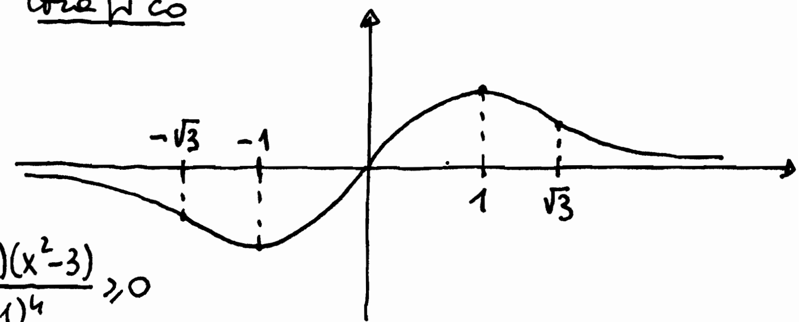
Compito di Matematica Generale del 24/3/2018 $\boxed{MG1}$

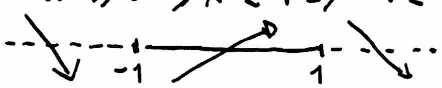
1) $f(x)=\dfrac{x}{x^2+1}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=0^-$; $\lim\limits_{x\to+\infty} f(x)=0^+$. $f(x)>0$ per $x>0$; $f(0)=0$.

$f'(x)=\dfrac{1\cdot(x^2+1)-x(2x)}{(x^2+1)^2}=\dfrac{1-x^2}{(x^2+1)^2}\geq 0$ per

Grafico

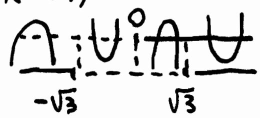


$1-x^2\geq 0 \Rightarrow x^2\leq 1 \Rightarrow -1\leq x\leq 1$



$f''(x)=\dfrac{-2x(x^2+1)^2-(1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}=\dfrac{2x(x^2+1)(x^2-3)}{(x^2+1)^4}\geq 0$

$\begin{cases} x\geq 0 \\ x^2-3\geq 0: x^2\geq 3 \Rightarrow x\leq-\sqrt{3} \cup x\geq\sqrt{3} \end{cases}$



2) $\lim\limits_{x\to 0}\dfrac{\sqrt{(1+x)^5}-1}{5x}=\dfrac{1}{5}\cdot\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{5}{2}}-1}{x}=\dfrac{1}{5}\cdot\dfrac{5}{2}=\dfrac{1}{2}$.

$\lim\limits_{x\to 0}\dfrac{x^2-\log(1+x)+2^x}{2^{-x}+2x-\operatorname{sen}x}=\dfrac{0-0+1}{1+0-0}=1$.

3) $f(x)=\dfrac{1-e^x}{2+e^x}$. C.E.: $\mathbb{R}$; $f\in\mathcal{C}(\mathbb{R})$. $f'(x)=\dfrac{-e^x(2+e^x)-e^x(1-e^x)}{(2+e^x)^2}=\dfrac{-3e^x}{(2+e^x)^2}<0\ \forall x\in\mathbb{R}$

$\lim\limits_{x\to-\infty}f(x)=\dfrac{1-0}{2+0}=\dfrac{1}{2}$; $\lim\limits_{x\to+\infty}f(x)=\dfrac{-1}{1}=-1$; $f:\mathbb{R}\to\left]-1;\frac{1}{2}\right[$ sempre invertibile su $\mathbb{R}$

in quanto monotona decrescente. $f^{-1}:\left]-1;\frac{1}{2}\right[\to\mathbb{R}$. $\dfrac{1-e^x}{2+e^x}=y \Rightarrow 1-e^x=2y+ye^x \Rightarrow$

$\Rightarrow e^x(y+1)=1-2y \Rightarrow e^x=\dfrac{1-2y}{1+y} \Rightarrow x=\log\left(\dfrac{1-2y}{1+y}\right)$. Inversa: $y=\log\left(\dfrac{1-2x}{1+x}\right)$.

4) $f(x)=e^{3x+1}$; $g(x)=3-2x$. $F(x)=f(g(x))-g(f(x))=f(3-2x)-g\left(e^{3x+1}\right)=$

$=F(x)=e^{3(3-2x)+1}-\left(3-2\left(e^{3x+1}\right)\right)=e^{10-6x}-3+2e^{3x+1}$. $F'(x)=-6e^{10-6x}+6e^{3x+1}$.

