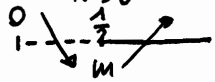
Compito di Matematica Generale del 15/1/2018 Compito A $\boxed{\text{MGA1}}$

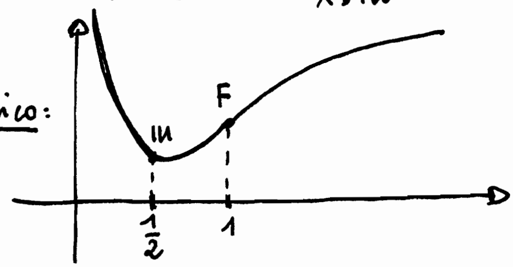
1) $f(x)=\dfrac{1}{2x}+\log 3x$. C.E.: $x>0$. $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\dfrac{1+2x\log 3x}{2x}=\left(\dfrac{\to 1+0}{\to 0^+}\right)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$f'(x)=-\dfrac{1}{2x^2}+\dfrac{1}{x}=\dfrac{2x-1}{2x^2}\geq 0$ per $x\geq\dfrac{1}{2}$.

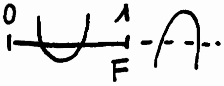


$f\left(\frac{1}{2}\right)=1+\log\frac{3}{2}>0$.

Grafico:



$f''(x)=-\dfrac{1}{2}\cdot(-2)\cdot\dfrac{1}{x^3}-\dfrac{1}{x^2}=\dfrac{1-x}{x^3}\geq 0: x\leq 1$.



Da $f\left(\frac{1}{2}\right)>0 \Rightarrow f(x)>0\ \forall x>0$.

2) $\lim\limits_{x\to 0}\dfrac{(1-\operatorname{sen}x)^7-1}{\log(1+3x)}=\lim\limits_{x\to 0}\dfrac{(1-\operatorname{sen}x)^7-1}{-\operatorname{sen}x}\cdot\dfrac{-\operatorname{sen}x}{3x}\cdot\dfrac{3x}{\log(1+3x)}=7\cdot\left(-\dfrac{1}{3}\right)\cdot 1=-\dfrac{7}{3}$.

$\lim\limits_{x\to+\infty}\left(\dfrac{1+x-\log x}{2x}\right)^{3x}=\lim\limits_{x\to+\infty}\left(\dfrac{x}{2x}\right)^{3x}=\left(\to\dfrac{1}{2}\right)^{(\to+\infty)}=0^+$.

3) $\lim\limits_{x\to 0}\dfrac{1-\cos(\alpha x)}{\operatorname{sen}^2 x}=\lim\limits_{x\to 0}\dfrac{1-\cos\alpha x}{\alpha^2x^2}\cdot\dfrac{\alpha^2x^2}{\operatorname{sen}^2x}=\dfrac{1}{2}\cdot\alpha^2=\lim\limits_{x\to+\infty}\left(1-\dfrac{\beta}{x}\right)^x=\lim\limits_{x\to+\infty}\left(1+\dfrac{(-\beta)}{x}\right)^x=$

$=e^{-\beta}\Rightarrow\dfrac{1}{2}\alpha^2=e^{-\beta}\Rightarrow\beta=-\log\dfrac{1}{2}\alpha^2$.

4) $F(x)=f(x)\cdot e^x\Rightarrow F'(x)=f'(x)\cdot e^x+f(x)\cdot e^x=\left(f'(x)+f(x)\right)\cdot e^x\Rightarrow\left(f'(x)+f(x)>0,\forall x\right)$

$\Rightarrow F''(x)=\left(f''(x)+f'(x)\right)e^x+\left(f'(x)+f(x)\right)\cdot e^x=\left(f''(x)+2f'(x)+f(x)\right)\cdot e^x$. Se allora

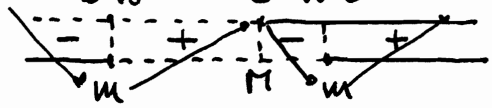
$f''(x)+f'(x)>0\ \forall x\Rightarrow F''(x)>0\ \forall x$ in quanto $f'(x)+f(x)>0\ \forall x$.

5) $\displaystyle\int_{-2}^{1} k-2x\,dx=4\Rightarrow\left(kx-x^2\right|_{-2}^{1}=(k-1)-(-2k-4)=3k+3=4\Rightarrow 3k=1\Rightarrow k=\frac{1}{3}\Rightarrow\left(0;\frac{1}{3}\right)$.

