TASKS of MATHEMATICS
for Economic Applications AA. 2017/18

Intermediate Test December 2017

I M 1) Using the trigonometric form of the complex numbers, calculate $\left(\dfrac{1+i}{1+\sqrt{3}\,i}\right)^4$.

I M 2) Given the matrix $\mathbb{A}=\left\|\begin{matrix}2&2&1\\0&k&0\\1&2&2\end{matrix}\right\|$, determine the values of the parameter k for which it admits a multiple eigenvalue and verify if, for these values of k, the matrix is a diagonalizable one.

I M 3) Given the matrix $\mathbb{A}=\left\|\begin{matrix}m&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&k\end{matrix}\right\|$, determine, on varying the parameters m and k, the dimensions of the Image and the Kernel of the linear map $f\colon\mathbb{R}^4\to\mathbb{R}^4$, $f(\mathbb{X})=\mathbb{A}\cdot\mathbb{X}$.

I M 4) Given the matrix $\mathbb{A}=\left\|\begin{matrix}1&2&3&1\\1&1&2&0\\2&1&3&-1\end{matrix}\right\|$, determine a basis for the Kernel of the linear map $f\colon\mathbb{R}^3\to\mathbb{R}^4$, $f(\mathbb{X})=\mathbb{A}^T\cdot\mathbb{X}$.

I M 5) Given the matrix $\mathbb{A}=\left\|\begin{matrix}1&0&1\\0&1&0\\1&0&1\end{matrix}\right\|$, verify that it has two eigenvalues equal to those of the matrix $\mathbb{A}^2=\mathbb{A}\cdot\mathbb{A}$.

I Winter Exam Session 2018

I M 1) Calculate the fourth roots of $z=e^{\log 16-i\pi}$.

I M 2) Given $\mathbb{A}=\left\|\begin{matrix}0&0&k\\0&k&0\\k&0&0\end{matrix}\right\|$ determine, on varying k, its eigenvalues and corresponding eigenvectors, their algebraic and geometric multiplicity, a basis for the eigenspace.

I M 3) For the linear map $f:\mathbb{R}^3\to\mathbb{R}^3$ generated by the matrix $\mathbb{A}=\left\|\begin{matrix}1&x_1&y_1\\2&x_2&y_2\\-1&x_3&y_3\end{matrix}\right\|$ it results $\begin{cases}f(1,1,1)=(3,3,1)\\f(1,1,-1)=(-1,5,-1)\end{cases}$. Find the dimensions of the Image and the Kernel of such linear map.

I M 4) In the base $\mathbb{W}:\{(1,0,1);(1,1,0);(1,1,1)\}$ the vector $\mathbb{X}$ has coordinates $(2,-1,1)$. Find its coordinates in the base $\mathbb{V}=\{(1,0,1);(1,1,0);(2,1,2)\}$.

II M 1) Given the equation $f(x,y)=y\log x+(x-1)\,e^y-y=0$ satisfied at the point $(1,0)$, verify that with it an implicit function $y=y(x)$ can be defined and then calculate, for this function, the expression of the Taylor polynomial of the second degree at the appropriate point.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 - y^2 \\ \text{u.c.} \begin{cases} x^2 + 4y^2 \leq 4 \\ 2y \leq 2 - x \end{cases} \end{cases}$.

II M 3) Given the function $f(x,y) = y^2 - x^3 - 2xy^2 + 27x$, analyze the nature of its stationary points.

II M 4) Given the function $f(x,y) = \alpha x^2 + \beta y^3 + 2x - 3y^2$, with $\alpha, \beta \in \mathbb{R}$, find the values of α and β if $f'_x(1,1) = f''_{y,y}(1,1)$ and $f'_y(-1,-1) = f''_{x,x}(-1,-1)$.

II Winter Exam Session 2018

I M 1) If $e^z = 1 + i$, determine z.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 3 & 2 & 1 \\ 1 & 4 & -9 \\ 1 & 2 & -3 \end{Vmatrix}$ determine its real and complex eigenvalues, and, for the real eigenvalue, the corresponding eigenvectors.

I M 3) Given the linear homogeneous system $\begin{cases} x_1 + mx_2 + mx_3 + 2x_4 = 0 \\ x_1 + x_2 + x_3 + 2mx_4 = 0 \end{cases}$, find, on varying the parameter m, the dimention of the linear space of its solutions, and when this dimention is maximum, find a basis for such space.

I M 4) Given the two orthogonal vectors $\mathbb{X}_1 = (1,2,-3)$ and $\mathbb{X}_2 = (1,1,1)$, determine a third vector $\mathbb{X}_3$ orthogonal to $\mathbb{X}_1$ and $\mathbb{X}_2$. With these three vectors realize a basis for $\mathbb{R}^3$. Finally determine the coordinates of the vector $\mathbb{Y} = (1,0,1)$ in such basis.

II M 1) With the system $\begin{cases} f(x,y,z) = x^2 + y^2 + z^3 - 3xy = 0 \\ g(x,y,z) = x^3 + y^3 - 3z^2 + xyz = 0 \end{cases}$ satisfied at the point $\mathrm{P} = (1,1,1)$ we can define an implicit function. Calculate its first order derivatives.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x\,(y+1) \\ \text{u.c.:} \begin{cases} x + y^2 \leq 1 \\ 0 \leq x \end{cases} \end{cases}$.