5) $f(x)=\log(1+e^{2x})$; $f'(x)=\dfrac{2e^{2x}}{1+e^{2x}}$; $f''(x)=\dfrac{4e^{2x}(1+e^{2x})-2e^{2x}\cdot 2e^{2x}}{(1+e^{2x})^2}=\dfrac{4e^{2x}}{(1+e^{2x})^2}$.

$f(0)=\log 2$; $f'(0)=1$; $f''(0)=1 \Rightarrow P_2(x;0)=\log 2+x+\dfrac{1}{2}x^2$.

6) $\int_1^2 2k - \frac{1}{x}\, dx = \left(2kx - \log x\right|_1^2 = (4k - \log 2) - (2k - 0) = 2k - \log 2 = 3\log 2 \Rightarrow k = 2\log 2 = \log 4.$

7) $f(x;y) = x^2 + y^2 + 2xy^2 - x$. $\nabla f = \underline{0} \Rightarrow \begin{cases} f'_x = 2x + 2y^2 - 1 = 0 \\ f'_y = 2y + 4xy = 2y(1+2x) = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = 0 \end{cases} \cup \begin{cases} x = -\frac{1}{2} \\ 2y^2 = 2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = -\frac{1}{2} \\ y = 1 \end{cases} \cup \begin{cases} x = -\frac{1}{2} \\ y = -1 \end{cases}$. $H(x;y) = \begin{Vmatrix} 2 & 4y \\ 4y & 2+4x \end{Vmatrix}$. $H(\frac{1}{2};0) = \begin{Vmatrix} 2 & 0 \\ 0 & 4 \end{Vmatrix} \Rightarrow \begin{cases} 2>0 ; 4>0 \\ 8-0>0 \end{cases}$: P. di Minimo.

$H(-\frac{1}{2};1) = \begin{Vmatrix} 2 & 4 \\ 4 & 0 \end{Vmatrix}$: $|H_2| = -16 < 0$: P. di Sella; $H(-\frac{1}{2};-1) = \begin{Vmatrix} 2 & -4 \\ -4 & 0 \end{Vmatrix}$: $|H_2| = -16 < 0$: P. di Sella.

8) A: $1 - x - x^2 < 0 \Rightarrow x^2 + x - 1 > 0 : x = \frac{-1 \pm \sqrt{1+4}}{2}$: $+ \quad \frac{-1-\sqrt{5}}{2} \quad - \quad \frac{-1+\sqrt{5}}{2} \quad +$ A è FALSA

B) $y = \frac{2}{3}x - \frac{1}{3}$ e $y = \frac{3}{2}x + \frac{1}{2}$: $m_1 \neq -\frac{1}{m_2}$: B è FALSA

C) $y = (\cos\pi)x - 1 = -x - 1 : y(1) = -2$: C è VERA.

A	B	C	non A	(non A o B)	(B ⇒ C)	[(non A o B) e (B ⇒ C)]	(C ⇒ A)	[(non A o B) e (B ⇒ C)] ⇔ (C ⇒ A)
0	0	1	1	1	1	1	0	0

La proposizione data è falsa.

9) $A \cdot A^T \cdot V = \begin{Vmatrix} 2 & 1 \\ -1 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 2 & -1 \\ 1 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \end{Vmatrix} = \begin{Vmatrix} 5 & -1 \\ -1 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \end{Vmatrix} = \begin{Vmatrix} 5x - y \\ -x + 2y \end{Vmatrix}$

$A \cdot A^T \cdot V \perp Y \Rightarrow (5x - y ; -x + 2y) \cdot (1;2) = 5x - y - 2x + 4y = 3x + 3y = 0 \Rightarrow y = -x;$

$A \cdot A^T \cdot V = (6x ; -3x) \Rightarrow \|A \cdot A^T \cdot V\| = \sqrt{36x^2 + 9x^2} = \sqrt{45x^2} = \sqrt{5} \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm\frac{1}{3}.$

Ci sono due soluzioni: $(2;-1)$ e $(-2;1)$.

10) $df(x_0) = f'(x_0) \cdot dx$; $f(x) = e^{3x-1}$; $f'(x) = 3e^{3x-1}$.

$f'(0) = 3 \cdot e^{-1} = \frac{3}{e} \Rightarrow df(0) = f'(0) \cdot dx \Rightarrow \frac{1}{10} = \frac{3}{e} \cdot dx \Rightarrow dx = \frac{e}{30}.$