6) $f(x)=x^2\cdot e^x\cdot(x-1)^3$. $f'(x)=2x\cdot e^x\cdot(x-1)^3+x^2\cdot e^x\cdot(x-1)^3+x^2\cdot e^x\cdot 3(x-1)^2\Rightarrow$

$\Rightarrow f'(x)=e^x\cdot x\cdot(x-1)^2\cdot\left(2(x-1)+x(x-1)+3x\right)=e^x\cdot x\cdot(x-1)^2\cdot\left(x^2+4x-2\right)\geq 0\Rightarrow$

$\Rightarrow\begin{cases}x\geq 0\\ x^2+4x-2\geq 0\end{cases}\Rightarrow\begin{cases}x\geq 0\\ x\leq-2-\sqrt{6}\cup x\geq-2+\sqrt{6}\end{cases}$



In $x=0$ punto di Massimo; in $x=-2-\sqrt{6}$ e in $x=\sqrt{6}-2$ punti di minimo.

7) $f(x;y)=\left(x^2-2x-4y\right)\cdot e^{-y}$. Funzione differenziabile $\forall(x;y)\in\mathbb{R}^2$.

MGA2

$$\begin{cases} f'_x = (2x-2)e^{-y} = 0 \\ f'_y = -4e^{-y} - (x^2-2x-4y)e^{-y} = 0 \end{cases} \Rightarrow \begin{cases} 2(x-1)e^{-y} = 0 \\ (4y-x^2+2x-4)e^{-y} = 0 \end{cases} \Rightarrow \begin{cases} x=1 \\ 4y-3=0 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=\frac{3}{4} \end{cases}.$$

$$H(x;y) = \begin{Vmatrix} 2e^{-y} & (2-2x)e^{-y} \\ (2-2x)e^{-y} & 4e^{-y}-(4y-x^2+2x-4)e^{-y} \end{Vmatrix} ; H\left(1;\frac{3}{4}\right) = \begin{Vmatrix} 2e^{-\frac{3}{4}} & 0 \\ 0 & 4\cdot e^{-\frac{3}{4}} \end{Vmatrix} \Rightarrow \begin{cases} |H_1|>0 \\ |H_2|>0 \end{cases} : \text{Minimo.}$$

8)

A	B	C	$(A \Leftrightarrow B)$	$(A \Leftrightarrow B) \Rightarrow C$	$(B \Rightarrow C)$	$A \Leftrightarrow (B \Rightarrow C)$	
1	1	1	1	1	1	1	
1	1	0	1	0	0	0	
1	0	1	0	1	1	1	
1	0	0	0	1	1	1	
0	1	1	0	1	1	0	*
0	1	0	0	1	0	1	
0	0	1	1	1	1	0	*
0	0	0	1	0	1	0	

Dalla 5ª e dalla 7ª riga si vede che le due proposizioni non sono logicamente equivalenti.

9) $A\cdot V = 3V \Rightarrow \begin{Vmatrix} 2 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \\ z \end{Vmatrix} = \begin{Vmatrix} 3x \\ 3y \\ 3z \end{Vmatrix} \Rightarrow \begin{cases} 2x+2y+z=3x \\ y=3y \\ x+2y+2z=3z \end{cases} \Rightarrow \begin{cases} x=z \\ y=0 \\ x=z \end{cases}$. Quindi

$V=(x;0;x)$. $\|V\| = \sqrt{x^2+0+x^2} = \sqrt{2x^2} = \sqrt{2} \Rightarrow x^2=1 \Rightarrow x=\pm 1.$

Ci sono due vettori: $V_1=(1;0;1)$ e $V_2=(-1;0;-1)$.

10) $f(x)=x^3-3x^2+3kx-1$; $f'(x)=3x^2-6x+3k$; $f''(x)=6x-6=0$ per $x=1$.

Quindi dovrà essere $f'(1)=3 \Rightarrow 3-6+3k=3 \Rightarrow 3k=6 \Rightarrow k=2$.

Quindi $f(x)=x^3-3x^2+6x-1$; $f'(x)=3x^2-6x+6$; $f''(x)=6x-6$. $f''(1)=0$.

