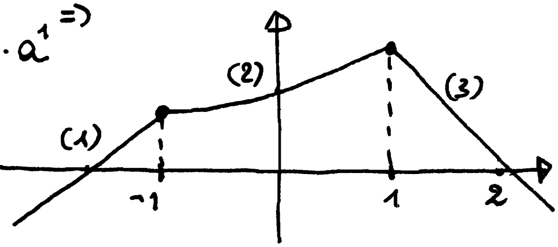
Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito A1}}$

1) $f(x)=\begin{cases} x+\frac{3}{2} & : x<-1 \\ k\cdot a^x & : -1\le x\le 1 \\ 6-3x & : 1<x \end{cases}$. Per la continuità dovrà risultare: $\begin{cases} -1+\frac{3}{2}=k\cdot a^{-1} \\ 6-3\cdot 1=k\cdot a^1 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} \frac{k}{a}=\frac{1}{2} \\ k\cdot a=3 \end{cases} \Rightarrow \begin{cases} k=\frac{1}{2}a \\ \frac{1}{2}a^2=3 \end{cases} \Rightarrow \begin{cases} a=\sqrt{6} \\ k=\frac{1}{2}\sqrt{6} \end{cases} \Rightarrow y=\frac{1}{2}\sqrt{6}\cdot(\sqrt{6})^x$. Grafico:



2) $\lim_{x\to 0}\frac{(1+x^2)^3-\cos 2x}{2x^2}=\lim_{x\to 0}\frac{(1+x^2)^3-1}{2x^2}+\frac{1-\cos 2x}{4x^2}\cdot 2=3\cdot\frac{1}{2}+\frac{1}{2}\cdot 2=\frac{3}{2}+1=\frac{5}{2}$.

$\lim_{x\to+\infty}\left(\frac{1+x+x^3}{\operatorname{sen}x+x^2}\right)^{\frac{x-1}{1-2x}}=\lim_{x\to+\infty}\left(\frac{x^3}{x^2}\right)^{\frac{x}{-2x}}=\left(\to+\infty\right)^{\left(\to-\frac{1}{2}\right)}=0^+$.

3) $f(x)=3^{1-2x}=y\Rightarrow 1-2x=\log_3 y\Rightarrow 2x=1-\log_3 y\Rightarrow x=\frac{1}{2}\left(1-\log_3 y\right);\ f^{-1}: y=\frac{1}{2}-\frac{1}{2}\log_3 x$.

$g(x)=\frac{x+1}{2x-1}=y\Rightarrow x+1=2xy-y\Rightarrow x(2y-1)=y+1\Rightarrow x=\frac{y+1}{2y-1};\ g^{-1}: y=\frac{x+1}{2x-1}$.

$g^{-1}(f^{-1}(x))=g^{-1}\left(\frac{1}{2}-\frac{1}{2}\log_3 x\right)=\frac{\frac{1}{2}-\frac{1}{2}\log_3 x+1}{1-\log_3 x-1}=\frac{\log_3 x-3}{2\log_3 x}$.

4) $f(x)=\log\left(\frac{1+x}{1-x}\right)$. c.e. $\frac{1+x}{1-x}>0$ $\begin{cases} 1+x>0: x>-1 \\ 1-x>0: x<1 \end{cases}$ $-\ \ +\ \ -$. C.E.: $-1<x<1$.

La proposizione P_1 è falsa.

$\lim_{x\to 0}f(x)=\lim_{x\to 0}2^{-\frac{1}{|x|}}=\left(2^{-\frac{1}{\to 0^+}}\right)=\left(2^{(\to-\infty)}\right)=0^+$. La proposizione P_2 è vera.

P_1	P_2	$(P_2\Rightarrow P_1)$	non P_2	$(P_1\Rightarrow$ non $P_2)$	$(P_2\Rightarrow P_1)\vee(P_1\Rightarrow$ non $P_2)$
0	1	0	0	1	1

La proposizione $(P_2\Rightarrow P_1)\vee(P_1\Rightarrow$ non $P_2)$ è vera.

