Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito A2}}$

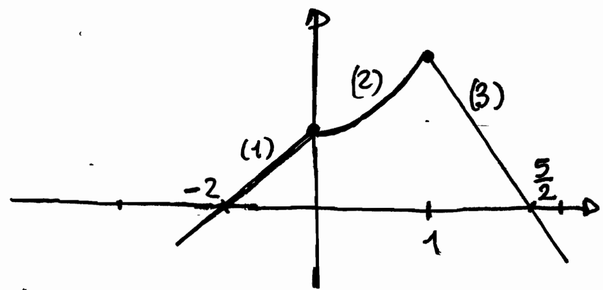
$$f(x)=\begin{cases} 2+x & : \; x<0 \\ a^x-K & : \; 0\le x\le 1 \\ 5-2x & : \; 1<x \end{cases}$$

Per la continuità dovrà risultare:

$$\begin{cases} 2+0=1-K \\ 5-2=a-K \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} K=-1 \\ a=3-1=2 \end{cases} \Rightarrow y=2^x+1.$$

Grafico:



2) $$\lim_{x\to 0}\frac{(1+x^2)^2+\cos x-2}{3x^2}=\lim_{x\to 0}\frac{(1+x^2)^2-1}{x^2}\cdot\frac{1}{3}-\frac{1-\cos x}{x^2}\cdot\frac{1}{3}=2\cdot\frac{1}{3}-\frac{1}{2}\cdot\frac{1}{3}=\frac{2}{3}-\frac{1}{6}=\frac{1}{2}.$$

$$\lim_{x\to+\infty}\left(\frac{1+x+x^2}{\operatorname{sen}x+2x^2}\right)^{\frac{x^2-1}{1-2x}}=\lim_{x\to+\infty}\left(\frac{x^2}{2x^2}\right)^{\left(\frac{x^2}{-2x}\right)}=\left(\to\frac{1}{2}\right)^{(\to-\infty)}=+\infty.$$

3) $$f^{-1}(x)=3^{2x+1}=y\Rightarrow 2x+1=\log_3 y\Rightarrow 2x=\log_3 y-1\Rightarrow x=\frac{1}{2}(\log_3 y-1);\; f: y=\frac{1}{2}(\log_3 x-1).$$

$$g(x)=\frac{1-x}{3x-1}=y\Rightarrow 1-x=3xy-y\Rightarrow x(3y+1)=1+y\Rightarrow x=\frac{1+y}{3y+1};\; g^{-1}: y=\frac{1+x}{3x+1}.$$

$$g^{-1}(f(x))=g^{-1}\left(\frac{1}{2}(\log_3 x-1)\right)=\frac{1+\frac{1}{2}(\log_3 x-1)}{\frac{3}{2}(\log_3 x-1)+1}=\frac{1+\log_3 x}{3\log_3 x-1}.$$

4)

A	B	C	$(A\Rightarrow B)$	(non C)	$(A \Leftrightarrow \text{non } C)$	$(A\Rightarrow B)$ e $(A\Leftrightarrow \text{non } C)$
1	0	1	0	0	0	0
0	1	1	1	0	1	1
0	1	0	1	1	0	0
0	0	1	1	0	1	1
0	0	0	1	1	0	0

5) $$\lim_{x\to 0}\frac{\log(1-\operatorname{sen}(2x^3))}{-\operatorname{sen}(2x^3)}\cdot\frac{-\operatorname{sen}(2x^3)}{2x^3}=1\cdot(-1)=-1. \text{ Quindi } \alpha=3.$$

