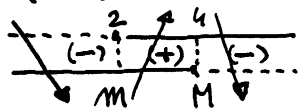
Compito di Matematica Generale del 12/2/2018 Compito A $\boxed{MGA1}$

1) $f(x)=(x-2)^2\cdot e^{1-x}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=0^+$. $f(x)\geq 0\ \forall x\in$ C.E.

$f(x)=0$ per $x=2$.

$f'(x)=2(x-2)e^{1-x}-(x-2)^2\cdot e^{1-x}=e^{1-x}\cdot(x-2)\cdot(4-x)\geq 0$

$\begin{cases} x-2\geq 0: x\geq 2 \\ 4-x\geq 0: x\leq 4\end{cases}$



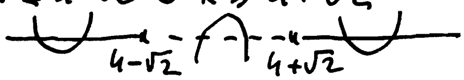
$f''(x)=D\left(e^{1-x}\cdot(6x-x^2-8)\right)=$

$=-e^{1-x}(6x-x^2-8)+e^{1-x}(6-2x)=$

$=e^{1-x}\cdot(x^2-8x+14)\geq 0$

$x=4\pm\sqrt{16-14}=4\pm\sqrt{2}$: $f''(x)\geq 0$ per

$x\leq 4-\sqrt{2}\ \cup\ x\geq 4+\sqrt{2}$



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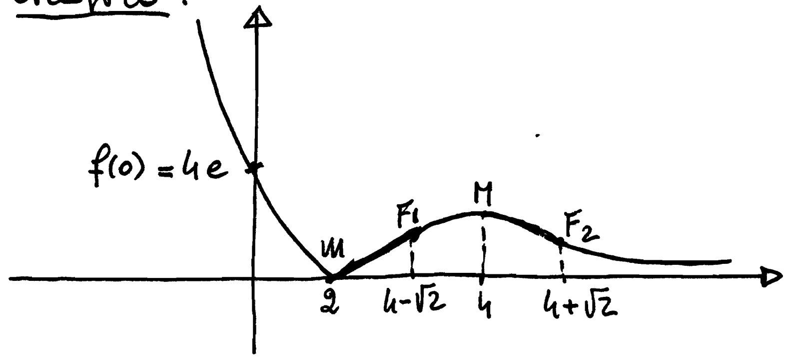


2) $\lim_{x\to 0}\frac{1-\cos 2x-\sin^2 x}{3x^2}=\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}\cdot\frac{4}{3}-\frac{1}{3}\frac{\sin^2 x}{x^2}=\frac{1}{2}\cdot\frac{4}{3}-\frac{1}{3}\cdot 1=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$.

$\lim_{x\to+\infty}\frac{x^2-\log x+2^x}{2^{-x}+2x-\sin x}=\lim_{x\to+\infty}\frac{2^x}{2x}=+\infty$ $\left(x^2=o(2^x);\ \log x=o(2^x);\ 2x=o(2^x);\ 2^{-x}\to 0\right.$ $\left.\sin x=o(2x)\right)$.

3) $f(x)=o(g(x))$ se $\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{e^{2-x}}{x-1}=0$: vera per $x\to+\infty$;

$g(x)=o(f(x))$ se $\lim_{x\to x_0}\frac{g(x)}{f(x)}=\lim_{x\to x_0}\frac{x-1}{e^{2-x}}=0$: vera per $x\to-\infty$ e per $x\to 1$.

4) $f(x)=\frac{x-1}{x+2}$; $f(g(x))=\frac{g(x)-1}{g(x)+2}=3-\log x\Rightarrow g(x)-1=3g(x)+6-g(x)\log x-2\log x\Rightarrow$

$\Rightarrow g(x)(\log x-2)=7-2\log x\Rightarrow g(x)=\frac{7-2\log x}{\log x-2}$.