5) $\lim_{x\to 0}\frac{\log(1-\operatorname{sen}^2 3x)}{k\cdot x^\alpha}\Rightarrow\lim_{x\to 0}\frac{\log(1+(-\operatorname{sen}^2 3x))}{(-\operatorname{sen}^2 3x)}\cdot\frac{-\operatorname{sen}^2 3x}{(3x)^2}=1\cdot(-1)=-1$

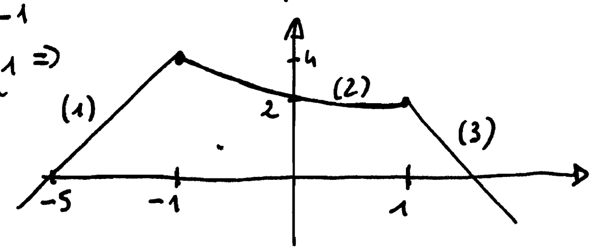
Deve quindi essere $\alpha=2$. $\lim_{x\to 0}\frac{\log(1+(-\operatorname{sen}^2 3x))}{(-\operatorname{sen}^2 3x)}\cdot\frac{(-\operatorname{sen}^2 3x)}{kx^2}=$

$=1\cdot\lim_{x\to 0}\frac{-\operatorname{sen}^2 3x}{9x^2}\cdot\frac{9}{k}=-\frac{9}{k}=2$ se $k=-\frac{9}{2}$.

Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito B1}}$

1) $f(x)=\begin{cases}5+x : x<-1\\ K\cdot a^x : -1\le x\le 1.\\ 3-2x : 1<x\end{cases}$ Per la Continuità dovrà risultare: $\begin{cases}5+(-1)=K\cdot a^{-1}\\ 3-2\cdot 1=K\cdot a^{1}\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}\frac{K}{a}=4\\ K\cdot a=1\end{cases}\Rightarrow\begin{cases}K=4a\\ 4a^2=1\end{cases}\Rightarrow\begin{cases}a=\frac{1}{2}\\ K=2\end{cases}\Rightarrow y=2\cdot\left(\frac{1}{2}\right)^x.$ Grafico:



2) $\lim\limits_{x\to 0}\dfrac{3^{x^2}-\cos 3x}{2x^2}=\lim\limits_{x\to 0}\dfrac{3^{x^2}-1}{2\cdot x^2}+\dfrac{1-\cos 3x}{9x^2}\cdot\dfrac{9}{2}=\dfrac{1}{2}\log 3+\dfrac{1}{2}\cdot\dfrac{9}{2}=\log\sqrt{3}+\dfrac{9}{4}.$

$\lim\limits_{x\to+\infty}\left(\dfrac{1+x+2x^2}{\cos x+x^3}\right)^{\frac{1-x}{1+x}}=\lim\limits_{x\to+\infty}\left(\dfrac{2x^2}{x^3}\right)^{\frac{-x}{+x}}=\left(\to 0^+\right)^{(\to -1)}=+\infty.$

3) $f(x)=3^{2x-1}=y\Rightarrow 2x-1=\log_3 y\Rightarrow 2x=1+\log_3 y\Rightarrow x=\frac{1}{2}\left(1+\log_3 y\right);\ f^{-1}: y=\frac{1}{2}\left(1+\log_3 x\right).$

$g(x)=\dfrac{2-x}{2x+1}=y\Rightarrow 2-x=2xy+y\Rightarrow x(2y+1)=2-y\Rightarrow x=\dfrac{2-y}{2y+1};\ g^{-1}: y=\dfrac{2-x}{2x+1}.$

$f^{-1}(g^{-1}(x))=f^{-1}\left(\dfrac{2-x}{2x+1}\right)=\dfrac{1}{2}+\dfrac{1}{2}\log_3\left(\dfrac{2-x}{2x+1}\right).$

4) $f(x)=\log\left(\dfrac{1}{1-e^{1-x}}\right)$; C.E.: $1-e^{1-x}>0\Rightarrow e^{1-x}<1\Rightarrow 1-x<0\Rightarrow x>1$. La Proposizione P_1 è vera.