$f'(1)=3-6+6=3$; $f(1)=1-3+6-1=3$.

Equazione retta tangente in $x=1$: $y-3=3(x-1) \Rightarrow y=3x$.

Compito di Matematica Generale del 15/1/2018 Compito B $\boxed{MGB1}$

$f(x)=\frac{1}{3x}+\log 4x$. C.E.: $x>0$. $\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{1+3x\cdot\log 4x}{3x}=\left(\frac{\to 1+0}{\to 0^+}\right)=+\infty$; $\lim_{x\to+\infty} f(x)=+\infty$.

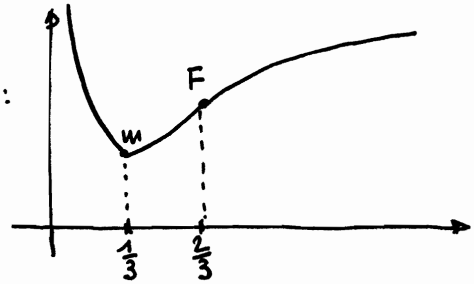
$f'(x)=-\frac{1}{3x^2}+\frac{1}{x}=\frac{3x-1}{3x^2}\geq 0$ per $x\geq\frac{1}{3}$.

$f\left(\frac{1}{3}\right)=1+\log\frac{4}{3}>0$.

$f''(x)=-\frac{1}{3}(-2)\frac{1}{x^3}-\frac{1}{x^2}=\frac{2-3x}{3x^3}\geq 0$ per $x\leq\frac{2}{3}$.

Da $f\left(\frac{1}{3}\right)>0 \Rightarrow f(x)>0\ \forall x>0$.

Grafico:



2) $\lim_{x\to0}\frac{\log(1-3x)}{(1+\operatorname{tg}x)^6-1}=\lim_{x\to0}\frac{\log(1-3x)}{-3x}\cdot\frac{-3x}{\operatorname{tg}x}\cdot\frac{\operatorname{tg}x}{(1+\operatorname{tg}x)^6-1}=1\cdot(-3)\cdot\frac{1}{6}=-\frac{1}{2}$.

$\lim_{x\to+\infty}\left(\frac{1+2x-\operatorname{sen}x}{x}\right)^{3x}=\lim_{x\to+\infty}\left(\frac{2x}{x}\right)^{3x}=(\to 2)^{(\to+\infty)}=+\infty$.

3) $\lim_{x\to0}\frac{1-\cos x}{\operatorname{sen}^2\alpha x}=\lim_{x\to0}\frac{1-\cos x}{x^2}\cdot\frac{\alpha^2x^2}{\operatorname{sen}^2\alpha x}\cdot\frac{1}{\alpha^2}=\frac{1}{2}\cdot1\cdot\frac{1}{\alpha^2}=\frac{1}{2\alpha^2}=\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)^{\beta x}=e^{2\beta}\Rightarrow$

$\Rightarrow e^{2\beta}=\frac{1}{2\alpha^2}\Rightarrow\beta=\frac{1}{2}\log\left(\frac{1}{2\alpha^2}\right)=-\frac{1}{2}\log(2\alpha^2)$.

4) $F(x)=x+e^{f(x)}\Rightarrow F'(x)=1+f'(x)\cdot e^{f(x)}\Rightarrow F''(x)=f''(x)\cdot e^{f(x)}+(f'(x))^2\cdot e^{f(x)}\Rightarrow$

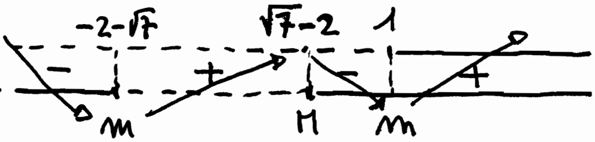
$\Rightarrow F''(x)=\left(f''(x)+(f'(x))^2\right)\cdot e^{f(x)}$. Se $f''(x)>0\ \forall x\Rightarrow F''(x)>0\ \forall x$.

5) $\int_{-1}^{2}3x+k\,dx=5\Rightarrow\left(\frac{3}{2}x^2+kx\right|_{-1}^{2}=(6+2k)-\left(\frac{3}{2}-k\right)=3k+\frac{9}{2}=5\Rightarrow 3k=\frac{1}{2}\Rightarrow k=\frac{1}{6}\Rightarrow\left(0;\frac{1}{6}\right)$.