5) $\log(1+x)=x-\frac{x^2}{2}+o(x^2)\Rightarrow\log(1-2x)=\log(1+(-2x))=(-2x)-\frac{(-2x)^2}{2}+o(x^2)\Rightarrow$

$\Rightarrow\log(1-2x)=-2x-2x^2+o(x^2)\Rightarrow x^3\cdot\log(1-2x)=x^3(-2x-2x^2+o(x^2))\Rightarrow$

$\Rightarrow x^3\cdot\log(1-2x)=-2x^4-2x^5+o(x^5)\Rightarrow P_2(x;0)=-2x^4-2x^5$.

6) $\int_0^1 kx-e^{2-x}\,dx=\left(\frac{1}{2}kx^2+e^{2-x}\right|_0^1=\left(\frac{1}{2}k+e\right)-\left(0+e^2\right)=e\Rightarrow\frac{1}{2}k=e^2\Rightarrow k=2e^2$.

7) $f(x;y)=x^3-y^2+2xy^2-3x\Rightarrow\nabla f=\mathbb{O}\Rightarrow\begin{cases}f'_x=3x^2+2y^2-3=0\\ f'_y=-2y+4xy=2y(2x-1)=0\end{cases}\Rightarrow$

$$\Rightarrow \begin{cases} y=0 \\ 3x^2=3 \end{cases} \Rightarrow \begin{cases} x^2=1 \\ y=0 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=0 \end{cases} \cup \begin{cases} x=-1 \\ y=0 \end{cases} \cup \begin{cases} x=\frac{1}{2} \\ 2y^2=3-\frac{3}{4} \end{cases} \Rightarrow \begin{cases} x=\frac{1}{2} \\ y^2=\frac{9}{8} \end{cases} \Rightarrow \begin{cases} x=\frac{1}{2} \\ y=\frac{3}{2\sqrt{2}} \end{cases} \cup \begin{cases} x=\frac{1}{2} \\ y=-\frac{3}{2\sqrt{2}} \end{cases}.$$

Ci sono 4 punti stazionari: $P_1=(1;0)$; $P_2=(-1;0)$; $P_3=\left(\frac{1}{2};\frac{3}{2\sqrt{2}}\right)$; $P_4=\left(\frac{1}{2};-\frac{3}{2\sqrt{2}}\right)$.

$$H(x;y)=\begin{Vmatrix} 6x & 4y \\ 4y & 4x-2 \end{Vmatrix}.\ H(P_1)=\begin{Vmatrix} 6 & 0 \\ 0 & 2 \end{Vmatrix}: \begin{cases} |H_1|>0 \\ |H_2|>0 \end{cases}: \underline{MIN};\ H(P_2)=\begin{Vmatrix} -6 & 0 \\ 0 & -6 \end{Vmatrix}: \begin{cases} |H_1|<0 \\ |H_2|>0 \end{cases}: \underline{MAX}$$

$$H(P_3)=\begin{Vmatrix} 3 & \frac{6}{\sqrt{2}} \\ \frac{6}{\sqrt{2}} & 0 \end{Vmatrix}: |H_2|=-9<0: \underline{P.\ Sella};\ H(P_4)=\begin{Vmatrix} 3 & -\frac{6}{\sqrt{2}} \\ -\frac{6}{\sqrt{2}} & 0 \end{Vmatrix}: |H_2|=-9<0: \underline{P.\ Sella}.$$

8) $A: 2x+3y=1 \Rightarrow y=-\frac{2}{3}x+\frac{1}{3} \Rightarrow m=-\frac{2}{3}$: A è VERA;

$B: y=3x-1$ e $y=\frac{1}{3}x+1$: $m_1=3$; $m_2=\frac{1}{3} \Rightarrow m_1\cdot m_2=1\neq -1$: B è FALSA;

$C: y=ex-1$: $y\left(\frac{1}{e}\right)=e\cdot\frac{1}{e}-1=0$; $y(2)=2e-1\neq e$: C è FALSA.

A B C	$(A\Rightarrow B)$	$(B\Rightarrow C)$	$[(A\Rightarrow B)\Rightarrow(B\Rightarrow C)]$	$(A\Rightarrow C)$	$[(A\Rightarrow B)\Rightarrow(B\Rightarrow C)]\Leftrightarrow(A\Rightarrow C)$
1 0 0	0	1	1	0	0

la proposizione data è FALSA.