$\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}3^{1-\frac{1}{|x|}}=\left(3^{\to 1-\frac{1}{\to 0^+}}\right)=3^{(\to-\infty)}=0^+$. La Proposizione P_2 è vera.

P_1	P_2	non P_2	($P_1\Rightarrow$ non P_2)	($P_2\Rightarrow P_1$)	($P_1\Rightarrow$ non P_2) o ($P_2\Rightarrow P_1$)
1	1	0	0	1	1

La Proposizione ($P_1\Rightarrow$ non P_2) o ($P_2\Rightarrow P_1$) è Vera.

5) $\lim\limits_{x\to 0}\dfrac{\operatorname{sen}^2(\operatorname{sen}^2 2x)}{K\cdot x^\alpha}\Rightarrow\lim\limits_{x\to 0}\dfrac{\operatorname{sen}(\operatorname{sen}^2 2x)}{\operatorname{sen}^2 2x}\cdot\dfrac{\operatorname{sen}(\operatorname{sen}^2 2x)}{\operatorname{sen}^2 2x}\cdot\dfrac{\operatorname{sen}^4 2x}{(2x)^4}=1\cdot 1\cdot 1=1\Rightarrow$

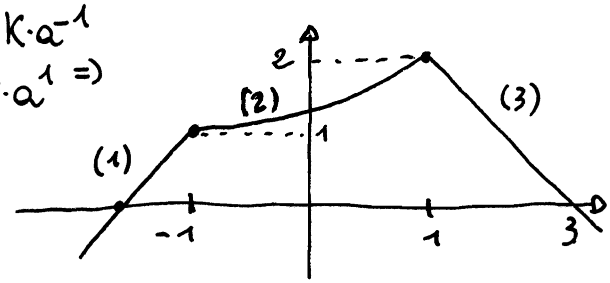
$\Rightarrow\alpha=4.\ \lim\limits_{x\to 0}\dfrac{\operatorname{sen}^2(\operatorname{sen}^2 2x)}{K\cdot x^4}=\lim\limits_{x\to 0}\dfrac{\operatorname{sen}^2(\operatorname{sen}^2 2x)}{\operatorname{sen}^4 2x}\cdot\dfrac{\operatorname{sen}^4 2x}{(2x)^4}\cdot\dfrac{16}{K}=1\Rightarrow K=16.$

Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito C1}}$

1) $f(x)=\begin{cases}2x+3 & : x<-1\\ K\cdot a^x & : -1\le x\le 1\\ 3-x & : 1<x\end{cases}$. Per la continuità dovrà risultare: $\begin{cases}2\cdot(-1)+3=K\cdot a^{-1}\\ 3-(1)=K\cdot a^{1}\end{cases}\Rightarrow$

$$\Rightarrow\begin{cases}\frac{K}{a}=1\\ K\cdot a=2\end{cases}\Rightarrow\begin{cases}K=a\\ a^2=2\end{cases}\Rightarrow\begin{cases}a=\sqrt2\\ K=\sqrt2\end{cases}\Rightarrow y=\sqrt2\cdot(\sqrt2)^x.$$

Grafico:



2) $\lim_{x\to0}\frac{\cos3x-\cos2x}{x^2}=\lim_{x\to0}\frac{1-\cos2x}{4x^2}\cdot4-\frac{1-\cos3x}{9x^2}\cdot9=\frac12\cdot4-\frac12\cdot9=2-\frac92=-\frac52.$