6) $f(x)=x^3\cdot e^x\cdot(x-1)^2$. $f'(x)=3x^2\cdot e^x\cdot(x-1)^2+x^3\cdot e^x\cdot(x-1)^2+x^3\cdot e^x\cdot2(x-1)\Rightarrow$

$\Rightarrow f'(x)=e^x\cdot x^2\cdot(x-1)\cdot\left(3(x-1)+x(x-1)+2x\right)=e^x\cdot x^2\cdot(x-1)\cdot(x^2+4x-3)\geq0\Rightarrow$

$\Rightarrow\begin{cases}x-1\geq0\\x^2+4x-3\geq0\end{cases}\Rightarrow\begin{cases}x\geq1\\x\leq-2-\sqrt7\ \cup\ x\geq-2+\sqrt7\end{cases}$



In $x=\sqrt7-2$ punto di Massimo; in $x=-2-\sqrt7$ e in $x=1$ punti di minimo.

7) $f(x;y)=(2x-y^2+4y)\cdot e^{-x}$. Funzione differenziabile $\forall(x;y)\in\mathbb{R}^2$.

MGB2

$$\begin{cases} f'_x = 2\cdot e^{-x} - (2x-y^2+4y)e^{-x} = 0 \\ f'_y = (4-2y)\cdot e^{-x} = 0 \end{cases} \Rightarrow \begin{cases} (y^2-4y-2x+2)e^{-x} = 0 \\ 2(2-y)e^{-x} = 0 \end{cases} \Rightarrow \begin{cases} -2-2x=0 \\ y=2 \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=2 \end{cases}.$$

$$H(x;y) = \begin{Vmatrix} -2e^{-x}-(y^2-4y-2x+2)e^{-x} & (2y-4)e^{-x} \\ (2y-4)e^{-x} & -2e^{-x} \end{Vmatrix} ; H(-1;2) = \begin{Vmatrix} -2e & 0 \\ 0 & -2e \end{Vmatrix} \Rightarrow \begin{cases} |H_1|<0 \\ |H_2|>0 \end{cases} : \text{Massimo.}$$

8)

A B C	$(A\Rightarrow B)$	$(A\Rightarrow B)\Leftrightarrow C$	$(B\Leftrightarrow C)$	$A\Rightarrow(B\Leftrightarrow C)$
1 1 1	1	1	1	1
1 1 0	1	0	0	0
1 0 1	0	0	0	0
1 0 0	0	1	1	1
0 1 1	1	1	1	1
0 1 0	1	0	0	1
0 0 1	1	1	0	1
0 0 0	1	0	1	1

Dalla 6ª e dalla 8ª riga si vede che le due * proposizioni non sono * logicamente equivalenti.

9) $$A\cdot V = V \Rightarrow \begin{Vmatrix} 2 & 2 & 1 \\ 0 & 3 & 0 \\ 1 & 2 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \\ z \end{Vmatrix} = \begin{Vmatrix} x \\ y \\ z \end{Vmatrix} \Rightarrow \begin{cases} 2x+2y+z = x \\ 3y = y \\ x+2y+2z = z \end{cases} \Rightarrow \begin{cases} x=-z \\ y=0 \\ x=-z \end{cases}.$$ Quindi

$V=(x;0;-x)$. $\|V\| = \sqrt{x^2+0+x^2} = \sqrt{2x^2} = \sqrt{8} \Rightarrow x^2=4 \Rightarrow x = \pm 2$.

Ci sono due vettori: $V_1=(2;0;-2)$ e $V_2=(-2;0;2)$.

10) $f(x)=x^3+3x^2-2kx+2$; $f'(x)=3x^2+6x-2k$; $f''(x)=6x+6=0$ per $x=-1$.

Quindi dovrà essere $f'(-1)=1 \Rightarrow 3-6-2k=1 \Rightarrow 2k=-4 \Rightarrow k=-2$.

Quindi $f(x)=x^3+3x^2+4x+2$; $f'(x)=3x^2+6x+4$; $f''(x)=6x+6$. $f''(-1)=0$.

$f'(-1)=3-6+4=1$; $f(-1)=-1+3-4+2=0$.

Equazione retta tangente in $x=-1$: $y-0=1(x+1) \Rightarrow y=x+1$.