9) $$A\cdot X+B\cdot Y=\begin{Vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 2 \\ 1 \\ m \end{Vmatrix}+\begin{Vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ 2 \\ 1 \\ k \end{Vmatrix}=\begin{Vmatrix} 2+0+m \\ 2+1+m \end{Vmatrix}+\begin{Vmatrix} 1+0+1+0 \\ 1+2+1+k \end{Vmatrix}=$$

$$=\begin{Vmatrix} 2+m \\ 3+m \end{Vmatrix}+\begin{Vmatrix} 2 \\ 4+k \end{Vmatrix}=\begin{Vmatrix} 4+m \\ 7+m+k \end{Vmatrix}=\begin{Vmatrix} 7 \\ 12 \end{Vmatrix} \Rightarrow \begin{cases} 4+m=7 \\ 7+m+k=12 \end{cases} \Rightarrow \begin{cases} m=3 \\ k=12-7-3 \end{cases} \Rightarrow \begin{cases} m=3 \\ k=2 \end{cases}.$$

10) $df(x_0)=f'(x_0)\cdot dx$. Se $f(x)=\log(2x+3) \Rightarrow f'(x)=\frac{2}{2x+3} \Rightarrow$

$$\Rightarrow df(x_0)=\frac{2}{2x_0+3}\cdot dx=\frac{2}{2x_0+3}\cdot\frac{1}{10}=\frac{1}{30} \Rightarrow \frac{2}{2x_0+3}=\frac{1}{3} \Rightarrow 2x_0+3=6 \Rightarrow$$

$$\Rightarrow 2x_0=3 \Rightarrow x_0=\frac{3}{2}.$$

$$f'\left(\frac{3}{2}\right)=\frac{2}{3+3}=\frac{1}{3} \Rightarrow \frac{1}{3}\cdot\frac{1}{10}=\frac{1}{30}.$$

Compito di Matematica Generale del 12/2/2018 Compito B $\boxed{MGB1}$

1) $f(x)=(x-1)^2\cdot e^{2-x}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=0^+$. $f(x)\geq 0\ \forall x\in$ C.E.

$f(x)=0$ per $x=1$.

$f'(x)=2(x-1)\cdot e^{2-x}-(x-1)^2\cdot e^{2-x}=e^{2-x}(x-1)(3-x)\geq 0$

$\begin{cases} x-1\geq 0 : x\geq 1 \\ 3-x\geq 0 : x\leq 3 \end{cases}$

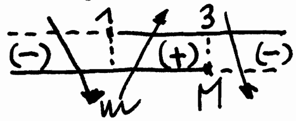
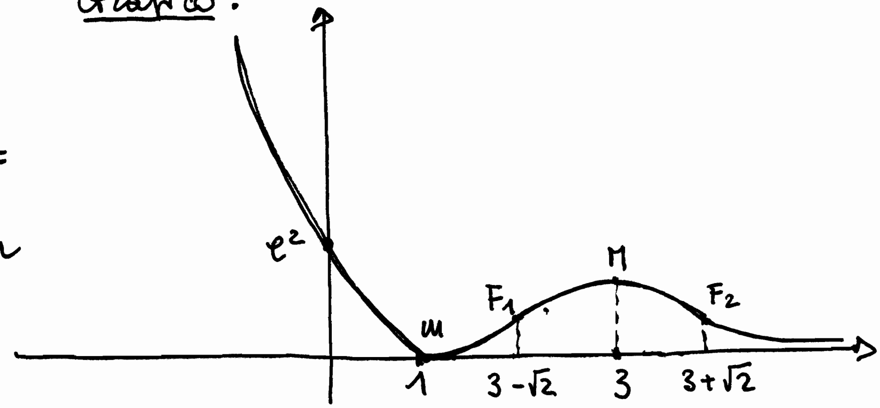


Grafico:



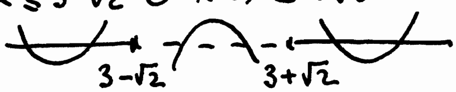
$f''(x)=D\left(e^{2-x}\cdot(4x-x^2-3)\right)=$