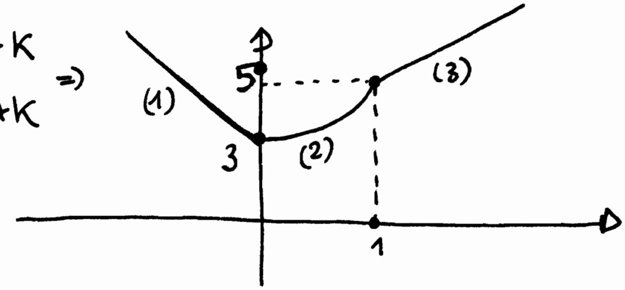
$$\lim_{x\to 0}\frac{\log(1-\operatorname{sen}(2x^3))}{k\cdot x^3}=\lim_{x\to 0}\frac{\log(1-\operatorname{sen}(2x^3))}{-\operatorname{sen}(2x^3)}\cdot\frac{-\operatorname{sen}(2x^3)}{2x^3}\cdot\frac{2}{K}=1\cdot(-1)\cdot\frac{2}{K}\Rightarrow$$

$$\Rightarrow -\frac{2}{K}=3\Rightarrow K=-\frac{2}{3}.$$

Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito B2}}$

1) $f(x)=\begin{cases} 3-x : x<0 \\ a^x+k : 0\le x\le 1 \\ 4+x : 1<x \end{cases}$ Per la continuità dovrà risultare: $\begin{cases} 3-0=1+k \\ 4+1=a+k \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} k=2 \\ a=5-2=3 \end{cases} \Rightarrow y=3^x+2.$ Grafico:



2) $\lim\limits_{x\to 0}\dfrac{3^{x^2}+\cos 2x-2}{3x^2}=\lim\limits_{x\to 0}\dfrac{3^{x^2}-1}{x^2}\cdot\dfrac{1}{3}-\dfrac{1-\cos 2x}{4x^2}\cdot\dfrac{4}{3}=\log 3\cdot\dfrac{1}{3}-\dfrac{1}{2}\cdot\dfrac{4}{3}=\dfrac{1}{3}\log 3-\dfrac{2}{3}.$

$\lim\limits_{x\to+\infty}\left(\dfrac{2+x}{1+x}\right)^{\frac{1-2x^2}{1+x}}=\lim\limits_{x\to+\infty}\left(\dfrac{1+x+1}{1+x}\right)^{\frac{1-2x^2}{1+x}}=\lim\limits_{x\to+\infty}\left[\left(1+\dfrac{1}{1+x}\right)^{1+x}\right]^{\frac{1-2x^2}{(1+x)^2}}=e^{-2}.$

3) $f(x)=3^{2x+1}=y\Rightarrow 2x+1=\log_3 y\Rightarrow 2x=\log_3 y-1\Rightarrow x=\dfrac{1}{2}\left(\log_3 y-1\right).\ f^{-1}: y=\dfrac{1}{2}\left(\log_3 x-1\right).$

$g^{-1}(x)=\dfrac{3-x}{2x-1}=y\Rightarrow 3-x=2xy-y\Rightarrow x(2y+1)=y+3\Rightarrow x=\dfrac{y+3}{2y+1}.\ g: y=\dfrac{x+3}{2x+1}.$

$f^{-1}(g(x))=f^{-1}\left(\dfrac{x+3}{2x+1}\right)=\dfrac{1}{2}\left(\log_3\left(\dfrac{x+3}{2x+1}\right)-1\right).$

4)

A B C	(non B)	(A ⇒ non B)	(A ⇔ C)	(A ⇒ non B) o (A ⇔ C)
1 1 1	0	0	1	1
1 1 0	0	0	0	0
1 0 1	1	1	1	1
1 0 0	1	1	0	1
0 0 1	1	1	0	1

5) $\lim\limits_{x\to 0}\dfrac{(1+\operatorname{sen}^2 x)^3-1}{\operatorname{sen}^2 x}\cdot\dfrac{\operatorname{sen}^2 x}{x^2}=3\cdot 1=3\Rightarrow$ quindi $\alpha=2.$

