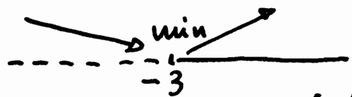
Compito di Matematica Generale del 11/6/2018 $\boxed{MG1}$

1) $f(x)=x^3\cdot e^{x-1}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=0^-$; $\lim\limits_{x\to+\infty} f(x)=+\infty$. $f(x)\geq 0$ per $x\geq 0$.

$f'(x)=3x^2\cdot e^{x-1}+x^3\cdot e^{x-1}=x^2\cdot e^{x-1}\cdot(3+x)\geq 0$

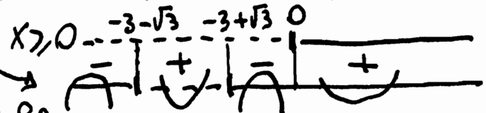
per $3+x\geq 0 \Rightarrow x\geq -3$:



$f''(x)=2x\,e^{x-1}(3+x)+x^2\cdot e^{x-1}\cdot(3+x)+x^2\cdot e^{x-1}=$

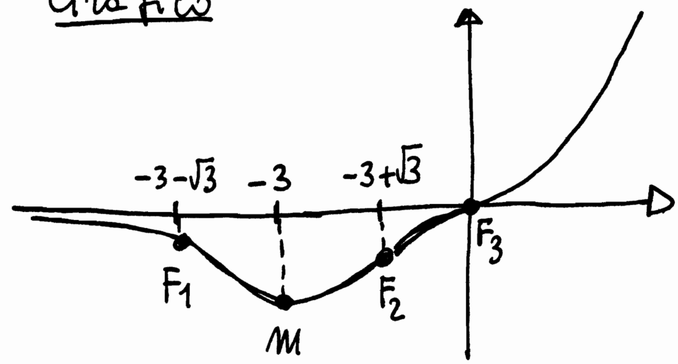
$=x\cdot e^{x-1}\cdot(6+2x+3x+x^2+x)=x\cdot e^{x-1}\cdot(x^2+6x+6)\geq 0$

per $\begin{cases} x\geq 0 \\ x^2+6x+6\geq 0 \end{cases}$ $\quad x=-3\pm\sqrt{9-6}=-3\pm\sqrt{3}$



Ci sono 3 punti di flesso.

Grafico



2) $\lim\limits_{x\to 0}\dfrac{\operatorname{sen}(\log(1+x))}{\log(1+\operatorname{sen}x)}=\lim\limits_{x\to 0}\dfrac{\operatorname{sen}(\log(1+x))}{\log(1+x)}\cdot\dfrac{\log(1+x)}{x}\cdot\dfrac{x}{\operatorname{sen}x}\cdot\dfrac{\operatorname{sen}x}{\log(1+\operatorname{sen}x)}=1\cdot1\cdot1\cdot1=1.$

$\lim\limits_{x\to+\infty}\left(\dfrac{5+x}{4+x}\right)^{1-x}=\lim\limits_{x\to+\infty}\left[\left(1+\dfrac{1}{4+x}\right)^{4+x}\right]^{\frac{1-x}{4+x}}=e^{-1}=\dfrac{1}{e}.$

3) $f(x)=\dfrac{x-3}{x+2}$; $g(x)=\dfrac{x+6}{x-1}$. $F(x)=f(g(x))=f\left(\dfrac{x+6}{x-1}\right)=\dfrac{\frac{x+6}{x-1}-3}{\frac{x+6}{x-1}+2}=\dfrac{x+6-3x+3}{x+6+2x-2}=\dfrac{9-2x}{3x+4}$, $(x\neq 1)$.

$F'(x)=\dfrac{-2(3x+4)-3(9-2x)}{(3x+4)^2}=-\dfrac{35}{(3x+4)^2}<0\ \forall x\neq-\dfrac{4}{3}$. C.E.: $x\neq-\dfrac{4}{3}$; sempre invertibile.

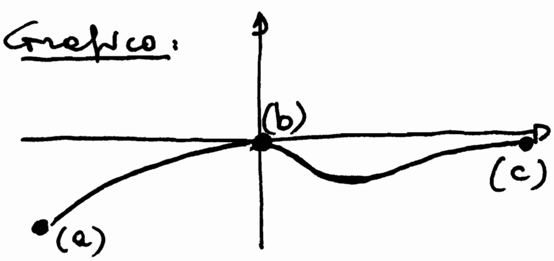
$\dfrac{9-2x}{3x+4}=y\Rightarrow 9-2x=3xy+4y\Rightarrow x(3y+2)=9-4y\Rightarrow x=\dfrac{9-4y}{3y+2}\Rightarrow F^{-1}(x)=y=\dfrac{9-4x}{3x+2}.$

4) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow f(x)<\varepsilon$: $\lim\limits_{x\to-\infty} f(x)=-\infty$;

$\forall\varepsilon>0\ \exists\delta(\varepsilon): |x|<\delta(\varepsilon)\Rightarrow|f(x)|<\varepsilon$: $\lim\limits_{x\to 0} f(x)=0$;

$\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow-\varepsilon<f(x)<0$: $\lim\limits_{x\to+\infty} f(x)=0^-$.

