

$$IM1) 1-i = \sqrt{2} \left( \frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right) = \sqrt{2} \cdot \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right);$$

$$1-i\sqrt{3} = 2 \left( \frac{1}{2} - i \cdot \frac{\sqrt{3}}{2} \right) = 2 \cdot \left( \cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi \right). \text{ Quindi:}$$

$$\left( \frac{1-i}{1-i\sqrt{3}} \right)^4 = \left( \frac{\sqrt{2}}{2} \right)^4 \cdot \left( \cos 4 \cdot \left( \frac{\pi}{4} - \frac{5}{3}\pi \right) + i \sin 4 \cdot \left( \frac{\pi}{4} - \frac{5}{3}\pi \right) \right) = \frac{1}{8} \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = \frac{1}{8} (1+i\sqrt{3}).$$

$$IM2) f(x,y) = \begin{cases} \frac{x^2|xy|}{x^2+y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}.$$

Per la continuità:  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2|xy|}{x^2+y^2} \Rightarrow \lim_{r \rightarrow 0} \frac{r^4 \cos^2 \vartheta |\cos \vartheta \cdot \sin \vartheta|}{r^2} = 0$  e la convergenza è uniforme in quanto  $r^2 \cdot \cos^2 \vartheta \cdot |\cos \vartheta \cdot \sin \vartheta| \leq r^2 \cdot 1 = r^2 < \varepsilon$ .

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \rightarrow 0} \frac{1}{h} \cdot \left( \frac{h^2 \cdot |h \cdot 0|}{h^2} - 0 \right) = \lim_{h \rightarrow 0} 0 = 0 = \frac{\partial f}{\partial y}(0,0). \text{ Per la differentiabilità:}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\frac{x^2|xy|}{x^2+y^2} - 0 - (0,0) \cdot (x-0, y-0)}{\sqrt{x^2+y^2}} \Rightarrow \lim_{r \rightarrow 0} \frac{\frac{r^4 \cos^2 \vartheta |\cos \vartheta \cdot \sin \vartheta|}{r^2} - 0}{r} =$$

$\lim_{r \rightarrow 0} r \cdot \cos^2 \vartheta \cdot |\cos \vartheta \cdot \sin \vartheta| = 0$  e la convergenza è uniforme in quanto  $r \cdot \cos^2 \vartheta \cdot |\cos \vartheta \cdot \sin \vartheta| \leq r \cdot 1 < \varepsilon$ . Quindi la funzione è differentiabile in  $(0,0)$ .

IM3)  $f(x,y) = x^3 - 3xy - y^3$  è un polinomio quindi funzione continua e differentiabile di qualsiasi ordine.  $v \rightarrow \left( \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$ ;  $w \rightarrow \left( \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right)$ .

$$\nabla f(x,y) = (3x^2 - 3y; -3x - 3y^2); H(x,y) = \begin{vmatrix} 6x & -3 \\ -3 & -6y \end{vmatrix}.$$

$$\mathcal{D}_{u,v}^2 f(x,y) = \left\| \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\| \cdot \left\| \begin{vmatrix} 6x & -3 \\ -3 & -6y \end{vmatrix} \right\| \cdot \left\| \frac{1}{\sqrt{5}} \right\| = \left\| \frac{1}{\sqrt{2}} (6x+3) \frac{1}{\sqrt{2}} (6y-3) \right\| \cdot \left\| \frac{1}{\sqrt{5}} \right\| = \frac{1}{\sqrt{10}} \cdot (6x+3) + \frac{2}{\sqrt{10}} (6y-3) =$$

$$= 3\sqrt{10} \Rightarrow 6x+3+12y-6=30 \text{ e dato che } y=x \Rightarrow 18x=33 \Rightarrow x=y=\frac{33}{18}=\frac{11}{6}.$$

$$IM4) f(x,y,z) = z \cdot e^{x-y} + x \cdot e^{y-z} - 2y \cdot e^{x-z} = 0. \quad f(1,1,1) = 1+1-2 = 0.$$

$$\nabla f(x,y,z) = (z e^{x-y} + e^{y-z} - 2y e^{x-z}; -x e^{x-y} + x e^{y-z} - 2 \cdot e^{x-z}; e^{x-y} - x e^{y-z} + 2y e^{x-z}).$$

$$\nabla f(1,1,1) = (1+1-2; -1+1-2; 1-1+2) = (0; -2; 2). \text{ Si può definire } (x,y) \rightarrow z.$$

Équation plan tangent à  $z = z(x; y)$  in  $(1; 1)$ :  $z'_x = -\frac{0}{2} = 0; z'_y = -\frac{-2}{2} = 1$ .

$$z = z(1; 1) + \nabla z(1; 1) \cdot (x-1; y-1) \Rightarrow z = 1 + \left(-\frac{0}{2}; -\frac{-2}{2}\right)(x-1; y-1) \Rightarrow$$

$$\Rightarrow z = 1 + (0; 1)(x-1; y-1) \Rightarrow z = y-1+1 = y.$$

$$H(f(x; y; z)) = \begin{vmatrix} ze^{x-y} - 2ye^{x-z} & -ze^{x-y} + e^{y-z} & -2e^{x-z} \\ -ze^{x-y} + e^{y-z} & 2e^{x-z} & ze^{x-y} + xe^{y-z} \\ e^{x-y} - e^{y-z} + 2ye^{x-z} & -e^{x-y} - xe^{y-z} + 2e^{x-z} & xe^{y-z} - 2ye^{x-z} \end{vmatrix}$$

$$H(f(1; 1; 1)) = \begin{vmatrix} 1-2 & -1+1-2 & 1-1+2 \\ -1+1-2 & 1+1 & -1-1+2 \\ 1-1+2 & -1-1+2 & 1-2 \end{vmatrix} = \begin{vmatrix} -1 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & -1 \end{vmatrix}.$$

$$dz = z'_x \cdot dx + z'_y \cdot dy = 0 \cdot dx + 1 \cdot dy \Rightarrow dz = dy.$$

$$d^2f = -1 \cdot (dx)^2 + 2(dy)^2 - 1(dz)^2 - 4dx dy + 4dx dz + 0 dy dz.$$

$$d^2z = -\frac{d^2f(x; y; z)}{f'_z} = \frac{(dx)^2 - 2(dy)^2 + (dz)^2 + 4dx dy - 4dx dz}{2} = (dx dz = dy) \Rightarrow$$

$$\Rightarrow d^2z = \frac{1}{2}(dx)^2 - \frac{1}{2}(dy)^2.$$

IM5)  $f: \mathbb{R} \rightarrow \mathbb{R}^3; t \rightarrow (e^{t^2-t}; \log(t^2-t+1); \sin t) = X(t)$ .

Équation recte tangente:  $X(t_0) + t \cdot X'(t_0)$ .

$$X(0) = (e^0; \log 1; \sin 0) = (1; 0; 0).$$

$$X'(t) = \left( (2t-1) \cdot e^{t^2-t}; \frac{2t-1}{t^2-t+1}; \cos t \right).$$

$$X'(0) = (-1e^0; \frac{-1}{1}; \cos 0) = (-1; -1; 1).$$

Équation recte tangente in  $t=0$ :  $t \rightarrow (1; 0; 0) + t \cdot (-1; -1; 1) = (1-t; -t; t)$ .