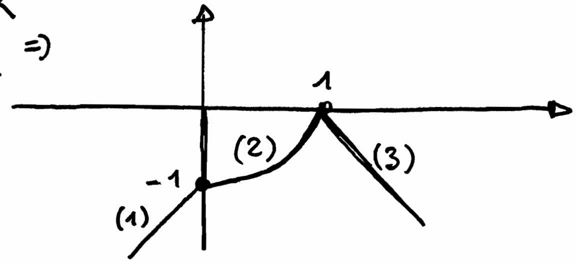
$\lim\limits_{x\to 0}\dfrac{(1+\operatorname{sen}^2 x)^3-1}{k\cdot x^2}=\lim\limits_{x\to 0}\dfrac{(1+\operatorname{sen}^2 x)^3-1}{\operatorname{sen}^2 x}\cdot\dfrac{\operatorname{sen}^2 x}{x^2}\cdot\dfrac{1}{k}=3\cdot 1\cdot\dfrac{1}{k}\Rightarrow$

$\Rightarrow \dfrac{3}{k}=1\Rightarrow k=3.$

Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito C2}}$

1) $f(x)=\begin{cases}2x-1 : x<0\\ a^x-k : 0\le x\le 1\\ 1-x : 1<x\end{cases}$. Per la continuità dovrà risultare: $\begin{cases}2\cdot 0-1=1-k\\ 1-1=a-k\end{cases} \Rightarrow$

$\Rightarrow \begin{cases}k=2\\ a=2\end{cases} \Rightarrow y=2^x-2$. Grafico:



2) $\lim\limits_{x\to 0}\dfrac{2-\cos 2x-\cos 3x}{3x^2}=\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{4x^2}\cdot\dfrac{4}{3}+\dfrac{1-\cos 3x}{9x^2}\cdot\dfrac{9}{3}=\dfrac{1}{2}\cdot\dfrac{4}{3}+\dfrac{1}{2}\cdot 3=\dfrac{2}{3}+\dfrac{3}{2}=\dfrac{13}{6}.$

$\lim\limits_{x\to+\infty}\left(\dfrac{1+2x}{\cos x+x^2}\right)^{\frac{x-x^2}{2+x}}=\lim\limits_{x\to+\infty}\left(\dfrac{2x}{x^2}\right)^{\frac{-x^2}{x}}=\left(\to 0^+\right)^{(\to-\infty)}=\left(\to+\infty\right)^{(\to+\infty)}=+\infty.$

3) $f^{-1}(x)=\dfrac{x-1}{2x+1}=y\Rightarrow x-1=2xy+y\Rightarrow x(1-2y)=y+1\Rightarrow x=\dfrac{y+1}{1-2y}$. $f: y=\dfrac{x+1}{1-2x}$.

$g(x)=2^{2-x}=y\Rightarrow 2-x=\log_2 y\Rightarrow x=2-\log_2 y$. $g^{-1}: y=2-\log_2 x$.

$g^{-1}(f(x))=g^{-1}\left(\dfrac{x+1}{1-2x}\right)=2-\log_2\left(\dfrac{x+1}{1-2x}\right).$

4)

A	B	C	$(A\Rightarrow B)$	(non A)	(non A $\Leftrightarrow$ C)	$(A\Rightarrow B)$ e (non A $\Leftrightarrow$ C)
1	1	0	1	0	1	1
1	0	1	0	0	0	0
1	0	0	0	0	1	0
0	0	1	1	1	1	1
0	0	0	1	1	0	0

5) $\lim\limits_{x\to 0}\dfrac{2^{\log(1+3x^2)}-1}{\log(1+3x^2)}\cdot\dfrac{\log(1+3x^2)}{3x^2}=\log 2\cdot 1=\log 2$. Quindi $\alpha=2$.