$=-e^{2-x}(4x-x^2-3)+e^{2-x}\cdot(4-2x)=$

$=e^{2-x}(x^2-6x+7)\geq 0$

$x=3\pm\sqrt{9-7}=3\pm\sqrt{2}$: $f''(x)\geq 0$ per

$x\leq 3-\sqrt{2}\ \cup\ x\geq 3+\sqrt{2}$



2) $\lim_{x\to 0} \frac{1-\cos 3x-\sin^2 x}{2x^2}=\lim_{x\to 0} \frac{1-\cos 3x}{9x^2}\cdot\frac{9}{2}-\frac{1}{2}\cdot\frac{\sin^2 x}{x^2}=\frac{1}{2}\cdot\frac{9}{2}-\frac{1}{2}=\frac{1}{2}\cdot\frac{7}{2}=\frac{7}{4}$.

$\lim_{x\to+\infty} \frac{x^2+\cos x-2^{-x}}{2^x+x-\log x}=\lim_{x\to+\infty}\frac{x^2}{2^x}=0^+$. $\left(\cos x=o(x^2); 2^{-x}\to 0; x=o(2^x); \log x=o(2^x); x^2=o(2^x)\right)$.

3) $f(x)=o(g(x))$ se $\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{e^{x-1}}{2-x}=0$: vera per $x\to-\infty$;

$g(x)=o(f(x))$ se $\lim_{x\to x_0}\frac{g(x)}{f(x)}=\lim_{x\to x_0}\frac{2-x}{e^{x-1}}=0$: vera per $x\to+\infty$ e per $x\to 2$.

4) $f(x)=\frac{x+1}{x-3}$; $f(g(x))=\frac{g(x)+1}{g(x)-3}=2-\log x \Rightarrow g(x)+1=2g(x)-6-g(x)\cdot\log x+3\log x \Rightarrow$

$\Rightarrow g(x)(\log x-1)=3\log x-7 \Rightarrow g(x)=\frac{3\log x-7}{\log x-1}$.

5) $\log(1+x)=x-\frac{x^2}{2}+o(x^2) \Rightarrow \log(1-3x)=\log(1+(-3x))=(-3x)-\frac{(-3x)^2}{2}+o(x^2)\Rightarrow$

$\Rightarrow \log(1-3x)=-3x-\frac{9}{2}x^2+o(x^2) \Rightarrow x^2\cdot\log(1-3x)=x^2\cdot\left(-3x-\frac{9}{2}x^2+o(x^2)\right)\Rightarrow$

$\Rightarrow x^2\cdot\log(1-3x)=-3x^3-\frac{9}{2}x^4+o(x^4) \Rightarrow P_2(x;0)=-3x^3-\frac{9}{2}x^4$.

6) $\int_0^2 e^{3-x}+kx\,dx=\left(\frac{1}{2}kx^2-e^{3-x}\right|_0^2=(2k-e)-(0-e^3)=e^3 \Rightarrow 2k=e \Rightarrow k=\frac{1}{2}\cdot e$.

7) $f(x;y)=x^2-y^3-2x^2y+3y \Rightarrow \nabla f = 0 \Rightarrow \begin{cases} f'_x = 2x-4xy = 2x(1-2y)=0 \\ f'_y = -3y^2-2x^2+3 = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x=0 \\ 3y^2=3 \end{cases} \Rightarrow \begin{cases} x=0 \\ y^2=1 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=1 \end{cases} \cup \begin{cases} x=0 \\ y=-1 \end{cases} \cup \begin{cases} 2x^2=3-\frac{3}{4} \\ y=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x^2=\frac{9}{8} \\ y=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x=\frac{3}{2\sqrt{2}} \\ y=\frac{1}{2} \end{cases} \cup \begin{cases} x=-\frac{3}{2\sqrt{2}} \\ y=\frac{1}{2} \end{cases}$.

Ci sono quattro punti stazionari: $P_1=(0;1)$; $P_2=(0;-1)$; $P_3=\left(\frac{3}{2\sqrt{2}};\frac{1}{2}\right)$; $P_4=\left(-\frac{3}{2\sqrt{2}};\frac{1}{2}\right)$.