II M 3) Given the function $f(x,y) = \log\left(\dfrac{x-y+1}{y-x^2+1}\right)$ graphically represent its existence field and then calculate the gradient $\nabla f(0,0)$.

II M 4) Given the function $f(x,y) = k(x^2 - y^2) + x - y$ and the unit vectors $e_1 = (1,0)$, $e_2 = (0,1)$, if the first order directional derivatives $D_{e_1} f(1,0)$ and $D_{e_2} f(0,1)$ are equal, find the value of k and then calculate the second order directional derivative $D^2_{e_1,e_2} f(1,1)$.

I Additional Exam Session 2018

I M 1) Find the cubic (third) roots of z if $z = e^{1+4\pi i} \cdot e^{-1+3\pi i}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} x+1 & x & x \\ y & y+1 & y \\ -y & -y & 1-y \end{Vmatrix}$, find the relationship that must exist between x and y so that the matrix admits an eigenvalue whose algebraic multiplicity is equal to 3.

I M 3) Given the linear map:
$f\colon \mathbb{R}^3 \to \mathbb{R}^3$, with $f(x_1,x_2,x_3) = (ax_1, x_1 + bx_2, x_1 + x_2 + cx_3)$,
since $f(1,1,1) = (0,1,2)$, calculate the values of the parametrs a, b and c and then find a basis for the kernel and a basis for the image of f.

I M 4) Given the matrices $\mathbb{A}=\left\|\begin{matrix}1&1&1\\0&1&1\\0&0&1\end{matrix}\right\|$ and $\mathbb{B}=\left\|\begin{matrix}1&0&0\\2&1&0\\4&2&1\end{matrix}\right\|$, calculate the matrix $\left(\mathbb{A}+\mathbb{B}^T\right)^{-1}$, the inverse of the sum of the matrix $\mathbb{A}$ with the transpose of the matrix $\mathbb{B}$.
II M 1) Given the equation $f(x,y,z)=xe^{y-z}-ye^{z-x}+ze^{x-y}=0$ satisfied at point $(0,0,0)$, verify that an implicit function $z=z(x,y)$ can be defined and then calculate the equation of the tangent plane for this function at the point $(0,0)$.
II M 2) Solve the problem $\begin{cases}\text{Max/min } f(x,y)=x^2-y^2\\ \text{u.c.: } x^2+4y^2\le 4\end{cases}$.
II M 3) Given $f(x,y)=x\cdot e^y+y\cdot e^x$ and the unit vector $v=(\cos\alpha,\sin\alpha)$, determine the values for α for which the directional derivatives $\mathcal{D}_v f(0,0)$ are equal to zero and then calculate $\mathcal{D}^2_{v,v} f(0,0)\,,\forall\,\alpha$.
II M 4) Given $f:\mathbb{R}\to\mathbb{R}^3, t\to\left(\sin\left(t^2\right),e^{2t},\log\left(1+t\right)\right)$, determine the equation of the tangent line to this curve at the point $t=0$.

I Summer Exam Session 2018

I M 1) Find the square roots of the complex number $z=\dfrac{1+i}{1-i}-\dfrac{1-i}{1+i}$.
I M 2) Given the matrix $\mathbb{A}=\left\|\begin{matrix}1&1&0&0\\1&1&0&0\\0&0&-1&-1\\0&0&-1&-1\end{matrix}\right\|$, find its eigenvalues and study the corresponding associated eigenspaces.
I M 3) Given the linear map $\mathbb{R}^3\to\mathbb{R}^3$, $f(\mathbb{X})=\mathbb{A}\cdot\mathbb{X}$, with $\mathbb{A}=\left\|\begin{matrix}1&0&1\\0&k&-1\\1&-1&2\end{matrix}\right\|$, since the dimension of the Kernel of such map is equal to 1, check if the matrix is a diagonalizable one.
I M 4) Check if, on varying the parameters m and k, the vector $\mathbb{Y}=(-1,1,k)$ can be expressed with a linear combination of the vectors $\mathbb{X}_1=(1,1,0)$, $\mathbb{X}_2=(-1,0,-1)$, $\mathbb{X}_3=(1,1,m)$ and $\mathbb{X}_4=(-1,1,-2)$.
II M 1) Given the equation $f(x,y,z)=x^3+y^3+z^3+3xy+3yz=1$, satisfied at the point $P=(1,1,-1)$, determine the first order partial derivatives of the implicit function defined by such equation having z as its dependent variable.
II M 2) Solve the problem $\begin{cases}\text{Max/min } f(x,y)=x^4+y^4\\ \text{u.c.: } x^2+y^2\le 1\end{cases}$.
II M 3) Given the vectors $v=\left(\dfrac{\sqrt{2}}{2},\dfrac{\sqrt{2}}{2}\right)$ and $w=\left(-\dfrac{\sqrt{2}}{2},-\dfrac{\sqrt{2}}{2}\right)$ and the function $f(x,y)=\log\left(e^{x^2+y^2}+e^{x^2-y^2}\right)$, calculate the first order directional derivatives $\mathcal{D}_v f(0,0)$ and $\mathcal{D}_w f(-1,1)$.
II M 4) Given the function $f(x,y)=(x-y)^2+(y-3x)^2$ analyze the nature of its stationary points.