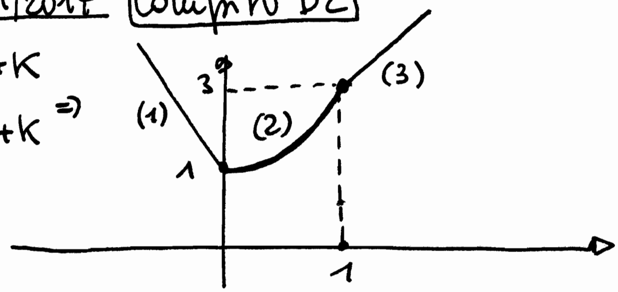
$\lim\limits_{x\to 0}\dfrac{2^{\log(1+3x^2)}-1}{k\cdot x^2}=\lim\limits_{x\to 0}\dfrac{2^{\log(1+3x^2)}-1}{\log(1+3x^2)}\cdot\dfrac{\log(1+3x^2)}{3x^2}\cdot\dfrac{3}{k}=\log 2\cdot 1\cdot\dfrac{3}{k}\Rightarrow$

$\Rightarrow \dfrac{3}{k}\cdot\log 2=3\Rightarrow k=\log 2.$

Prova Intermedia di Matematica Generale del 6/11/2017 $\boxed{\text{Compito D2}}$

1) $f(x)=\begin{cases}1-2x : x<0\\ a^x+K : 0\le x\le 1\\ x+2 : 1<x\end{cases}$. Per la continuità dovrà risultare: $\begin{cases}1-0=1+K\\ 1+2=a+K\end{cases} \Rightarrow$

$\Rightarrow \begin{cases}K=0\\ a=3\end{cases}$. $y=3^x$. Grafico:



2) $\lim_{x\to 0} \frac{(1+x)^5+3^x-2}{\operatorname{sen} 2x} = \lim_{x\to 0} \frac{(1+x)^5-1}{x}\cdot\frac{2x}{\operatorname{sen}2x}\cdot\frac{1}{2}+\frac{3^x-1}{x}\cdot\frac{2x}{\operatorname{sen}2x}\cdot\frac{1}{2} = 5\cdot\frac{1}{2}+\log 3\cdot\frac{1}{2}=\frac{5}{2}+\log\sqrt{3}.$

$\lim_{x\to+\infty}\left(\frac{\operatorname{sen}^2 x+x^2}{\operatorname{sen}x+2x}\right)^{\frac{\operatorname{sen}x-2x}{\operatorname{sen}x+x}} = \lim_{x\to+\infty}\left(\frac{x^2}{2x}\right)^{\frac{-2x}{x}} = (\to+\infty)^{(\to-2)} = 0^+.$

3) $f(x)=\frac{x+1}{1-2x}=y \Rightarrow x+1=y-2xy \Rightarrow x(1+2y)=y-1 \Rightarrow x=\frac{y-1}{1+2y}$. $f^{-1}: y=\frac{x-1}{1+2x}$.

$g^{-1}(x)=2^{2x-1}=y \Rightarrow 2x-1=\log_2 y \Rightarrow 2x=1+\log_2 y \Rightarrow x=\frac{1}{2}(1+\log_2 y)$. $g: y=\frac{1}{2}(1+\log_2 x)$.

$f^{-1}(g(x))=f^{-1}\left(\frac{1}{2}(1+\log_2 x)\right)=\frac{\frac{1}{2}+\frac{1}{2}\log_2 x-1}{1+1+\log_2 x}=\frac{\log_2 x-1}{4+2\log_2 x}.$

4)

A	B	C	(non A)	(non A ⇒ C)	(A ⇔ B)	(non A ⇒ C) ∨ (A ⇔ B)
1	1	1	0	1	1	1
1	0	1	0	1	0	1
0	1	1	1	1	0	1
0	1	0	1	0	0	0
0	0	1	1	1	1	1

5) $\lim_{x\to 0}\frac{1-\cos(\operatorname{sen}2x)}{\operatorname{sen}^2 2x}\cdot\frac{\operatorname{sen}^2 2x}{4x^2}=\frac{1}{2}\cdot 1=\frac{1}{2}$. Quindi $\alpha=2$.

$\lim_{x\to 0}\frac{1-\cos(\operatorname{sen}2x)}{\operatorname{sen}^2 2x}\cdot\frac{\operatorname{sen}^2 2x}{4x^2}\cdot\frac{4}{K}=\frac{1}{2}\cdot 1\cdot\frac{4}{K}=1 \Rightarrow \frac{2}{K}=1 \Rightarrow K=2.$