$\lim_{x\to+\infty}\left(\frac{1+2x+x^2}{\cos x+x}\right)^{\frac{1-x^2}{2+x}}=\lim_{x\to+\infty}\left(\frac{x^2}{x}\right)^{\frac{-x^2}{x}}=(\to+\infty)^{(\to-\infty)}=\left(\frac{1}{\to+\infty}\right)=0^+.$

3) $f(x)=\frac{x-1}{2x+1}=y\Rightarrow x-1=2xy+y\Rightarrow x(1-2y)=y+1\Rightarrow x=\frac{y+1}{1-2y};\ f^{-1}: y=\frac{x+1}{1-2x}.$

$g(x)=2^{1-3x}=y\Rightarrow 1-3x=\log_2 y\Rightarrow 3x=1-\log_2 y\Rightarrow x=\frac13(1-\log_2 y);\ g^{-1}: y=\frac13(1-\log_2 x).$

$g^{-1}(f^{-1}(x))=g^{-1}\left(\frac{x+1}{1-2x}\right)=\frac13\left(1-\log_2\left(\frac{x+1}{1-2x}\right)\right).$

4) $f(x)=\log\left(\frac{1-x}{1+x}\right)$. C.E.: $\frac{1-x}{1+x}>0 \begin{cases}1-x>0: x<1\\ 1+x>0: x>-1\end{cases}$

$-1 \quad 1$

$(-)\quad(+)\quad(-)$

C.E.: $-1<x<1$. La Proposizione P_1 è vera.

$\lim_{x\to0}f(x)=\lim_{x\to0}\log\left(1+\frac{1}{x^2}\right)=(\log(\to+\infty))=+\infty$: Disc. II^a specie. La Proposizione P_2 è vera.

P_1	P_2	$(P_1\Rightarrow P_2)$	$(\text{non } P_2)$	$(\text{non } P_2\Rightarrow P_1)$	$(P_1\Rightarrow P_2)$ e $(\text{non } P_2\Rightarrow P_1)$
1	1	1	0	1	1

La Proposizione $(P_1\Rightarrow P_2)$ e $(\text{non } P_2\Rightarrow P_1)$ è vera.

5) $\lim_{x\to0}\frac{\log(1+tg^3 2x)}{K\cdot x^\alpha}\Rightarrow\lim_{x\to0}\frac{\log(1+tg^3 2x)}{tg^3 2x}\cdot\frac{tg^3 2x}{(2x)^3}=1\cdot1=1.$

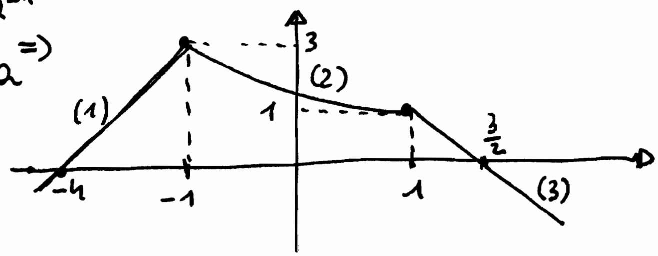
Quindi $\alpha=3$. $\lim_{x\to0}\frac{\log(1+tg^3 2x)}{tg^3 2x}\cdot\frac{tg^3 2x}{K\cdot x^3}=1\cdot\lim_{x\to0}\frac{tg^3 2x}{8x^3}\cdot\frac8K=\frac8K=3.$

Quindi $\frac8K=3\Rightarrow K=\frac83.$

Prove Intermedie di Matematica Generale del 6/11/2017 $\boxed{\text{Compito D1}}$

1) $f(x)=\begin{cases} x+4 : x<-1 \\ K\cdot a^x : -1\leq x\leq 1 \\ 3-2x : 1<x \end{cases}$ Per la continuità dovrà risultare: $\begin{cases} -1+4=K\cdot a^{-1} \\ 3-2\cdot 1 = K\cdot a \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} \frac{K}{a}=3 \\ Ka=1 \end{cases} \Rightarrow \begin{cases} K=3a \\ 3a^2=1 \end{cases} \Rightarrow \begin{cases} a=\frac{1}{\sqrt{3}} \\ K=\sqrt{3} \end{cases} : y=\sqrt{3}\cdot\left(\frac{1}{\sqrt{3}}\right)^x$. Grafico:



2) $$\lim_{x\to 0}\frac{(1+x)^{10}-2^x}{\operatorname{sen}x}=\lim_{x\to 0}\frac{(1+x)^{10}-1}{x}\cdot\frac{x}{\operatorname{sen}x}-\frac{2^x-1}{x}\cdot\frac{x}{\operatorname{sen}x}=10\cdot 1-\log 2\cdot 1=10-\log 2.$$

$$\lim_{x\to+\infty}\left(\frac{\operatorname{sen}^2x+x^2}{\operatorname{sen}x+2x^2}\right)^{\frac{\operatorname{sen}x-x}{\operatorname{sen}x+x}}=\lim_{x\to 0}\left(\frac{x^2}{2x^2}\right)^{\frac{-x}{x}}=\left(\to\frac{1}{2}\right)^{(\to -1)}=2.$$

3) $f(x)=\frac{x+2}{1-2x}=y\Rightarrow x+2=y-2xy\Rightarrow x(1+2y)=y-2\Rightarrow x=\frac{y-2}{1+2y}$; $f^{-1}: y=\frac{x-2}{1+2x}$.

$g(x)=2^{3x-2}=y\Rightarrow 3x-2=\log_2 y\Rightarrow 3x=2+\log_2 y\Rightarrow x=\frac{1}{3}(2+\log_2 y)$; $g^{-1}: y=\frac{1}{3}(2+\log_2 x)$.

$$f^{-1}(g^{-1}(x))=f^{-1}\left(\frac{1}{3}(2+\log_2 x)\right)=\frac{\frac{2}{3}+\frac{1}{3}\log_2 x-2}{1+\frac{4}{3}+\frac{2}{3}\log_2 x}=\frac{\log_2 x-4}{2\log_2 x+7}.$$

4) $f(x)=\log(x-x^2)$. C.E.: $x-x^2=x(1-x)>0$ $\begin{cases} x>0 \\ 1-x>0: x<1\end{cases}$ $\quad$ 0 $\;(-)\;|\;(+)\;|\;(-)\;$ 1 $\quad$ C.E.: $0<x<1$. La Proposizione P_1 è falsa.

$$\lim_{x\to 1}f(x)=\lim_{x\to 1}\frac{x^3-1}{x-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}=3$$: Disc. III Sp. La Proposizione P_2 è falsa.

P_1	P_2	(non P_1)	($P_2 \Rightarrow$ non P_1)	($P_1 \Rightarrow P_2$)	($P_2 \Rightarrow$ non P_1) e ($P_1 \Rightarrow P_2$)
0	0	1	1	1	1

La proposizione ($P_2 \Rightarrow$ non P_1) e ($P_1 \Rightarrow P_2$) è Vera.

5) $$\lim_{x\to 0}\frac{3^{1-\cos 2x}-1}{k\cdot x^\alpha}\Rightarrow\lim_{x\to 0}\frac{3^{1-\cos 2x}-1}{1-\cos 2x}\cdot\frac{1-\cos 2x}{4x^2}=\log 3\cdot\frac{1}{2}=\frac{1}{2}\log 3.$$

Quindi $\alpha=2$. $$\lim_{x\to 0}\frac{3^{1-\cos 2x}-1}{1-\cos 2x}\cdot\frac{1-\cos 2x}{4x^2}\cdot\frac{4}{k}=\log 3\cdot\frac{1}{2}\cdot\frac{4}{k}=2\log 3\cdot\frac{1}{k}=5\Rightarrow$$

$\Rightarrow K=\frac{2}{5}\log 3.$