Grafico:



5) Da $(A\Rightarrow B)\Leftrightarrow(\text{non}A \text{ o } B)$ possiamo dedurre: $(A\Rightarrow\text{non}B)\Leftrightarrow(\text{non}A \text{ o non}B)\Rightarrow X$: non B.

A B	non B	(A ⇒ non B)	(non A)	(non B)	(non A o non B)
1 1	0	0	0	0	0
1 0	1	1	0	1	1
0 1	0	1	1	0	1
0 0	1	1	1	1	1

Quindi: $(A\Rightarrow\text{non}B)\Leftrightarrow(\text{non}A \text{ o non}B)$ è una tautologia.

$\boxed{MG2}$

6) $\displaystyle\int_0^K \frac{x}{1+x^2}\,dx = \frac{1}{2}\log(1+x^2)\Big|_0^K = \frac{1}{2}\Big(\log(1+K^2) - \underbrace{\log(1+0)}_{0}\Big) = 1 \Rightarrow \log(1+K^2) = 2 \Rightarrow$

$\Rightarrow 1+K^2 = e^2 \Rightarrow K^2 = e^2-1 \Rightarrow K = \sqrt{e^2-1}.$

7) $A\cdot B\cdot C + C\cdot B\cdot A = \begin{Vmatrix}1 & 1\\0 & 1\end{Vmatrix}\cdot\begin{Vmatrix}0 & 1\\1 & 1\end{Vmatrix}\cdot\begin{Vmatrix}1 & 0\\0 & K\end{Vmatrix} + \begin{Vmatrix}1 & 0\\0 & K\end{Vmatrix}\cdot\begin{Vmatrix}0 & 1\\1 & 1\end{Vmatrix}\cdot\begin{Vmatrix}1 & 1\\0 & 1\end{Vmatrix} = \begin{Vmatrix}1 & 1\\0 & 1\end{Vmatrix}\cdot\begin{Vmatrix}0 & K\\1 & K\end{Vmatrix} + \begin{Vmatrix}1 & 0\\0 & K\end{Vmatrix}\cdot\begin{Vmatrix}0 & 1\\1 & 2\end{Vmatrix} =$

$= \begin{Vmatrix}1 & 2K\\1 & K\end{Vmatrix} + \begin{Vmatrix}0 & 1\\K & 2K\end{Vmatrix} = \begin{Vmatrix}1 & 2K+1\\1+K & 3K\end{Vmatrix} = \begin{Vmatrix}1 & 5\\3 & 6\end{Vmatrix} \Rightarrow K = 2.$

8) $f(x;y) = 2xy^2 - 2x^2 + 3x - 6y^2$. $\nabla f(x;y) = (0;0) \Rightarrow$

$\Rightarrow \begin{cases} f'_x = 2y^2 - 4x + 3 = 0 \\ f'_y = 4xy - 12y = 4y(x-3) = 0 \end{cases} \Rightarrow \begin{cases} x = 3 \\ 2y^2 = 9 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = \pm\frac{3}{\sqrt{2}} \end{cases} \cup \begin{cases} 4x = 3 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{3}{4} \\ y = 0 \end{cases}.$

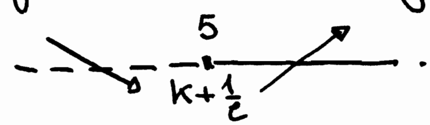
$H(x;y) = \begin{Vmatrix}-4 & 4y\\4y & 4x-12\end{Vmatrix}$. $H\left(\frac{3}{4};0\right) = \begin{Vmatrix}-4 & 0\\0 & -9\end{Vmatrix} \Rightarrow \begin{cases} -4<0;\ -9<0 \\ 36-0>0 \end{cases}$: Punto di Massimo.

$H\left(3;\frac{3}{\sqrt{2}}\right) = \begin{Vmatrix}-4 & 6\sqrt{2}\\6\sqrt{2} & 0\end{Vmatrix} : |H_2| = -72<0$: P. Sella; $H\left(3;-\frac{3}{\sqrt{2}}\right) = \begin{Vmatrix}-4 & -6\sqrt{2}\\-6\sqrt{2} & 0\end{Vmatrix} : |H_2| = -72<0$: P. Sella.

9) $f(x) = (x-K)\cdot\log(x-K)$. C.E.: $x > K$.

$f'(x) = 1\cdot\log(x-K) + (x-K)\cdot\frac{1}{x-K} = \log(x-K)+1 \geq 0 \Rightarrow \log(x-K) \geq -1 \Rightarrow$

$\Rightarrow x-K \geq e^{-1} = \frac{1}{e} \Rightarrow x \geq K+\frac{1}{e}$:



Se $K+\frac{1}{e} = 5 \Rightarrow K = 5-\frac{1}{e} = \frac{5e-1}{e}$ per avere un minimo in $x_0 = 5$.

10) Da $f(x) = \log(1+3x) \Rightarrow m = f'(x_0) = \frac{3}{1+3x_0}$. Quindi:

$\frac{3}{1+3x_0} = \frac{33}{56} \Rightarrow 1+3x_0 = \frac{56}{11} \Rightarrow 3x_0 = \frac{56}{11}-1 = \frac{45}{11} \Rightarrow x_0 = \frac{1}{3}\cdot\frac{45}{11} = \frac{15}{11}.$

Dato che $\frac{15}{11} \cong 1,36 \Rightarrow x_0 \in [1;2]$.