$H(x;y)=\begin{Vmatrix} 2-4y & -4x \\ -4x & -6y \end{Vmatrix}$. $H(P_1)=\begin{Vmatrix} -2 & 0 \\ 0 & -6 \end{Vmatrix} : \begin{cases} |H_1|<0 \\ |H_2|>0 \end{cases}$: $\underline{MAX}$; $H(P_2)=\begin{Vmatrix} 6 & 0 \\ 0 & 6 \end{Vmatrix} : \begin{cases} |H_1|>0 \\ |H_2|>0 \end{cases}$: $\underline{MIN}$;

$H(P_3)=\begin{Vmatrix} 0 & -\frac{6}{\sqrt{2}} \\ -\frac{6}{\sqrt{2}} & -3 \end{Vmatrix} : |H_2|=-9<0$: <u>P. Sella</u>; $H(P_4)=\begin{Vmatrix} 0 & \frac{6}{\sqrt{2}} \\ \frac{6}{\sqrt{2}} & -3 \end{Vmatrix} : |H_2|=-9<0$: <u>P. Sella</u>.

8) A: $y=2x+1$ e $y=-\frac{1}{2}x+2$: $m_1=2$; $m_2=-\frac{1}{2} \Rightarrow m_1 \cdot m_2 = -1$: A è VERA;

B: $3y-2x=1 \Rightarrow y=\frac{2}{3}x+\frac{1}{3} \Rightarrow m=\frac{2}{3}$: B è FALSA;

C: $y=ex-e$: $y\left(\frac{1}{e}\right)=e\cdot\frac{1}{e}-e=1-e\neq 0$; $y(2)=2e-e=e$: C è FALSA.

A B C	$(A\Rightarrow B)$	$(B\Rightarrow C)$	$[(A\Rightarrow B)\Leftrightarrow(B\Rightarrow C)]$	$(A\Rightarrow C)$	$[(A\Rightarrow B)\Leftrightarrow(B\Rightarrow C)] \Rightarrow (A\Rightarrow C)$
1 0 0	0	1	0	0	1

La proposizione data è vera.

9) $A\cdot X+B\cdot Y=\begin{Vmatrix} 1 & 2 & 1 \\ 0 & 1 & m \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ 2 \\ 3 \end{Vmatrix}+\begin{Vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ k \\ 1 \\ 2 \end{Vmatrix}=\begin{Vmatrix} 1+4+3 \\ 0+2+3m \end{Vmatrix}+\begin{Vmatrix} 1+k+1+0 \\ 0+k+1+2 \end{Vmatrix}=$

$=\begin{Vmatrix} 8 \\ 2+3m \end{Vmatrix}+\begin{Vmatrix} 2+k \\ k+3 \end{Vmatrix}=\begin{Vmatrix} 10+k \\ 3m+k+5 \end{Vmatrix}=\begin{Vmatrix} 12 \\ 10 \end{Vmatrix} \Rightarrow \begin{cases} 10+k=12 \\ 3m+k+5=10 \end{cases} \Rightarrow \begin{cases} k=2 \\ 3m=10-7 \end{cases} \Rightarrow \begin{cases} k=2 \\ m=1 \end{cases}$.

10) $df(x_0)=f'(x_0)\cdot dx$. Se $f(x)=\log(3x+2) \Rightarrow f'(x)=\frac{3}{3x+2} \Rightarrow$

$\Rightarrow df(x_0)=\frac{3}{3x_0+2}\cdot dx=\frac{3}{3x_0+2}\cdot\frac{2}{10}=\frac{1}{10} \Rightarrow \frac{3}{3x_0+2}=\frac{1}{2} \Rightarrow 3x_0+2=6 \Rightarrow$

$\Rightarrow 3x_0=4 \Rightarrow x_0=\frac{4}{3}$.

$f'\left(\frac{4}{3}\right)=\frac{3}{4+2}=\frac{1}{2} \Rightarrow \frac{1}{2}\cdot\frac{2}{10}=\frac{1}{10}$.